Int. J. Math. And Appl., 6(1–E)(2018), 895–902
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

A Sequence of Complete Cosmic Scenario from Inflation to Late Time Acceleration with Adiabatic Particle Creation

Shibaji Halder[1,*]

1 Department of Mathematics, Vidyasagar College, Kolkata, West Bengal, India.

Abstract: In the background of a flat FRW model, we assume the non-equilibrium thermodynamical process to be isentropic so that the entropy per particle does not change and consequently the dissipative pressure can be expressed linearly in terms of the particle creation rate. This paper deal with the Friedman equations and by a sequence of proper choices of the particle creation rate as a function of the Hubble parameter, it is possible to show a continuous cosmic evolution from inflation to late time acceleration.

Keywords: Particle creation, dissipative pressure, entropy production, isentropic process, unified cosmic evolution.

1. Introduction

The relativistic second order thermodynamic theories [1–6] play crucial role in describing the evolution of the Universe as a sequence of dissipative processes. The theory proposes that deviations from equilibrium described by bulk stress, heat flow and shear stress can be treated as independent dynamical variables bounded by average molecular speed thereby ensuring causality. In a homogeneous and isotropic FRW universe the bulk viscous pressure is the only possible mechanism for dissipative processes. The bulk viscous pressure can be attributed to particle number changing processes in an expanding universe [7–14] or it might due to coupling of the different components of the cosmic fluids [15–18]. Particle creation mechanism driving bulk viscous pressure has been extensively used to describe the dynamics and evolution of the early universe including early inflation and current accelerated expansion [19]. Particle creation has also been related to emergent universe [20].

In the present work it is consider that the cosmological implications of bulk viscous pressure due to particle creation mechanism in a universe with matter described by the perfect fluid and a sequence of proper choices of the particle creation rate as a function of the Hubble parameter, it is possible to show a continuous cosmic evolution from inflation to late time acceleration.

2. Bulk Viscous Universe with Particle Creation: Non Equilibrium M-I-S type Thermodynamic Theory

The energy-momentum tensor of a relativistic fluid having bulk viscosity as the only dissipative part is given by

$$T_{\mu\vartheta} = (\rho + p + \Pi)\, u_\mu u_\vartheta + (p + \Pi)\, g_{\mu\vartheta} \tag{1}$$

* *E-mail: shibajihalderrrm@gmail.com*

where as usual ρ, p and Π stands for the energy density, equilibrium pressure and bulk viscous pressure (connected with entropy production) and u^μ is the fluid 4-velocity. Considering the second order non equilibrium thermodynamics, the entropy flow vector S^μ is characterized by the equation [3]

$$S^\mu = sN^\mu - \frac{\tau\Pi^2}{2\zeta T}u^\mu \tag{2}$$

Where $N^\mu = nu^\mu$ is the particle flow vector with n being the particle number density, s, the entropy per particle, τ is the relaxation time, T is the temperature of the fluid and ζ is the coefficient of bulk viscosity. Now we consider a scenario where the non vanishing bulk viscous pressure is due to a change in fluid number density, which is characterized by the particle production rate $\Gamma = \frac{\dot{N}}{N}$, $N = na^3$ being the number of particles in co-moving volume a^3. For $G > 0$ we get particle creation while $G > 0$ usually means particle annihilation. The varying particle number density will cause a change of phase creating a entropy production density which will be given by:

$$S^\mu;_\mu = -\frac{\Pi}{T}\left[3H + \frac{\tau}{s}\dot{\Pi} + \frac{1}{2}\Pi T\left(\frac{\tau}{\zeta T}u^\mu\right);_\mu + \varepsilon\frac{n\Gamma}{\Pi}\right] \tag{3}$$

ε being the chemical potential. For the validity of second law of thermodynamics we must have $S^\mu;_\mu \geq \frac{\Pi^2}{\zeta T} \geq 0$. This gives the following non linear differential equation for bulk viscosity Π [14].

$$\Pi^2\left[1 + \frac{1}{2}T\left(\frac{\tau}{\zeta T}u^\mu\right);_\mu\right] + \tau\Pi\dot{\Pi} + 3\mathrm{H}\zeta\Pi = -\zeta n\Gamma. \tag{4}$$

Thus any deviation from equilibrium is characterized by the bulk viscous pressure Π in the presence of particle creation G, further the above equation asserts the existence of a single causal theory even with particle creation processes taken into account. Given the existence of particle creation G, the conservation equations are modified as

$$T^{\mu\vartheta};_\vartheta = 0 \text{ and } N^\mu;_\mu = \Gamma \tag{5}$$

has the explicit expressions

$$\dot{\rho} + \theta\left(\rho + p + \Pi\right) = 0 \text{ and } \dot{n} + \theta n = \Gamma \tag{6}$$

with $\dot{n} = n,_\mu u^\mu$ Comparing equation (6) with the Gibb's relation

$$T\,ds = d\left(\frac{\rho}{n}\right) + pd\left(\frac{1}{n}\right) \tag{7}$$

we get

$$nT\dot{s} = -3H\Pi - (p + \rho) \tag{8}$$

Considering that the pressure p and density ρ are related to the thermodynamic variables n and T by the equations $p = p\left(n, T\right)$ and $\rho = \rho\left(n, T\right)$ and using the conservation equation (6)together with

$$\frac{\partial\rho}{\partial n} = \frac{p + \rho}{n} - \frac{T}{n}\frac{dp}{dT} \tag{9}$$

we get the temperature evolution equation as

$$\frac{\dot{T}}{T} = -3H\left[\frac{\partial p/\partial T}{\partial\rho/\partial T} + \frac{\Pi}{T\partial\rho/\partial T}\right] + \Gamma\left[\frac{\partial p/\partial T}{\partial\rho/\partial T} - \frac{p + \rho}{T\partial\rho/\partial T}\right] \tag{10}$$

Using (8) the above relation can be written as

$$\frac{\dot{T}}{T} = -\left(3H - \Gamma\right)\frac{\partial p/\partial T}{\partial \rho/\partial T} + \frac{n\dot{s}}{\partial \rho/\partial T} \tag{11}$$

Thus it is easily observed that particle production affects the temperature with an effective viscous pressure Π together with a direct coupling. Considering isentropic particle production characterized by constant entropy $\dot{s} = 0$ the viscous pressure can be obtained directly in terms of particle production rate as [7].

$$\Pi = -\frac{\Gamma}{3H}\left(p + \rho\right) \tag{12}$$

From the above we can get a cosmic fluid characterized by changing particle number density. Also the fluid temperature is now given by

$$\frac{\dot{T}}{T} = -\left(3H - \Gamma\right)\frac{\partial p}{\partial \rho} \tag{13}$$

Further from (7) for isentropic particle production the evolution of n is given by

$$\frac{\dot{n}}{n} = -\left(3H - \Gamma\right) \tag{14}$$

3. Particle Creation in Viscous FRW and a Sequence of Unified Cosmic Evolution

As the particle number is not conserved, so we shall consider an open model of the universe. Here, for simplicity, we consider spatially flat FRW model of the universe having line element

$$ds^2 = -dt^2 + a^2\left(t\right)\left[dr^2 + r^2 d\Omega^2\right] \tag{15}$$

Now the Einstein field equations for the cosmic fluid having energy-momentum tensor given by Equation (1) are

$$\kappa\rho = 3H^2 \text{ and } \dot{H} = -\frac{\kappa}{2}\left(\rho + p + \Pi\right) \tag{16}$$

Where κ is the Einstein's gravitational constant and $H = \frac{\theta}{3}$ is the Hubble parameter. It should be noted that the cosmic fluid may be considered as a perfect fluid and the dissipative term Π is the effective bulk viscous pressure (see the previous section) due to particle production, i.e., the cosmic substratum is not a conventional dissipative fluid, rather a perfect fluid(equation of state $p = \left(\gamma - 1\right)\rho$) with varying particle number. In the context of cosmology, as the substratum is a perfect fluid, so we shall focus on isentropic particle production, i.e., a process having constant entropy per particle. However, there will be entropy production due to the enlargement of the phase space of the system since the universe is expanding and also the number of fluid particles increases. Now using the Einstein field equations (16) into the isentropic condition (12), we have

$$\frac{\Gamma}{\theta} = 1 + \frac{2}{3\gamma}\frac{\dot{H}}{H^2} \tag{17}$$

Which turn into

$$\frac{dH}{H\left(\frac{\Gamma}{3H} - 1\right)} = \frac{3\gamma}{2}\frac{da}{a} \tag{18}$$

Again deceleration parameter is given by

$$q = -\left(1 + \frac{\dot{H}}{H^2}\right) \tag{19}$$

As thermodynamic parameters all vary with the particle creation rate G. Thus we need specific choices of G to trace the history of the cosmic evolution. Here particle creation rate G be taken as functions of Hubble parameter for the three phases. We have chosen the time instants $t = t_r$ (i.e $a = a_r$) and $t = t_f$ (i.e $a = a_f$) as the time of transitions from acceleration to deceleration (in the early phase) and again from deceleration to acceleration (in the late era). Thus we shall denote the particle creation rates as

(i). $\frac{\Gamma}{3H} = \beta \frac{H^k}{H_r^k}$ i.e $\Gamma = 3\beta \frac{H^{k+1}}{H_r^k}$ where $k \in N$, for $t \le t_r$ (i.e $a \le a_r$) (called phase I)

(ii). $\frac{\Gamma}{3H} = \beta$ i.e $\Gamma = 3\beta H$ for $t_r \le t \le t_f$ (i.e $a_r \le a \le a_f$) (called phase II)

(iii). $\frac{\Gamma}{3H} = \beta \frac{H_f^k}{H^k}$ i.e $\Gamma = 3\beta \frac{H_f^k}{H^{k-1}}$ where $k \in N$, for $t \ge t_f$ (i.e $a \ge a_f$) (called phase III)

Phase I: For the above choice (i), integrate Equation (18) we get

$$\log\left(\beta \frac{H^k}{H_r^k} - 1\right) - k \log H + \log c_1 = \frac{3\gamma k}{2} \log a$$

where c_1 is integrating constant. If we take, when $a = a_r$, $H = H_r$, we get

$$\left(\beta \frac{H^k}{H_r^k} - 1\right) \frac{H_r^k}{H^k} = -(1-\beta)\left(\frac{a}{a_r}\right)^{\frac{3\gamma k}{2}}$$
$$\Rightarrow H^k = \frac{H_r^k}{\beta + (1-\beta)\left(\frac{a}{a_r}\right)^{\frac{3\gamma k}{2}}} \tag{20a}$$

Now from Equation (17),

$$\frac{\dot{H}}{H^2} = \frac{3\gamma}{2}\left(\frac{\Gamma}{3H} - 1\right) \Rightarrow \frac{\dot{H}}{H^2} = \frac{3\gamma}{2}\left(\beta \frac{H^k}{H_r^k} - 1\right)$$
$$q = -1 + \frac{3\gamma}{2} \frac{(1-\beta)\left(\frac{a}{a_r}\right)^{\frac{3\gamma k}{2}}}{\beta + (1-\beta)\left(\frac{a}{a_r}\right)^{\frac{3\gamma k}{2}}} \tag{20b}$$

From Equation (14)

$$\frac{dn}{n} = -\left(1 - \frac{G}{3H}\right) 3H dt = -\frac{3(1-\beta)\left(\frac{a}{a_r}\right)^{\frac{3\gamma k}{2}}}{\beta + (1-\beta)\left(\frac{a}{a_r}\right)^{\frac{3\gamma k}{2}}} \frac{da}{a}$$

Integrating and using $a = a_r$, $n = n_r$, we get

$$n = n_r \left(\beta + (1-\beta)\left(\frac{a}{a_r}\right)^{\frac{3\gamma k}{2}}\right)^{-\frac{2}{\gamma k}} \tag{20c}$$

From Equation (6), we get

$$\dot{\rho} = -(\theta - \Gamma)(p + \rho) = -(\theta - \Gamma)\gamma\rho$$
$$\Rightarrow \frac{d\rho}{\rho} = -\frac{3\gamma(1-\beta)\left(\frac{a}{a_r}\right)^{\frac{3\gamma k}{2}}}{\beta + (1-\beta)\left(\frac{a}{a_r}\right)^{\frac{3\gamma k}{2}}} \frac{da}{a}$$

Integrating and using $a = a_r$, $\rho = \rho_r$, we get

$$\rho = \rho_r \left(\beta + (1-\beta)\left(\frac{a}{a_r}\right)^{\frac{3\gamma k}{2}}\right)^{-\frac{2}{k}} \tag{20d}$$

From Equation (13), we get

$$\frac{\dot{T}}{T}=-\left(3H-\Gamma\right)\left(\gamma-1\right)\Rightarrow\frac{dT}{T}=-\left(\gamma-1\right)\frac{3\left(1-\beta\right)\left(\frac{a}{a_r}\right)^{\frac{3\gamma k}{2}}}{\beta+\left(1-\beta\right)\left(\frac{a}{a_r}\right)^{\frac{3\gamma k}{2}}}\frac{da}{a}$$

Integrating and using $a=a_r$, $T=T_r$ we get,

$$T=T_r\left(\beta+\left(1-\beta\right)\left(\frac{a}{a_r}\right)^{\frac{3\gamma k}{2}}\right)^{-\frac{2(\gamma-1)}{\gamma k}} \tag{20d}$$

Phase II: For the above choice (ii), we get

$$H=H_r\left(\frac{a}{a_r}\right)^{-\frac{3\gamma}{2}(1-\beta)} \tag{21a}$$

$$q=-1+\frac{3\gamma}{2}\left(1-\beta\right) \tag{21b}$$

$$n=n_r\left(\frac{a}{a_r}\right)^{3(1-\beta)} \tag{21c}$$

$$\rho=\rho_r\left(\frac{a}{a_r}\right)^{3\gamma(1-\beta)} \tag{21c}$$

$$T=T_r\left(\frac{a}{a_r}\right)^{3(\gamma-1)(1-\beta)} \tag{21c}$$

Phase III: For the above choice (iii), integrate Equation (18) we get,

$$\log\left(H^k-\beta H_f^k\right)+\log c_2=-\frac{3\gamma k}{2}\log a$$

where c_2 is integrating constant. If we take, when $a=a_f$, $H=H_f$, we get

$$\frac{H^k-\beta H_f^k}{H_f^k\left(1-\beta\right)}=\left(\frac{a}{a_f}\right)^{-\frac{3\gamma n}{2}}$$

$$\Rightarrow H^k=H_r^k\left(\beta+\left(1-\beta\right)\left(\frac{a}{a_r}\right)^{-\frac{3\gamma k}{2}}\right) \tag{22a}$$

Now from Equation (17),

$$\frac{\dot{H}}{H^2}=\frac{3\gamma}{2}\left(\frac{\Gamma}{3H}-1\right)\Rightarrow\frac{\dot{H}}{H^2}=\frac{3\gamma}{2}\left(\beta\frac{H_f^k}{H^k}-1\right) \tag{20}$$

$$q=-1+\frac{3\gamma}{2}\frac{\left(1-\beta\right)\left(\frac{a}{a_f}\right)^{-\frac{3\gamma k}{2}}}{\beta+\left(1-\beta\right)\left(\frac{a}{a_f}\right)^{-\frac{3\gamma k}{2}}} \tag{22b}$$

From Equation (14)

$$\frac{dn}{n}=-\left(1-\frac{\Gamma}{3H}\right)3Hdt=-\frac{3\left(1-\beta\right)\left(\frac{a}{a_f}\right)^{-\frac{3\gamma k}{2}}}{\beta+\left(1-\beta\right)\left(\frac{a}{a_f}\right)^{-\frac{3\gamma k}{2}}}\cdot\frac{da}{a}$$

Integrating and using $a=a_f$, $n=n_f$, we get

$$n=n_f\left(\beta+\left(1-\beta\right)\left(\frac{a}{a_f}\right)^{-\frac{3\gamma k}{2}}\right)^{\frac{2}{\gamma k}} \tag{22c}$$

From Equation (6), we get

$$\dot{\rho} = -(\theta - \Gamma)(p + \rho) = -(\theta - \Gamma)\gamma\rho$$
$$\Rightarrow \frac{d\rho}{\rho} = -\frac{3\gamma(1-\beta)\left(\frac{a}{a_f}\right)^{-\frac{3\gamma k}{2}}}{\beta + (1-\beta)\left(\frac{a}{a_f}\right)^{-\frac{3\gamma k}{2}}} \cdot \frac{da}{a}$$

Integrating and using $a = a_f$, $\rho = \rho_f$, we get

$$\rho = \rho_f \left(\beta + (1-\beta)\left(\frac{a}{a_f}\right)^{-\frac{3\gamma k}{2}}\right)^{\frac{2}{k}} \tag{22d}$$

From Equation (13), we get

$$\frac{\dot{T}}{T} = -(3H - \Gamma)(\gamma - 1) \Rightarrow \frac{dT}{T} = -(\gamma - 1)\frac{3(1-\beta)\left(\frac{a}{a_f}\right)^{-\frac{3\gamma k}{2}}}{\beta + (1-\beta)\left(\frac{a}{a_f}\right)^{-\frac{3\gamma k}{2}}} \cdot \frac{da}{a}$$

Integrating and using $a = a_f$, $T = T_f$ we get,

$$T = T_f \left(\beta + (1-\beta)\left(\frac{a}{a_f}\right)^{-\frac{3\gamma k}{2}}\right)^{\frac{2(\gamma-1)}{\gamma k}} \tag{22d}$$

The above values of q in terms of a are shown in the figures below for different values of k.

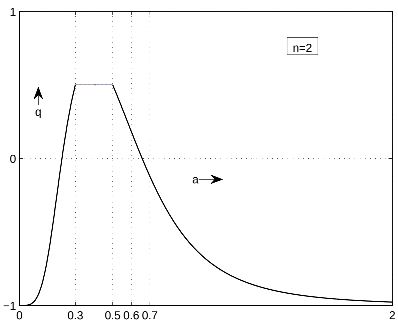

Figure 1.

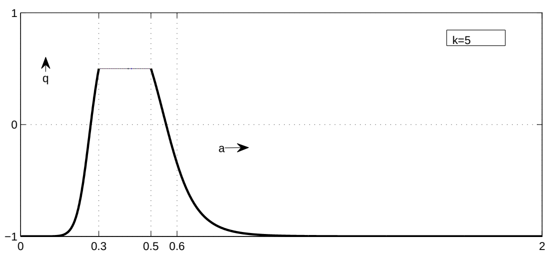

Figure 2.

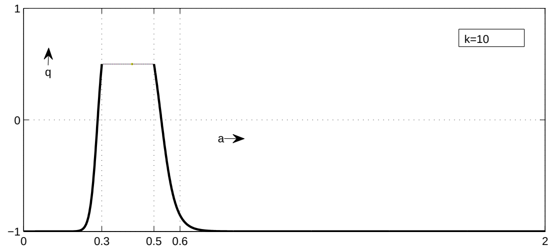

Figure 3.

Figures are drawn of a-q curves of this sequence of a models for $k = 2$ (Figure 1), $k = 5$ (Figure 2) and $k = 10$ (Figure 3). Where $a_r = 0.3$ and $a_f = 0.5$ and taking the values of the parameters $\gamma = \frac{4}{3}$ and $\beta = \frac{1}{4}$.

4. Summary of the Results

This paper deals with non-equilibrium thermodynamics in the context of cosmology having perfect fluid as cosmic substratum. The dissipative phenomenon occurs due to particle creation mechanism and behaves as a bulk viscous pressure. For simplicity, we have assumed the thermodynamical process to be adiabatic, i.e., the entropy per particle remains constant and as a result the dissipative pressure is related linearly to the particle creation rate. It is assumed that the particle creation rate as a function of the Hubble parameter. By proper choice of the functional form and considering a flat FRW model (open themrodynamical system), we are able to show the complete continuous cosmological solution as well as the relevant thermodynamical parameters.

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