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Types of Irregular Product Vague Line Graphs

G. Thamizhendhi[1,*]

1 Department of Mathematics, Vellalar College for Women (Autonomous), Erode, Tamilnadu, India.

Abstract: In this paper, neighborly irregular, highly irregular and totally irregular product vague line graphs are defined. Also, some properties of neighborly irregular and totally irregular product vague line graphs are given. Further, the applications of product vague line graph to model road network have been discussed with an example.

Keywords: Vague line graphs, Product vague line graph, Neighborly irregular, Totally irregular.

1. Introduction

Gau and Buehrer [5] proposed the concept of *vague set* and vague relation by replacing the value of an element in a set with a subinterval of [0,1]. Namely a true membership function $t_v(x)$ and a false membership function $f_v(x)$ are used to describe the boundaries of the membership degree. These two boundaries form a subinterval $[t_v(x), 1 - f_v(x)]$ of $[0,1]$. Ramakrishna [12] introduced the concept of *vague graphs* and studied some of their properties. Akram, Feng, Sarwar and Jun [2] defined types of irregular vague graphs and discussed some properties. Akram, Dudek, and Yousaf [1] are introduced the concepts of vague intersection graphs and vague line graphs. Rashmanlou and Borzooei [14] introduced the concepts of product vague graphs, complete product vague graphs density and balanced irregular vague graphs.

In this paper, product vague line graphs are defined and types of irregular product vague line graphs are introduced. Also some properties of product vague line graphs are investigated. Finally, an application of product vague line graphs in travel time networks were discussed with an example.

2. Preliminaries

Definition 2.1 ([4]). *Let a set E be fixed. An intuitionistic fuzzy set (IFS) A in E is an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in E\}$, where the function $\mu_A : E \to [0,1]$ and $\nu_A : E \to [0,1]$ determine the degree of membership and the degree of non-membership of the element $x \in E$, respectively and , $0 \le \mu_A(x) + \nu_A(x) \le 1$, for every $x \in E$.*

Definition 2.2 ([5]). *A vague set A in an ordinary finite nonempty set X is a pair (t_A, f_A), where $t_A(x) : X \to [0,1]$, $f_A(x) : X \to [0,1]$ are true and false membership functions respectively such that $0 \le t_A(x) + f_A(x) \le 1$ for all $x \in X$, the functions t_A and f_A should satisfy the condition $t_A \le 1 - f_A$.*

* *E-mail: gthamil@rediffmail.com*

Example 2.3. *Let X be the set of integers. The vague set A is defined as $A = \{$set of even numbers less than 10$\}$. The true membership function is defined as*

$$t_A(x) = \begin{cases} \frac{1}{5x^2+2}; & \text{if } x \in (1, 10) \\ 0; & \text{otherwise} \end{cases}$$

$x = 2 \Rightarrow t_A(x) = 0.045$; $x = 4 \Rightarrow t_A(x) = 0.012$; $x = 6 \Rightarrow t_A(x) = 0.005$; $x = 8 \Rightarrow t_A(x) = 0.003$. *Hesitancy value* $(V_A(x) = 0.1)$

$$f_A(x) = 1 - t_A(x) - V_A(x)$$
$$t_A(x) = 0.045;\ f_A(x) = 0.855;\ V_A(x) = 0.1$$
$$t_A(x) = 0.012;\ f_A(x) = 0.888;\ V_A(x) = 0.1$$
$$t_A(x) = 0.005;\ f_A(x) = 0.895;\ V_A(x) = 0.1$$
$$t_A(x) = 0.003;\ f_A(x) = 0.855;\ V_A(x) = 0.1$$

Definition 2.4 ([7])**.** *A pair $G = (V, E)$ be a vague graph, where $V = \langle t_A, f_A \rangle$ is a vague set on V and $E = \langle t_B, f_B \rangle$ is a vague set on $E \subseteq V \times V$ such that $t_B(v_i, v_j) \leq \min(t_A(v_i), t_A(v_j))$ and $f_B(v_i, v_j) \geq \max(f_A(v_i), f_B(v_j))$ for each $(v_i, v_j) \in E$.*

Example 2.5. *Let $G = (V, E)$ be a vague graph with $V = \{v_1, v_2, v_3, v_4\}$, $E = \{(v_1, v_2), (v_2, v_4), (v_3, v_4), (v_1, v_3)(v_2, v_3)(v_1, v_4)\}$,*

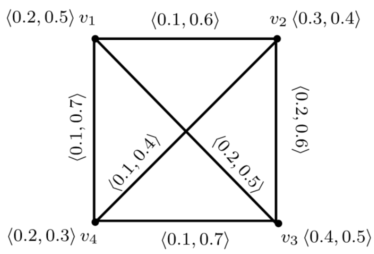


Figure 1: Vague graph

Definition 2.6. *A product vague graph $G = (V, E)$ is a pair of $G = (A, B)$, where $A = \langle t_A, f_A \rangle$ is an vague set in V and $B = \langle t_B, f_B \rangle$ is a vague set on E such that $t_B(v_i v_j) \leq t_A(v_i) \times t_A(v_j)$ and $f_B(v_i v_j) \geq f_A(v_i) \times f_A(v_j)$ for all $v_i, v_j \in V$.*

Example 2.7. *Let $G = (V, E)$ where $V = \{v_1, v_2, v_3, v_4\}$ and $E = \{v_1 v_4, v_2 v_3\}$. The product vague graph G is displayed in Figure 2,*

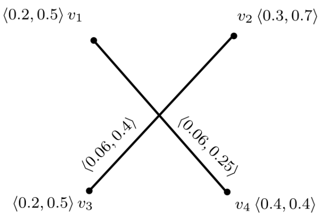


Figure 2: Product vague graph

Definition 2.8. *Let $G = (V, E)$ be a product vague graph with vague subsets $A = \langle t_A, f_A \rangle$ on V and $B = \langle t_B, f_B \rangle$ on E. Then the product vague line graph $L(G) = (V^{'}, E^{'})$ of G is defined as follows:*

(1). $V^{'} = \left\{ v_i^{'} \in V^{'} | v_i^{'} = (v_i, v_j),\ \forall\ (v_i, v_j) \in E \right\}$

(2). $E^{'} = \left\{ (v_i^{'}, v_j^{'}) \in E^{'} | v_i^{'} \cap v_j^{'} \neq \phi, \right\}$

(3). $A_1 = \langle t_{A_1}, f_{A_1} \rangle$ *and* $B_1 = \langle t_{B_1}, f_{B_1} \rangle$ *are vague subsets of* $V^{'}$ *and* $E^{'}$ *respectively,*

(4). $t_{A_1}(v_i^{'}) = t_B(v_i, v_j),\ f_{A_1}(v_i^{'}) = f_B(v_i, v_j),$

(5). $t_{B_1}(v_i^{'}, v_j^{'}) = t_{A_1}(v_i^{'}) \times t_{A_1}(v_j^{'}),\ f_{B_1}(v_i^{'}, v_j^{'}) = f_{A_1}(v_i^{'}) \times f_{A_1}(v_j^{'}).$

Example 2.9. *Consider the product vague graph $G = (V, E)$ where $V = \{v_1, v_2, v_3, v_4\}$ and $E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_1), (v_3, v_1)\}$. The product vague line graph $L(G) = (V^{'}, E^{'})$ of G such that $V^{'} = \left\{ v_1^{'} = (v_1, v_2), v_2^{'} = (v_2, v_3), v_3^{'} = (v_3, v_4), v_4^{'} = (v_4, v_1), v_5^{'} = (v_3, v_1) \right\}$ and $E^{'} = \left\{ (v_1^{'}, v_2^{'}),\ (v_2^{'}, v_3^{'}),\ (v_3^{'}, v_4^{'}),\ (v_4^{'}, v_1^{'}), (v_5^{'}, v_1^{'}), (v_5^{'}, v_2^{'}), (v_5^{'}, v_3^{'}), (v_5^{'}, v_4^{'}) \right\}$.*

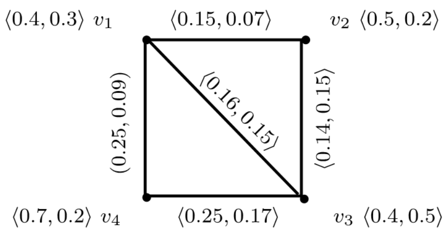


Figure 3: Product vague graph

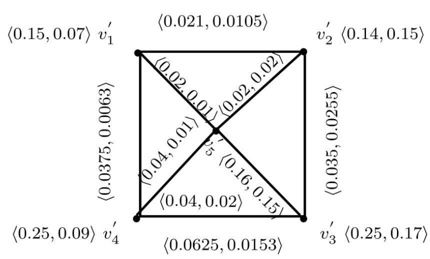


Figure 4: Product vague line graph

Definition 2.10. *A product vague graph $G = (V, E)$ is said to be strong if $t_B(v_i, v_j) = t_A(v_i) \times t_A(v_j)$ and $f_B(v_i, v_j) = f_A(v_i) \times f_A(v_j)$ for all $(v_i, v_j) \in E$.*

Definition 2.11. *Let $G = (V, E)$ be a product vague graph the open neighbourhood degree of a vertex v in G is defined by $deg(v_i) = (deg_t(v_i), deg_f(v_i))$, where $deg_t(v_i) = \sum\limits_{v_i \neq v_j, (v_i, v_j) \in E} t_B((v_i, v_j))$ and $deg_f(v_i) = \sum\limits_{v_i \neq v_j, (v_i, v_j) \in E} f_B((v_i, v_j))$. If all the vertices of G have same open neighborhood degree (r_1, r_2), then G is called (r_1, r_2)-regular product vague graph.*

Example 2.12. *The regular product vague displayed in Figure 5*

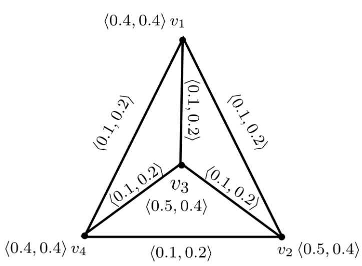


Figure 5: Regular product vague graph

In Figure 5, $deg(v_1) = deg(v_2) = deg(v_3) = deg(v_4) = (0.3, 0.6)$. Hence, G is regular product vague graph.

Definition 2.13. *Let $G = (V, E)$ be a product vague graph. The closed neighbourhood degree of a vertex v is defined by $deg_t[v_i] = (deg_t[v_i], deg_f[v_i])$, where $deg_t[v_i] = deg_t(v_i) + t_A(v_i)$ and $deg_f[v_i] = deg_f(v_i) + f_A(v_i)$. If each vertex of G has the same closed neighborhood degree (g_1, g_2), then G is called (g_1, g_2)-totally regular product vague graph.*

Example 2.14. *Totally regular product vague graph is shown in Figure 6*

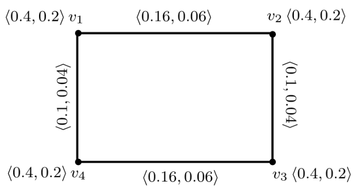


Figure 6: Totally regular product vague graph

In Figure 6, $deg[v_1] = deg[v_2] = deg[v_3] = deg[v_4] = (0.67, 0.28)$. Hence, G is totally regular product vague graph.

3. Types of Irregular Product Vague Line Graphs

Definition 3.1. *Let $L(G) = (V^{'}, E^{'})$ be a product vague line graph. Then $L(G)$ is irregular product vague line graph if there is a vertex which is adjacent to vertices with distinct degrees.*

Example 3.2.

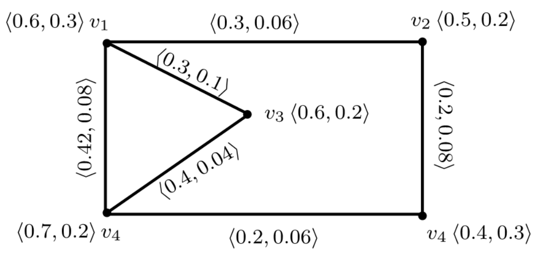


Figure 7: Irregular product vague graph

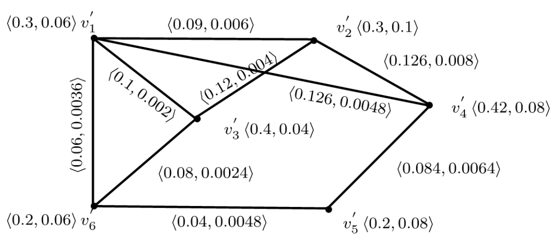


Figure 8: Irregular product vague line graph L(G)

In Figure 8, $deg(v_1^{'}) = (0.396, 0.017)$; $deg(v_2^{'}) = (0.336, 0.018)$; $deg(v_3^{'}) = (0.32, 0.009)$; $deg(v_4^{'}) = (0.336, 0.0192)$; $deg(v_5^{'}) = (0.124, 0.0112)$; $deg(v_6^{'}) = (0.18, 0.0108)$. Here $v_1^{'}$ adjacent to $v_2^{'}$, $v_3^{'}$, $v_4^{'}$ and $v_6^{'}$ which are having distinct degrees then, $L(G)$ is irregular product vague line graph.

Definition 3.3. *Let $L(G) = (V', E')$ be a product vague line graph. $L(G)$ is said to be a neighborly irregular product vague line graph if every two adjacent vertices of $L(G)$ have distinct degrees.*

Example 3.4.

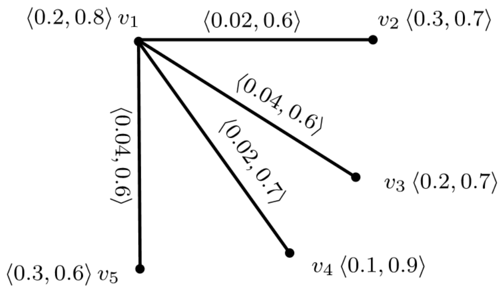


Figure 9: Neighbourly irregular product vague graph

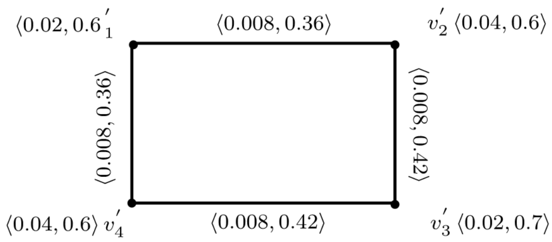
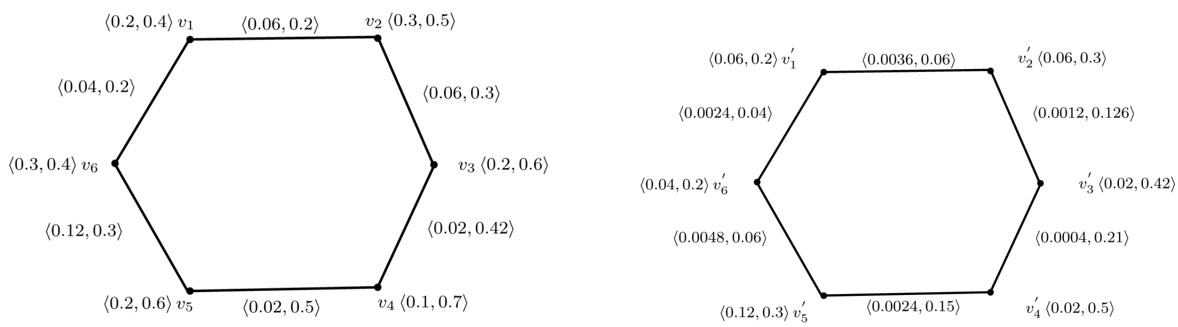


Figure 10: Neighbourly irregular product vague line graph L(G)

In Figure 10, $deg\left(v_1^{'}\right) = (0.0012, 0.72)$; $deg\left(v_2^{'}\right) = (0.0016, 0.78)$; $deg\left(v_3^{'}\right) = (0.0016, 0.84)$; $deg\left(v_4^{'}\right) = (0.0016, 0.78)$.

Definition 3.5. *Let $L(G) = (V', E')$ be a product vague line graph. Then $L(G)$ is said to be a highly irregular product vague line graph if every vertex of $L(G)$ is adjacent to vertices with distinct degrees. Every vertex of product vague line graph is adjacent to vertices with distinct degrees.*

Example 3.6. *Highly irregular product vague and line graph is displayed in Figure 11(a) and 11(b)*



(a) Highly irregular product vague graph

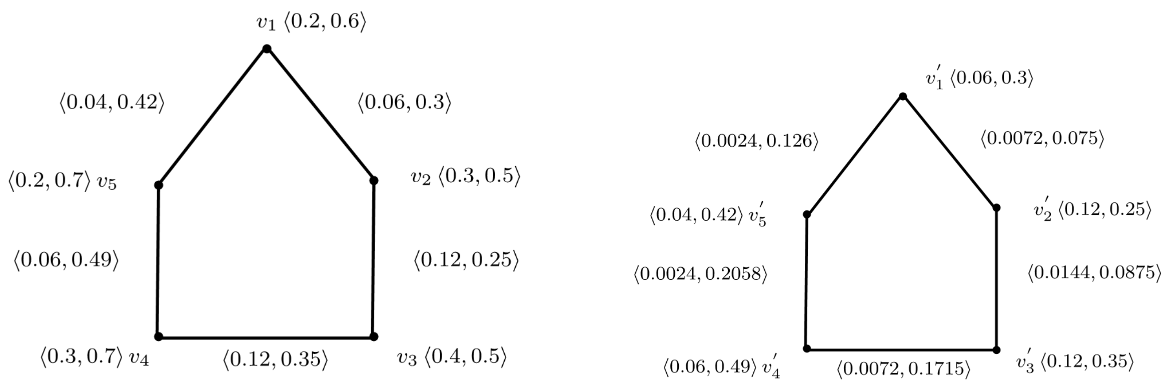
(b) Highly irregular product vague line graph L(G)

Figure 11:

In Figure 11(b), $deg\left(v_1^{'}\right) = (0.006, 0.1)$; $deg\left(v_2^{'}\right) = (0.0048, 0.186)$; $deg\left(v_3^{'}\right) = (0.0016, 0.336)$; $deg\left(v_4^{'}\right) = (0.0028, 0.36)$; $deg\left(v_5^{'}\right) = (0.0072, 0.21)$; $deg\left(v_6^{'}\right) = (0.0072, 0.1)$. *We see that every vertex of $L(G)$ is adjacent to vertices with distinct degree. So $L(G)$ is highly irregular product vague line graph.*

Definition 3.7. *Let $L(G) = (V^{'}, E^{'})$ be a product vague line graph. Then $L(G)$ is said to be a totally irregular product vague line graph if there is a vertex which is adjacent to vertices with distinct total degrees.*

Example 3.8.



(a) Totally irregular product vague graph

(b) Totally irregular product vague line graph L(G)

Figure 12:

In Figure 12(b), deg $\left[v_1^{'}\right] = (0.0696, 0.501)$*; deg* $\left[v_2^{'}\right] = (0.1416, 0.4125)$*; deg* $\left[v_3^{'}\right] = (0.1416, 0.609)$*; deg* $\left[v_4^{'}\right] = (0.0696, 0.8673)$*; deg* $\left[v_5^{'}\right] = (0.0448, 0.7518)$.

Theorem 3.9. *Let $L(G)$ be a product vague graph. Then $L(G)$ is highly irregular product vague line graph and neighbourly irregular product vague line graph if and only if the neighbourhood degrees of all the vertices of $L(G)$ are distinct.*

Proof. Let $L(G)$ be a product vague line graph with n-vertices $v_1, v_2, \cdots v_n$. Assume that $L(G)$ is highly irregular product vague line graph and neighbourly irregular product vague line graph. To claim that neighbourhood degrees of all vertices of $L(G)$ are distinct. Let $deg(v_i) = (k_i, l_i),\ i = 1, 2, \cdots, n$. Let the adjacent vertices of v_1 be $v_2, v_3, \ldots, v_n$ with neighbourhood degrees $(k_2, l_2), (k_3, l_3), \ldots, (k_n, l_n)$ respectively. Then $k_2 \neq k_3 \neq \cdots \neq k_n$ and $l_2 \neq l_3 \neq \cdots \neq l_n$, since $L(G)$ is highly irregular. Also $k_1 \neq k_2 \neq k_3 \neq \cdots \neq k_n$ and $l_1 \neq l_2 \neq l_3 \neq \cdots \neq l_n$, since $L(G)$ is neighbourly irregular. Hence, the neighbourhood degree of all the vertices of $L(G)$ are distinct.

Conversely, assume that the neighbourhood degrees of all the vertices of $L(G)$ are distinct. $L(G)$ is highly irregular and neighbourly irregular product vague line graph. Let $deg(v_i) = (k_i, l_i),\ i = 1, 2, \ldots, n$. Given that, $k_1 \neq k_2 \neq k_3 \neq \cdots \neq k_n$ and $l_1 \neq l_2 \neq l_3 \neq \cdots \neq l_n$, which implies that every two adjacent vertices have distinct neighbourhood degrees and to every vertex, the adjacent vertices have distinct neighbourhood degrees. □

Theorem 3.10. *A product vague line graph $L(G)$ is a cycle with vertices three is neighbourly irregular and highly irregular product vague line graph if and only if the true membership and false membership value of the vertices between every pair of vertices are all distinct.*

Proof. Assume that true membership and false membership value of the vertices are all distinct. $L(G)$ is neighbourly irregular and highly irregular product vague line graph. Let $v_i, v_j, v_k \in V$, given that, $t_A(v_i) \neq t_A(v_j) \neq t_A(v_k)$ and $f_A(v_i) \neq f_A(v_j) \neq f_A(v_k)$, which implies that $\sum x \in N(x) t_A(v_i) \neq \sum x \in N(x) t_A(v_j) \neq \sum x \in N(x) t_A(v_k)$ and $\sum x \in N(x) f_A(v_i) \neq \sum x \in N(x) f_A(v_j) \neq \sum x \in N(x) f_A(v_k)$. That is, $deg(v_i) \neq deg(v_j) \neq deg(v_k)$. Hence G is neighbourly irregular and highly irregular product vague line graph.

Conversely, assume that $L(G)$ is neighbourly irregular and highly irregular. True membership and false membership value of the vertices are all distinct. Let $deg(v_i) = (k_i, l_i),\ i = 1, 2, \ldots, n$. Suppose that true membership and false membership value of any two vertices are same. Let $v_1, v_2 \in V$. Let $t_A(v_1) = t_A(v_2)$ and $f_A(v_1) = f_A(v_2)$. Then $deg(v_1) = deg(v_2)$, since $L(G)$ is cycle, which is contradiction. To the fact that $L(G)$ is neighbourly irregular and highly irregular product vague line graph. Hence true membership and false membership value of the vertices are all distinct. □

Proposition 3.11. *Let $L(G)$ be a product vague line graph. If $L(G)$ is neighbourly irregular product vague line graph and (t_A, f_A) is a constant function, then $L(G)$ is a neighbourly total irregular product vague line graph.*

Proof. Assume that $L(G)$ is a neighbourly irregular product vague line graph. That is the neighbourhood degrees of every two adjacent vertices are distinct. Let $v_i, v_j \in V$, where v_i and v_j are adjacent vertices with distinct neighbourhood degrees (k_1, l_1) and (k_2, l_2) respectively. That is $deg(v_i) = (k_1, l_1)$ and $deg(v_j) = (k_2, l_2)$, where $k_1 \neq k_2$, $l_1 \neq l_2$. Let as assume that $(t_1(v_i), f_1(v_i)) = (t_1(v_j), f_1(v_j)) = (c_1, c_2)$, where c_1, c_2 are constant and $c_1, c_2 \in [0, 1]$. Therefore, $deg_t\,[v_i] = deg_t(v_i) + t_1(v_i) = k_1 + c_1$ and $deg_f\,[v_i] = deg_f(v_i) + f_1(v_i) = l_1 + c_2$; $deg_t\,[v_j] = deg_t(v_j) + t_1(v_j) = k_2 + c_1$ and $deg_f\,[v_j] = deg_f(v_j) + f_1(v_j) = l_2 + c_2$.

Claim: $deg_t\,[v_i] \neq deg_t\,[v_j]$ and $deg_f\,[v_i] \neq deg_f\,[v_j]$ suppose that,$deg_t\,[v_i] = deg_t\,[v_j]$ and $deg_f\,[v_i] = deg_f\,[v_j]$. Consider,

$$deg_t\,[v_i] = deg_t\,[v_j]$$
$$k_1 + c_1 = k_2 + c_1$$
$$k_1 - k_2 = c_1 - c_1 = 0$$
$$k_1 = k_2,$$

which is a contradiction to $k_1 \neq k_2$. Therefore,$deg_t\,[v_i] \neq deg_t\,[v_j]$. Similarly, consider

$$deg_f\,[v_i] = deg_f\,[v_j]$$
$$l_1 + c_2 = l_2 + c_2$$
$$l_1 - l_2 = c_2 - c_2 = 0$$
$$l_1 = l_2,$$

which is a contradiction to $l_1 \neq l_2$. Therefore, $deg_f\,[v_i] \neq deg_f\,[v_j]$. Hence $L(G)$ is a neighbourly total irregular product vague line graph. □

Proposition 3.12. *Let $L(G)$ be a product vague line graph. If $L(G)$ is a neighbourly total irregular and (t_A, f_A) is a constant function, then $L(G)$ is a neighbourly irregular product vague line graph.*

Proof. Assume that $L(G)$ is a neighbourly total irregular product vague line graph. That is the closed neighbourhood degrees of every two adjacent vertices are distinct. Let $v_i, v_j \in V$ and $deg\,[v_i] = (k_1, l_1)$,$deg\,[v_j] = (k_2, l_2)$, where $k_1 \neq k_2$ and $l_1 \neq l_2$. Assume that, $(t_1(v_i), f_1(v_i)) = (c_1, c_2)$ and $(t_1(v_j), f_1(v_j)) = (c_1, c_2)$, where $c_1, c_2 \in [0, 1]$ are constant and $deg\,[v_i] \neq deg\,[v_j]$.

Claim: $deg(v_i) \neq deg(v_j)$. Given that $deg\,[v_i] \neq deg\,[v_j]$ which implies $deg_t\,[v_i] \neq deg_t\,[v_j]$ and $deg_f\,[v_i] \neq deg_f\,[v_j]$. Consider,

$$deg_t\,[v_i] \neq deg_t\,[v_j]$$
$$k_1 + c_1 \neq k_2 + c_1$$
$$k_1 \neq k_2$$

Consider

$$deg_f\,[v_i] \neq deg_f\,[v_j]$$
$$l_1 + c_2 \neq l_2 + c_2$$
$$l_1 \neq l_2$$

That is the neighbourhood degrees of adjacent vertices of $L(G)$ are distinct. Hence neighbourhood degree of every pair of adjacent vertices is distinct in $L(G)$. □

4. Applications of Product Vague Line Graphs

Product vague line graphs are used in database theory, expert systems, neural networks, decision making problems, and geographical systems. The main problem in networks is to find shortest path between two nodes. In many situations the length of the road may be uncertain.The vague sets are used to define the uncertain concepts. A road network can be represented by a product vague line graph $L(G) = (V^{'}, E^{'})$, in which $V^{'}$ represents nodes corresponds to crossings and $E^{'}$ represents edges corresponds to roads. The weight of the roads defined by vague numbers.

Example 4.1.

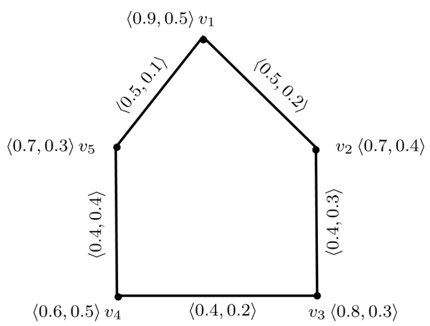


Figure 13: Product vague graph of a road network

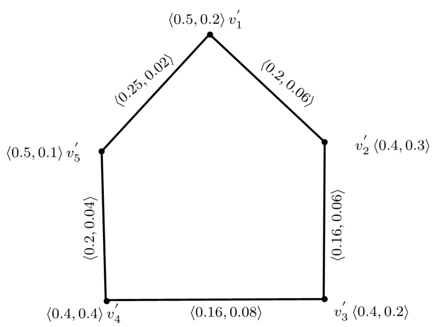


Figure 14: Product vague line graph of a road network

Figure 14, shows a model of a road network represented as an product vague line graph $L(G) = (V^{'}, E^{'})$, when $V^{'}$ is a an vague set of crossings at which the traffic density is calculated.

$$V^{'} = \left\{ \left\langle v_1^{'}, 0.5, 0.2 \right\rangle, \left\langle v_2^{'}, 0.4, 0.3 \right\rangle, \left\langle v_3^{'}, 0.4, 0.2 \right\rangle, \left\langle v_4^{'}, 0.4, 0.4 \right\rangle, \left\langle v_5^{'}, 0.5, 0.1 \right\rangle \right\}$$

and $E^{'}$ is an vague set of roads between two crossing. The product vague line graph $L(G)$ of the road network is represented by the adjacency matrix given below

$$L(G) = \begin{bmatrix} (0.0, 1.0) & (0.2, 0.06) & (0.0, 1.0) & (0.0, 1.0) & (0.0, 1.0) \\ (0.0, 1.0) & (0.0, 1.0) & (0.16, 0.06) & (0.0, 1.0) & (0.0, 1.0) \\ (0.0, 1.0) & (0.0, 1.0) & (0.0, 1.0) & (0.16, 0.08) & (0.0, 1.0) \\ (0.0, 1.0) & (0.0, 1.0) & (0.0, 1.0) & (0.0, 1.0) & (0.2, 0.04) \\ (0.25, 0.02) & (0.0, 1.0) & (0.0, 1.0) & (0.0, 1.0) & (0.0, 1.0) \end{bmatrix}$$

The final weights on edges can be calculated by finding the ranks as $S_i = t_A(x) - f_A(x) \times V_A(x)$*. Here* S_i *represents the shortest path between two vertices. The calculation of the weighted adjacency matrix is*

$$S_1 = 0.2 - 0.06 \times 0.74 = 0.1556$$

$$S_2 = 0.16 - 0.06 \times 0.78 = 0.1132$$

$$S_3 = 0.16 - 0.08 \times 0.76 = 0.0992$$

$$S_4 = 0.2 - 0.04 \times 0.76 = 0.1696$$

$$S_5 = 0.25 - 0.02 \times 0.73 = 0.2354$$

$$WL(G) = \begin{bmatrix} 0 & 0.1556 & 0 & 0 & 0 \\ 0 & 0 & 0.1132 & 0 & 0 \\ 0 & 0 & 0 & 0.0992 & 0 \\ 0 & 0 & 0 & 0 & 0.1696 \\ 0.2354 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The optimal path between two vertices can be find from weighted adjacency matrix. Hence the calculated weight between the two vertices is $v_1^{'}$ *and* $v_2^{'}$ *is 0.1556,* $v_2^{'}$ *and* v'_3 *is 0.1132,* v'_3 *and* v'_4 *is 0.0992,* v'_4 *and* v'_5 *is 0.1696,* v'_5 *and* v'_1 *is 0.2354.*

5. Conclusion

Graph theory has several interesting applications in system analysis, operations research, computer applications, and economics. Since most of the time the aspects of graph problems are uncertain. It is nice to deal with these aspects via the methods of vague systems. This paper discussed about the product vague line graphs and types of irregular product vague line graphs. Also, some interesting properties of these new concepts are proved. Finally an application of product vague line graph in travel time in decision support systems are discussed.

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