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Edge Version of Multiplicative Atom Bend Connectivity Index of Certain Nanotubes and Nanotorus

V. R. Kulli[1,*]

1 Department of Mathematics, Gulbarga University, Gulbarga, Karnataka, India.

Abstract: We define the edge version of multiplicative atom bond connectivity index of a molecular graph. In this paper, we compute this index for different chemically interesting nanotubes and nanotorus.

MSC: 05C05, 05C012.

Keywords: Multiplicative atom bond connectivity index, nanotubes, nanotorus.

1. Introduction

Let G be a finite, simple, connected graph with vertex set *V(G)* and edge set *E(G)*. A molecular graph is a finite simple graph such that its vertices correspond to the atoms and the edges to the bonds. A topological index is a numerical parameter mathematically derived from the graph structure. There are several topological indices that have some applications in theoretical chemistry, especially in *QSPR/QSAR* study [1, 2]. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. The degree of an edge $e = uv$ in G is defined by $d_G(e) = d_G(u) + d_G(v) - 2$. The line graph $L(G)$ of G is the graph whose vertex set corresponds to the edges of G such that two vertices of *L(G)* are adjacent if the corresponding edges of G are adjacent. We refer to [3] for undefined term and notation. The multiplicative atom bond connectivity index [4] of a graph G is defined as

$$ABCII\,(G) = \prod_{uv \in E(G)} \sqrt{\frac{d_G\,(u) + d_G\,(v) - 2}{d_G\,(u)\, d_G\,(v)}}$$

This index was also studied, for example, in [5, 6]. We now define the edge version of multiplicative atom bond connectivity index of a graph G as follows:

The edge version of multiplicative atom bond connectivity index of a molecular graph G is defined as

$$ABCII_e\,(G) = \prod_{ef \in E(L(G))} \sqrt{\frac{d_{L(G)}\,(e) + d_{L(G)}\,(f) - 2}{d_{L(G)}\,(e)\, d_{L(G)}\,(f)}}.$$

Many other multiplicative topological indices were studied, for example, in [7–19]. Edge version of indices were also studied, for example, in [20, 21]. In this paper, we compute the edge version of multiplicative atom bond connectivity index of $TUC_4C_6C_8[p, q]$, $TUSC_4C_8(S)[m, n]$, $NPHX[m, n]$ nanotubes and $C_4C_6C_8[p, q]$, $TC_4C_8(S)[p, q]$ nanotorus. For more information about nanotubes and nanotori see [22].

* *E-mail: vrkulli@gmail.com*

2. Results for Nanotubes

2.1. $TUC_4C_6C_8[p,q]$ Nanotube

We consider the graph 2-D lattice of $TUC_4C_6C_8[1,1]$ nanotube as depicted in Figure 1(a). The line graph of $TUC_4C_6C_8[1,1]$ is shown in Figure 1(b). Also we consider the graph of $TUC_4C_6C_8[p,q]$ nanotube with p columns and q rows. The graph of $TUC_4C_6C_8[4,5]$ is shown in Figure 1(c).

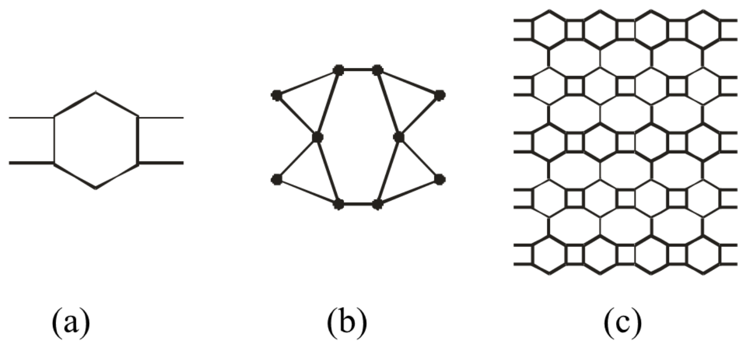


Figure 1.

We determine the edge version of multiplicative atom bond connectivity index for $TUC_4C_6C_8[p,q]$ nanotube.

Theorem 2.1. *The edge version of multiplicative atom bond connectivity index of* $TUC_4C_6C_8[p,q]$ *nanotube is*

$$ABCII_e\left(TUC_4C_6C_8\left[p,q\right]\right) = \left(\frac{2}{3}\right)^{2p}\left(\frac{5}{12}\right)^{4p}\left(\frac{3}{8}\right)^{9pq-7p}.$$

Proof. Let G be the graph of $TUC_4C_6C_8[p,q]$ nanotube. By algebraic method, the line graph of $TUC_4C_6C_8[p,q]$ has $18pq-4p$ edges. Also we obtain that the edge set $E(L(G))$ can be divided into three partitions as given in Table 1.

$d_{L(G)}(e), d_{L(G)}(f)\backslash ef \in E(L(G))$	(3, 3)	(3, 4)	(4, 4)
Number of edges	$2p$	$8p$	$18pq-14p$

Table 1. Partitions of the edge set $E(L(G))$

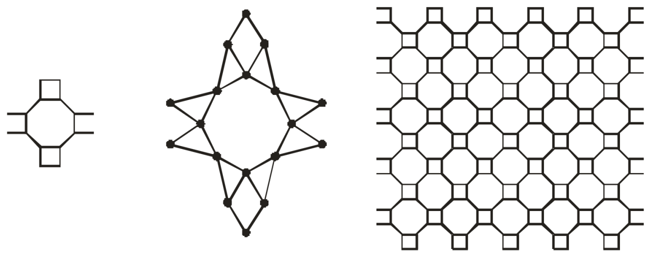
Thus, using the definition of edge version of multiplicative atom bond connectivity index, we obtain

$$\begin{aligned} ABCII_e\left(TUC_4C_6C_8\left[p,q\right]\right) &= \left(\sqrt{\frac{3+3-2}{3\times 3}}\right)^{2p}\left(\sqrt{\frac{3+4-2}{3\times 4}}\right)^{8p}\left(\sqrt{\frac{4+4-2}{4\times 4}}\right)^{18pq-14p} \\ &= \left(\frac{2}{3}\right)^{2p}\left(\frac{5}{12}\right)^{4p}\left(\frac{3}{8}\right)^{9pq-7p}. \end{aligned}$$

□

2.2. $TUSC_4C_8(S)[m,n]$ Nanotube

We consider the graph 2-D lattice of $TUSC_4C_8(S)[1,1]$ nanotube as depicted in Figure 2(a). The line graph of $TUSC_4C_8(S)[1,1]$ is shown in Figure 2(b). Also we consider the graph of $TUSC_4C_8(S)[m,n]$ nanotube with m columns and n rows. The graph of $TUSC_4C_8(S)[m,n]$ is shown in Figure 2(c).



(a) (b) (c)

Figure 2.

In the following theorem, we determine the edge version of multiplicative atom bond connectivity index for $TUSC_4C_8(S)[m,n]$ nanotube.

Theorem 2.2. *The edge version of multiplicative atom bond connectivity index of $TUSC_4C_8(S)[m,n]$ nanotube is*

$$ABCII_e\left(TUSC_4C_8\left(S\right)\left[m,n\right]\right) = \left(\frac{1}{2}\right)^{2m}\left(\frac{5}{12}\right)^{4m}\left(\frac{3}{8}\right)^{12mn-4m}.$$

Proof. Let G be the graph of $TUSC_4C_8(S)[m,n]$ nanotube. By algebraic method, the line graph of $TUSC_4C_8(S)[m,n]$ has $24mn+4m$ edges. Also we obtain that the edge set $E(L(G))$ can be divided into three partitions as given in Table 2.

$d_{L(G)}(e), d_{L(G)}(f) \backslash ef \in E(L(G))$	(2, 3)	(3, 4)	(4, 4)
Number of edges	$4m$	$8m$	$24mn-8m$

Table 2. **Partitions of the edge set E (L(G))**

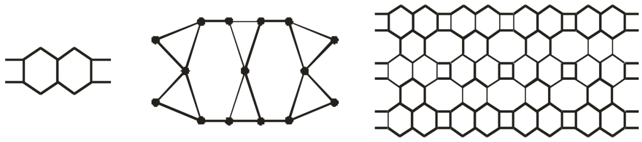
Thus using the definition of edge version of multiplicative atom bond connectivity index, we obtain

$$\begin{aligned} ABCII_e\left(TUSC_4C_8\left(S\right)\left[m,n\right]\right) &= \left(\sqrt{\frac{2+3-2}{2\times 3}}\right)^{4m}\left(\sqrt{\frac{3+4-2}{3\times 4}}\right)^{8m}\left(\sqrt{\frac{4+4-2}{4\times 4}}\right)^{24mn-8m} \\ &= \left(\frac{1}{2}\right)^{2m}\left(\frac{5}{12}\right)^{4m}\left(\frac{3}{8}\right)^{12mn-4m}. \end{aligned}$$

□

2.3. H-Naphtalenic $NPHX[m,n]$ Nanotube

We consider the graph 2-D lattice of H-Naphtalenic $NPHX[1,1]$ nanotube as shown in Figure 3(a). The line graph of H-Naphtalenic $NPHX[1,1]$ is shown in Figure 3(b). Also we consider the graph of H-Naphtalenic $NPHX[4,3]$ nanotube as shown in Figure 3(c).



(a) (b) (c)

Figure 3.

In the following theorem, we determine the edge version of multiplicative atom bond connectivity index for $NPHX[m,n]$ nanotube.

Theorem 2.3. *The edge version of multiplicative atom bond connectivity index of $NPHX[m,n]$ nanotube is*

$$ABCII_e\left(NPHX\left[m,n\right]\right)=\left(\frac{2}{3}\right)^{6m}\left(\frac{5}{12}\right)^{6m}\left(\frac{3}{8}\right)^{15mn-13m}.$$

Proof. Let G be the graph of $NPHX[m,n]$ nanotube. By algebraic methed, the line graph of $NPHX[m,n]$ nanotube has $30mn-8m$ edges. Also we obtain that the edge set $E(L(G))$ can be divided into three partitions as given in Table 3.

$d_{L(G)}(e),\, d_{L(G)}(f)\backslash ef\in E(L(G))$	(3, 3)	(3, 4)	(4, 4)
Number of edges	$6m$	$12m$	$30mn-26m$

Table 3. **Partitions of edge set** $E(L(G))$

Thus, using the definition of edge version of multiplicative atom bond connectivity index, we obtain

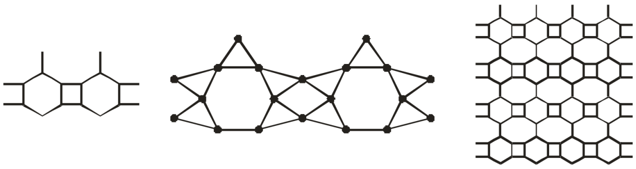
$$\begin{aligned}ABCII_e\left(NPHX\left[m,n\right]\right)&=\left(\sqrt{\frac{3+3-2}{3\times 3}}\right)^{6m}\left(\sqrt{\frac{3+4-2}{3\times 4}}\right)^{12m}\left(\sqrt{\frac{4+4-2}{4\times 4}}\right)^{30mn-26m}\\&=\left(\frac{2}{3}\right)^{6m}\left(\frac{5}{12}\right)^{6m}\left(\frac{3}{8}\right)^{15mn-13m}.\end{aligned}$$

□

3. Results for Nanotorus

3.1. $C_4C_6C_8[p,q]$ Nanotori

Consider the graph of 2-D lattice of $C_4C_6C_8[2,1]$ nanotori as shown in Figure 4 (a). The line graph of $C_4C_6C_8[2,1]$ nanotori is shown in Figure 4(b). Also we consider the graph of 2-D lattice of $C_4C_6C_8[p,q]$ nanotori as shown in Figure 4(c).



(a) (b) (c)

Figure 4.

Theorem 3.1. *The edge version of multiplicative atom bond connectivity index of $C_4C_6C_8[p,q]$ is*

$$ABCII_e\left(C_4C_6C_8\left[p,q\right]\right)=\left(\frac{1}{2}\right)^{p}\left(\frac{2}{3}\right)^{p}\left(\frac{5}{12}\right)^{2p}\left(\frac{\sqrt{6}}{4}\right)^{18pq-9p}.$$

Proof. Let G be the graph of $C_4C_6C_8[p,q]$ nanotori. By algebraic method, the line graph of $C_4C_6C_8[p,q]$ nanotori has $18pq-2p$ edges. Also we obtain that the edge set $E(L(G))$ can be divided into four partitions as given in Table 4.

$d_{L(G)}(e),\, d_{L(G)}(f)\backslash ef\in E(L(G))$	(2, 4)	(3, 3)	(3, 4)	(4, 4)
Number of edges	$2p$	p	$4p$	$18pq-9p$

Table 4. **Partitions of the edge set** $E(L(G))$

Thus using the definition of edge version of multiplicative atom bond connectivity index, we obtain

$$ABCII_e\left(C_4C_6C_8\left[p,q\right]\right)=\left(\sqrt{\frac{2+4-2}{2\times 4}}\right)^{2p}\left(\sqrt{\frac{3+3-2}{3\times 3}}\right)^{p}\left(\sqrt{\frac{3+4-2}{3\times 4}}\right)^{4p}\left(\sqrt{\frac{4+4-2}{4\times 4}}\right)^{18pq-9p}$$

$$= \left(\frac{1}{2}\right)^{p}\left(\frac{2}{3}\right)^{p}\left(\frac{5}{12}\right)^{2p}\left(\frac{\sqrt{6}}{4}\right)^{18pq-9p}.$$

□

3.2. $TC_4C_8(S)[p,q]$ Nanotori

Consider the graph of 2-D lattice of $TC_4C_8(S)[1,1]$ nanotori as shown in Figure 5(a). The line graph of $TC_4C_8(S)[1,1]$ manotori is shown in Figure 5(b). Also we consider the graph of 2-D lattice of $TC_4C_8(S)[5,3]$ nanotori as shown in Figure 5(c).

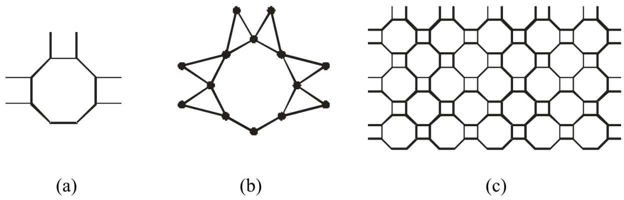


Figure 5.

In the following theorem, we determine the edge version of multiplicative atom bond connectivity index for $TC_4C_8(S)[p,q]$ nanotori.

Theorem 3.2. *The edge version of multiplicative atom bond connectivity index of $TC_4C_8(S)[p,q]$ nanotori is*

$$ABCII_e\left(TC_4C_8\left(S\right)\left[p,q\right]\right) = \left(\frac{1}{2}\right)^{3p}\left(\frac{5}{12}\right)^{4p}\left(\frac{3}{8}\right)^{12pq-7p}.$$

Proof. Let G be the graph of $TC_4C_8(S)[p,q]$ nanotori. By algebraic method, the line graph of $TC_4C_8(S)[p,q]$ nanotori has $24pq-4p$ edges. Also we obtain that the edge set $E(L(G))$ can be divided into four partitions as given in Table 5.

$d_{L(G)}(e), d_{L(G)}(f)\backslash ef \in E(L(G))$	(2, 3)	(2, 4)	(3, 4)	(4, 4)
Number of edges	$2p$	$4p$	$4p$	$24pq-14p$

Table 5. **Partitions of the edge set** $E(L(G))$

Thus, using the definition of edge version of multiplicative atom bond connectivity index, we obtain

$$ABCII_e\left(TC_4C_8\left(S\right)\left[p,q\right]\right) = \left(\sqrt{\frac{2+3-2}{2\times 3}}\right)^{2p}\left(\sqrt{\frac{2+4-2}{2\times 4}}\right)^{4p}\left(\sqrt{\frac{3+4-2}{3\times 4}}\right)^{4p}\left(\sqrt{\frac{4+4-2}{4\times 4}}\right)^{24pq-14p}$$
$$= \left(\frac{1}{2}\right)^{3p}\left(\frac{5}{12}\right)^{2p}\left(\frac{3}{8}\right)^{12pq-7p}.$$

□

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