Int. J. Math. And Appl., 6(1–E)(2018), 1085–1092
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Fixed Point of Cyclic Contraction Mappings in Banach Spaces Via C-Class Function

Arslan Hojat Ansari[1], R. Krishnakumar[2], K. Dinesh[2,*] and D. Dhamodharan[3]

1 Department of Mathematics, Karaj Branch, Islamic Azad University, Karaj, Iran.

2 PG and Research Department of Mathematics, Urumu Dhanalakshmi College, Trichy, Tamil Nadu, India.

3 Department of Mathematics, Jamal Mohamed College (Autonomous), Trichy, Tamil Nadu, India.

Abstract: In this paper, we discuss some fixed point theorems for cyclic mapping with contractive and non-expansive condition and $\varphi-\psi-$ contractive mappings in Banach space and analogous to the results presented in the literature.

MSC: 15A09, 15B05,15A99,15B99

Keywords: Fixed Point, Contractive Mapping, Cyclic Contraction, Banach spaces, C-class function

1. Introduction

Kirk introduced the notation of a cyclic representation and characterized the Banach contraction principle in context of a cyclic mapping. In 2011 Karapinar and Erhan introduced Kannan type cyclic contraction and Chatterjea type cyclic contraction. Moreover, they derive some fixed point theorems for such cyclic contractions in complete metric spaces. Note on $\varphi-\psi-$contractive type mappings and related fixed point are proved by Arslan Hojat Ansari [15]. The fixed point of cyclic contraction functions in Banach spaces are proved by R.Krishnakumar and K. Dinesh [10]. In this paper we investigate the fixed point theorems for cyclic mapping with contractive and nonexpansive condition and $\varphi-\psi-$contractive mappings in Banach space

Theorem 1.1 (Fixed point theorem for Kannan-type cyclic contraction)**.** *Let A and B be nonempty subsets of metric spaces (X,d) and a cyclic mapping $T : A\cup B \to A\cup B$ satisfies*

$$d(Tx,Ty) \leq k[d(x,Tx)+d(y,Ty)]\ \forall\ x\in A, y\in B$$

Where $0\leq k<\frac{1}{2}$. Then T has a unique fixed point in $A\cap B$.

Theorem 1.2 (Fixed point theorem for Chatterjea-type cyclic contraction)**.** *Let A and B be a nonempty subsets of metric space (X,d) and a cyclic mapping $T : A\cup B\to A\cup B$ satisfies*

$$d(Tx,Ty) \leq k[d(x,Tx)+d(y,Ty)]\ \forall\ x\in A, y\in B$$

* *E-mail: dinesh.skksv93@gmail.com*

Definition 1.3. *Let K be subset of a Banach space X. An operator T defined on K is said to belong to class $D(a,b)$ if*

$$\|Tx - Ty\| \leq a\|x-y\| + b[\|x-Tx\| + \|y-Ty\|] \tag{1}$$

for all x and y in k, where, if an operator T is in class $D(k,0)$ with $0<k<1$, then T is a contraction with $0<k<1$.

Definition 1.4. *Let K_1 and K_2 be closed subsets of a Banach space X. An operator T defined on K is said to belong to class $D(a,b,c)$ if*

$$\|Tx - Ty\| \leq a\|x-y\| + b[\|x-Tx\| + \|y-Ty\|] + c[\|x-Ty\| + \|Tx-y\|] \tag{2}$$

for all x and y in K where $0 \leq a,b,c \leq 1$, $a+2b+2c \leq 1$ and $b>0$.

Definition 1.5. *Let K_1 and K_2 be closed subsets of a Banach space X. An operator $T : K_1 \cup K_2 \to K_1 \cup K_2$ with $T(K_1) \subset K_2$ and $T(K_2) \subset K_1$ is said to belong to class $D(a,b)$ if it satisfies*

$$\|Tx - Ty\| \leq a\|x-y\| + b[\|x-Tx\| + \|y-Ty\|] \tag{3}$$

for all $x \in K_1$ and $y \in K_2$, where $0 \leq a,b \leq 1$. It is clear that if T belongs to the class $D(k,0)$ with $0<k<1$, then T is a cyclic contraction.

Example 1.6. *Let $K_1 = [0, \frac{1}{2}]$ and $K_2 = [\frac{1}{3}, 1]$. Define the operator T as follows*

$$T(x) = \begin{cases} \frac{2}{5}, & \text{if } 0 \leq x \leq \frac{1}{2} \\ \frac{2}{3}(1-x), & \text{if } \frac{1}{2} \leq x \leq 1 \end{cases}$$

We will show that T is in the class $D(\frac{1}{4}, \frac{1}{4})$ take $x \in \left[0, \frac{1}{2}\right]$ and $y \in \left[\frac{1}{3}, \frac{1}{2}\right]$. Then $\|Tx - Ty\| = \left|\frac{1}{5} - \frac{2}{5}\right| = 0$. Now $x \in \left[0, \frac{1}{2}\right]$ and $y \in \left[\frac{1}{2}, 1\right]$ then

$$\begin{aligned} \|Tx - Ty\| &= \left|\frac{2}{5} - \frac{2}{3} + \frac{2}{3}y\right| \\ &= \left|\frac{2}{3}y - \frac{4}{15}\right| \\ &= \left|\frac{1}{4}x - \frac{1}{4}y + \frac{1}{4}x - \frac{1}{10} + \frac{5}{12}y - \frac{1}{6}\right| \\ &\leq \frac{1}{4}\left|x-y\right| + \frac{1}{4}\left(\left|x - \frac{2}{5}\right| + \left|\frac{5}{3}y - \frac{2}{3}\right|\right) \\ &= \frac{1}{4}\left|x-y\right| + \frac{1}{4}\left(\left|x - Tx\right| + \left|y - Ty\right|\right) \end{aligned}$$

Observe that T has a unique fixed point $p = \frac{2}{5}$.

Definition 1.7. *Let K_1 and K_2 be closed subsets of a Banach space X. An operator $T : K_1 \cup K_2 \to K_1 \cup K_2$ with $T(K_1) \subset K_2$ and $T(K_2) \subset K_1$ is said to belong to class $D(a,b,c)$ if it satisfies*

$$\|Tx - Ty\| \leq a\|x-y\| + b[\|x-Tx\| + \|y-Ty\|] + c[\|x-Ty\| + \|Tx-y\|] \tag{4}$$

For all $x \in K_1$ and $y \in K_2$ where $0 \leq a,b \leq 1$. It is clear that if T belongs to the class $D(k,0,0)$ with $0<k<1$. Then T is cyclic contraction.

Definition 1.8. *A function $\psi : [0,\infty)^2 \to [0,\infty)$ is called an altering distance function if the following properties are satisfied:*

(i) ψ *is non-decreasing and continuous*

(ii) $\psi(t) = 0$ *if and only if* $t = 0$

Definition 1.9. *An ultra altering distance function is a continuous, non decreasing mapping* $\varphi : [0, \infty) \to [0, \infty)$ *such that* $\varphi(t) > 0, t > 0$ *and* $\varphi(0) \geq 0$. *We denote this set with* Φ_u.

Definition 1.10. *A mapping* $f : [0, \infty)^2 \to [0, \infty)$ *is called cone* $C-$*class function if it is continuous and satisfies following axioms:*

(i). $F(s, t) < s$;

(ii). $F(s, t) = s$ *implies that either* $s = 0$ *or* $t = 0$; *for all* $s, t \in P$.

We denote $C-$*class functions as* $\mathcal{C}$.

Example 1.11. *The following functions* $F : [0, \infty)^2 \to [0, \infty)$ *are elements of* C, *for all* $s, t \in [0, \infty)$:

(i) $F(s, t) = s - t$

(ii) $F(s, t) = ks$, *where* $0 < k < 1$,

(iii) $F(s, t) = s\beta(s)$, *where* $\beta : [0, \infty) \to [0, 1)$,

(iv) $F(s, t) = \psi(s)$, *where* $\psi : [0, \infty) \to [0, \infty), \psi(0) = 0, \psi(s) > 0$ *for all* $s \in [0, \infty)$ *with* $s \neq 0$ *and* $\psi(s) \leq s$ *for all* $s \in [0, \infty)$

(v) $F(s, t) = s - \varphi(s)$, *where* $\varphi : [0, \infty) \to [0, \infty)$ *is a continuous function such that* $\varphi(t) = 0 \ \Leftrightarrow t = 0$;

(vi) $F(s, t) = s - h(s, t)$, *whereh* $: [0, \infty) \times [0, \infty) \to [0, \infty)$ *is a continuous function such that* $h(s, t) = 0 \Leftrightarrow t = 0$ *for all* $t, s > 0$

(vii) $F(s, t) = \varphi(s), F(s, t) = s \Rightarrow s = 0$, *here* $\varphi : [0, \infty) \to [0, \infty)$ *is a upper semi continuous function such that* $\varphi(0) = 0$ *and* $\varphi(t) < t$ *for* $t > 0$

Lemma 1.12. *Let* ψ *and* φ *are altering distance and ultra altering distance functions respectively,* $F \in \mathcal{C}$ *and* $\{s_n\}$ *a decreasing sequence in* R *such that*

$$\psi(s_(n+1)) \leq F(\psi(s_n), \varphi(s_n))$$

For all $n \geq 1$. *Then* $\lim_{n \to \infty} s_n = 0$.

2. Main Results

Proposition 2.1. *Let* K_1 *and* K_2 *be closed subsets of a Banach space* X. *an operator* $T : K_1 \cup K_2 \to K_1 \cup K_2$ *with* $T(K_1) \subset K_2$ *and* $T(K_2) \subset K_1$ *and satisfies*

$$\|Tx - Ty\| \leq a\|x - y\| + b[\|x - Tx\| + \|y - Ty\|] + c[\|x - Ty\| + \|Tx - y\|]$$

with $0 \leq a, b, c < 1$ *and* $a + 2b + 2c < 1$. *If* $F(T) = \{x \in K_1 \cup K_2 Tx = x\} \neq \phi$, *then* $F(T)$ *consists of a single point.*

Proof. Assume the contrary, that is, let $a, w \in K_1 \cup K_2$ be two distinct fixed points of T, Then

$$\|z-w\| = \|Tz-Tw\| \le a\|z-w\| + b[\|z-Tz\| + \|w-Tw\|] + c[\|z-Tw\| + \|Tz-w\|] = (a+2c)\|z-w\|$$

Which implies $z = w$, since $a + 2c < 1$. □

Theorem 2.2. *Let K_1 and K_2 be closed subsets of a Banach space X. Suppose that the operator $T : K_1 \cup K_2 \to K_1 \cup K_2$ with $T(K_1) \subset K_2 and T(K_2) \subset K_1$ and satisfies*

$$\begin{aligned} \psi(\|Tx-Ty\|) \le F(\psi(a\|x-y\| + b[\|x-Tx\| + \|y-Ty\|] + c[\|x-Ty\| + \|Tx-y\|]), \\ \varphi(a\|x-y\| + b[\|x-Tx\| + \|y-Ty\|] + c[\|x-Ty\| + \|Tx-y\|])) \end{aligned} \tag{5}$$

with $0 \le a, b, c < 1$ and $a + 2b + 3c \le 1$. ψ and φ are altering distance and ultra altering distance functions respectively, $F \in \mathcal{C}$ such that $\psi(t+s) \le \psi(t) + \psi(s)$. Then the sequence $\{x_n\}$ in $K_1 \cup K_2$ satisfies $\lim_{n\to\infty} \|x_n - Tx_n\| = 0$ and the sequence $\{x_n\}$ converges to the unique fixed point of T.

Proof. Let $x_0 \in K_1$. Define $x_n = Tx_{n-1} = T^n x_0$. Then By (5) implies that

$$\begin{aligned} \psi(\|x_n - x_{n+1}\|) &= \psi(\|Tx_{n-1} - Tx_n\|) \\ &\le F(\psi(a\|x_{n-1} - x_n\| + b[\|x_{n-1} - Tx_{n-1}\| + \|x_n - Tx_n\|] \\ &+ c[\|x_{n-1} - Tx_n\| + \|Tx_{n-1} - x_n\|]), \varphi(a\|x_{n-1} - x_n\| + b[\|x_{n-1} - Tx_{n-1}\| + \|x_n - Tx_n\|] \\ &+ c[\|x_{n-1} - Tx_n\| + \|Tx_{n-1} - x_n\|])) \\ &\le F(\psi(a\|x_{n-1} - x_n\| + b[\|x_{n-1} - x_n\| + \|x_n - x_{n+1}\|] \\ &+ c[\|x_{n-1} - x_{n+1}\| + \|x_n - x_{n-1}\|]), \varphi(a\|x_{n-1} - x_n\| + b[\|x_{n-1} - x_n\| + \|x_n - x_{n+1}\|] \\ &+ c[\|x_{n-1} - x_{n+1}\| + \|x_n - x_{n-1}\|])) \\ &\le \psi((a+b+2c)\|x_{n-1} - x_n\| + (b+c)\|x_n - x_{n+1}\|) \\ \Rightarrow \big||x_n - x_{n+1}|\big| &\le h\|x_{n-1} - x_n|| \end{aligned} \tag{6}$$

where $h = \frac{a+b+2c}{1-b-c}$. Implies that the sequence $\{\big||x_{n+1} - x_n|\big|\}$ is monotonic decreasing and continuous. There exists a real number, say $r \ge 0$ such that

$$\lim_{n\to\infty} \big||x_{n+1} - x_n|\big| = r,$$

As $n \to \infty$, equation (6) implies,

$$\psi(r) \le F(\psi(r), \varphi(r))$$

So, $\psi(r) = 0$ or $\varphi(r) = 0$ which is only possible if $r = 0$. Thus

$$\lim_{n\to\infty} \big||x_{n+1} - x_n|\big| = 0$$

Claim: $\{x_n\}$ is a Cauchy sequence.

Suppose $\{x_n\}$ is not a Cauchy sequence. There exists an $\epsilon > 0$ and sub sequence $\{n_i\}$ and $\{m_i\}$ such that $m_i < n_i < m_{n+1}$; $\|x_{m_i} - x_{n_i}\| \ge \epsilon$ and $\|x_{m_i} - x_{n_i}\| \le \epsilon$

$$\epsilon \le \|x_{m_i} - x_{n_i}\| \le \|x_{m_i} - x_{n_{i-1}}\| + \|x_{n_{i-1}} - x_{n-i}\|$$

Therefore $\lim_{i\to\infty} \|x_{m_i} - x_{n_i}\| = \epsilon$. Now

$$\begin{aligned}\epsilon &\leq \|x_{m_{i-1}} - x_{n_{i-1}}\| \\ &\leq \|x_{m_{i-1}} - x_{m_i}\| + \|x_{m_i} - x_{n_{i-1}}\|\end{aligned}$$

by taking $\lim_{i\to\infty}$ we get, $\lim_{i\to\infty} \|x_{m_{i-1}} - x_{n_{i-1}}\| = \epsilon$.

$$\begin{aligned}\psi(\epsilon) \leq \psi(\|x_{m_i} - x_{n_i}\|) &= \psi(\|Tx_{m_i} - Tx_{n_i}\|) \\ &\leq F\Big(\psi\big(\lambda(x_{m_i}, x_{n_i})\big), \varphi\big(\lambda(x_{m_i}, x_{n_i})\big)\Big) \qquad (7)\end{aligned}$$

Where,

$$\begin{aligned}\lambda(x_{m_i}, x_{n_i}) &\leq a\|x_{m_i} - x_{n_i}\| + b[\|x_{m_i} - Tx_{m_i}\| + \|x_{n_i} - Tx_{n_i}\|] + c[\|x_{m_i} - Tx_{n_i}\| + \|Tx_{m_i} - x_{n_i}\|] \\ &\leq a\|x_{m_i} - x_{n_i}\| + b[\|x_{m_i} - x_{m_{i-1}}\| + \|x_{n_i} - x_{n_{i-1}}\|] + c[\|x_{m_i} - x_{n_{i-1}}\| + \|x_{m_{i-1}} - x_{n_i}\|]\end{aligned}$$

Taking limit as $i \to \infty$, we get $\lim_{i\to\infty} \lambda(x_{m_i}, x_{n_i}) \leq a\epsilon + b(0+0) + c(\epsilon+\epsilon) \Rightarrow \lim_{i\to\infty} \lambda(x_{m_i}, x_{n_i}) \leq \epsilon$. From (6) we get, $\psi(\epsilon) \leq F(\psi(\epsilon), \varphi(\epsilon))$ so, $\psi(\epsilon) = 0$ or $\varphi(\epsilon) = 0$. This is a contradiction because > 0. Therefore $\{x_n\}$ is Cauchy sequence in X. Since $K_1 \cup K_2$ is complete, then it converges to a limit, say $z \in K_1 \cup K_2$. That is $\lim_{n\to\infty} Tx_n = z$. Note that the subsequence $\{x_n\} \in K_1$ and the subsequence $\{x_{n+1}\} \in K_2$ thus $z \in K_1 \cup K_2 \neq \phi$. Then we employ the triangle inequality and the fact that $a + 2b + 3c < 1$ to get

$$\begin{aligned}\psi(\|z - Tz\|) &= \psi(\|Tz - Tx_n\|) \\ &\leq F(\psi(a\|z - x_n\| + b[\|z - Tz\| + \|x_n - Tx_n\|] + c[\|z - Tx_n\| + \|Tz - x_n\|]), \\ &\varphi(a\|z - x_n\| + b[\|z - Tz\| + \|x_n - Tx_n\|] + c[\|z - Tx_n\| + \|Tz - x_n\|])) \\ &\leq F(\psi((a+2c)\|z - x_n\| + (b+c)\|x_n - Tx_n\| + (b + c(\|z - Tz\|)), \\ &\varphi((a+2c)\|z - x_n\| + (b+c)\|x_n - Tx_n\| + (b + c(\|z - Tz\|)))\end{aligned}$$

It follows from $\lim_{n\to\infty} x_n = z$ and $\lim n \to \infty(Tx_n - x_n) = 0$ that z is the fixed point of T which is unique by the Proposition 3.1. □

Corollary 2.3. *Let K_1 and K_2 be closed subsets of a Banach space X. Suppose that the operator $T : K_1 \cup K_2 \to K_1 \cup K_2$ with $T(K_1) \subset K_2$ and $T(K_2) \subset K_1$ and satisfies*

$$\psi(\|Tx - Ty\|) \leq F(\psi(a\|x - y\| + b[\|x - Tx\| + \|y - Ty\|]), \varphi(a\|x - y\| + b[\|x - Tx\| + \|y - Ty\|])$$

with $0 \leq a, b < 1$ and $a + 2b = 1$. ψ and φ are altering distance and ultra altering distance functions respectively, $F \in \mathcal{C}$ such that $\psi(t+s) \leq \psi(t) + \psi(s)$. Then the sequence $\{x_n\}$ in $K_1 \cup K_2$ satisfies $\lim_{n\to\infty}(x_n - Tx_n) = 0$ and the sequence $\{x_n\}$ converges to the unique fixed point of T.

Proof. The proof of corollary follows immediate, by taking $c = 0$ in the above theorem. □

Theorem 2.4. *Let K_1 and K_2 be closed subsets of a Banach space X. Suppose that the operator $T : K_1 \cup K_2 \to K_1 \cup K_2$ with $T(K_1) \subset K_2$ and $T(K_2) \subset K_1$ and satisfies*

$$\psi(\|Tx - Ty\|) \leq F(\psi(a\|x - y\| + b[\|x - Tx\| + \|y - Ty\|] + c[\|x - Ty\| + \|Tx - y\|]),$$
$$\varphi(a\|x - y\| + b[\|x - Tx\| + \|y - Ty\|] + c[\|x - Ty\| + \|Tx - y\|]))$$

with $0 \leq a, b, c < 1$ and $a + 2b + 3c < 1$. Then

(i) T has a unique fixed point p in $K_1 \cap K_2$.

(ii) $\|Tx - p\| < \|x - p\|$ for all $x \in K_1 \cup K_2$ where p is the fixed point of T.

Proof.

(i) Take a point $x_0 \in K_1$. Define $x_{n+1} = Tx_n \; for \; n = 0, 1, 2, \dots$ then we have

$$\psi(\|x_{n+1} - Tx_{n+1}\|) = \psi(\|Tx_n - Tx_{n+1}\|)$$
$$\leq F(\psi(a\|x_n - x_{n+1}\| + b[\|x_n - Tx_n\| + \|x_{n+1} - Tx_{n+1}\|]$$
$$+ c[\|x_n - Tx_{n+1}\| + \|Tx_n - x_{n+1}\|]), \varphi(a\|x_n - x_{n+1}\| + b[\|x_n - Tx_n\| + \|x_{n+1} - Tx_{n+1}\|]$$
$$+ c[\|x_n - Tx_{n+1}\| + \|Tx_n - x_{n+1}\|]))$$

This inequality implies

$$\leq F(\psi(a\|x_n - x_{n+1}\| + b[\|x_n - Tx_n\| + \|x_{n+1} - Tx_{n+1}\|] + c[\|x_n - Tx_{n+1}\|$$
$$+ \|Tx_n - x_{n+1}\|]), \varphi(a\|x_n - x_{n+1}\| + b[\|x_n - Tx_n\| + \|x_{n+1} - Tx_{n+1}\|]$$
$$+ c[\|x_n - Tx_{n+1}\| + \|Tx_n - x_{n+1}\|]))$$
$$\psi(\|x_{n+1} - Tx_{n+1}\|) \leq F(\psi((a + b + c)\|x_n - Tx_{n+1}\| + (b + c)\|x_{n+1} - Tx_{n+1}\|),$$
$$\varphi((a + b + c)\|x_n - Tx_n\| + (b + c)\|x_{n+1} - Tx_{n+1}\|))$$

Hence, we obtain $\|x_n - Tx_n\| \to 0$ as $n \to \infty$, implying that $\lim_{n\to\infty} x_n = p$, where p is the fixed point of T. Since the subsequence $\{x_{2n}\} \in K_1$ and the subsequence $\{x_{2n+1}\} \in K_2$, then $p \in K_1 \cap K_2$. The uniqueness follows from Proposition 3.1.

(ii) Let p be the fixed point of T and $x \in K_1 \cap K_2$. Then using (4) and the triangle inequality, we have

$$\psi(\|Tx - p\|) \leq \psi(\|Tx - Tp\| + \|Tp - p\|)$$
$$\leq F(\psi(a\|x - p\| + b[\|x - Tx\| + \|p - Tp\|] + c[\|x - Tp\| + \|Tx - p\|]),$$
$$\varphi(a\|x - p\| + b[\|x - Tx\| + \|p - Tp\|] + c[\|x - Tp\| + \|Tx - p\|]))$$
$$\leq F(\psi(a\|x - p\| + b[\|x - p\| + \|p - Tx\|] + c[\|x - p\| + \|Tx - p\|]),$$
$$\varphi(a\|x - p\| + b[\|x - p\| + \|p - Tx\|] + c[\|x - p\| + \|Tx - p\|]))$$

The inequality implies

$$\leq F(\psi((a + b + c)\|x - p\| + (b + c)\|Tx - p\|), \varphi((a + b + c)\|x - p\| + (b + c)\|Tx - p\|)$$
$$\|Tx - p\| \leq \left(\frac{a + b + c}{1 - b - c}\right)\|x - p\| < \|x - p\|$$

As $\left(\frac{a+b+c}{1-b-c}\right) < 1$, which completes the proof. ∎

Corollary 2.5. *Let K_1 and K_2 be closed subsets of a Banach space X. Suppose that the operator $T : K_1 \cup K_2 \to K_1 \cup K_2$ with $T(K_1) \subset K_2$ and $T(K_2) \subset K_1$ and satisfies*

$$\psi(\|Tx - Ty\|) \leq F(\psi(a\|x - y\| + b[\|x - Tx\| + \|y - Ty\|]), \varphi(a\|x - y\| + b[\|x - Tx\| + \|y - Ty\|]))$$

with $0 \leq a, b < 1$ and $a + 2b < 1$. Then

(i) T has a unique fixed point p in $K_1 \cup K_2$.

(ii) $\|Tx - p\| < \|x - p\|$ for all $x \in K_1 \cup K_2$ where p is the fixed point of T.

Proof. The proof of corollary follows immediate, by taking $c = 0$ in the above theorem. □

3. Conclusion

We have proved some fixed point theorems for cyclic mapping with contractive and expansive conditions $\varphi - \psi-$contractive mappings in Banach space. The presented results generalize the results proved in various spaces and extend some results from the literature.

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