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Reliability of Insulin administered to Diabetic Patients

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Abstract: This paper works a new approach to estimate the Reliability of Insulin administered to diabetic patients. The aim of this work is to maintain Normoglycemia of diabetic patients by determining the Reliability of the insulin injected daily to them. We proposed a Mathematical model for the Type 1 Diabetic Mellitus Patient. The model takes into account Glucose concentration and Insulin concentration. The solutions arrived by solving the differential equations arising in our model, represent the variation of glucose and insulin concentration in diabetic patients and hence from these solutions the Reliability of Insulin, Mean Time to Failure and Hazard Rate are found.

Keywords: Diabetic Mellitus, Normoglycemia, Juvenile diabetes, Hyperglycemia, Insulin.

1. Introduction

Diabetes comes from Greek, and it means a “siphon”. In 1675, Thomas Wills added mellitus to the term, Mel in Latin means “honey”. Diabetes Mellitus could literally mean “siphoning off sweet water”. The chronic metabolic disorder diabetic mellitus is sweeping the globe as a silent epidemic largely contributing to the growing burden of non-communicable diseases and mainly encouraged by decreasing levels of activity and increasing prevalence of obesity. It is estimated that in 2010 there were globally 285 million people suffering from this disease. This number is estimated to increase to 430 million in the absence of better control or cure. In 2013, it was estimated that over 382 million people throughout the world had diabetes. Also approximately 10% of all the diabetic cases are type 1-IDDM-Insulin dependent. As a need, this paper analyses the reliability of Insulin that is injected using a Mathematical model.

2. Diabetes

Diabetes, often referred to by doctors as Diabetes Mellitus, describes a group of metabolic diseases in which the person has high blood glucose, either because insulin production is inadequate or because the body’s cells do not respond properly to insulin, or both. Normally, blood glucose is at 64.8 to 104.4 mg/dl. But it is important to note that blood glucose level is lowest in the morning before the first meal is taken. After two or three hours of a meal, blood glucose will elevate depending on what kind of food was taken. For a normal person without diabetes, blood sugar levels reach as high as 175 mg/dl after a meal is taken. This level will return to normal rate after sometime. The time consumed for the blood sugar level to return to normal rate is faster in normal healthy person than in person with diabetes.

Types of Diabetes: There are three types of Diabetes Mellitus.

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(1). Type 1

(2). Type 2

(3). Gestational Diabetes

Type 1: Type 1 diabetes is one of the most challenging medical disorder because of the demands it imposes on day-to-day life. It was formerly known as insulin dependent diabetes mellitus (IDDM) or Juvenile onset diabetes mellitus. Between 2001 and 2009, the prevalence of type 1 diabetes among the under 20s in the USA rose 23%, according to SEARCH for Diabetes in Youth data issued by the CDC (Centers for Disease Control and Prevention. In this type of diabetes, the pancreas undergoes an autoimmune attack by the body itself and is rendered incapable of making insulin. It is an autoimmune disorder, in which body's own immune system attacks the beta cells in the islets of Langerhans of the Pancreas destroying them or damaging them sufficiently to reduce insulin production. As a result, Patients with type 1 diabetes will need to take insulin injections for the rest of their life. They must also ensure proper blood-glucose levels by carrying out regular blood tests and following a special diet.

Symptoms: The early symptoms of diabetes are related to elevated blood sugar levels in the body and loss of glucose in the urine. Common sign and symptoms may include

- Thirstiness
- Urinating often
- Feeling very hungry or tired
- Losing weight
- Repeated or slow healing infections
- Having dry, itchy skin
- Extreme fatigue
- Blurred vision
- Tingling or loss of feeling in the hands or feet

2.1. Insulin

Insulin is a hormone, normally produced in sufficient amounts by the healthy human pancreas. Its role is to facilitate the final digestion of glucose by the human body. When insulin is lacking (diabetes), glucose remains undigested in the blood. The body isn't being fed, and the high blood sugar can cause damage, the ramifications of diabetes. Since the pancreas of the type 1 diabetic has ceased producing insulin, and insulin must be injected. This injected insulin is a replacement, and barring the transplantation of a healthy pancreas, will need to be continued for life. Insulin cannot be taken by mouth because it is digestible. Oral insulin would be obliterated in the stomach, long before it reached the bloodstream where it is needed. Once injected, it starts to work and is used up in a matter of hours. Depending on a number of factors, individuals vary insulin volume, type, and frequency, to optimize blood glucose management.

2.2. Types of Insulin

Rapid-Acting insulin: It starts working more quickly than other types of insulin. It begins working within 15 minutes and leaves your body after 3 to 5 hours. This includes such as insulin lispro, insulin aspart and insulin glulisine.

Short-Acting insulin: This is the regular insulin which starts working in 30 to 60 minutes and lasts 5 to 8 hours.

Intermediate-Acting insulin: This include such as NPH which starts working in 1 to 3 hours and lasts 12 to 16 hours.

Long-lasting insulin: Insulin such as insulin glargine and insulin detemir that starts working in about 1 hour and lasts 20 to 26 hours.

Premixed insulin: This is a combination of 2 types insulin (usually a rapid or short acting insulin and an intermediate acting insulin).

Insulin reaction: Hypoglycemia is the name for a condition in which the level of sugar in the blood is too low. Most people who take insulin have insulin reactions at some time. Signs of an insulin reaction and hypoglycemia include the following:

- Feeling very tired
- Yawning frequently
- Being unable to think or speak clearly
- Losing muscle coordination
- Sweating
- Twitching
- Having a seizure
- Becoming very pale
- Losing consciousness

People who have diabetes should carry atleast 15 grams of a fast-acting carbohydrate with them at all times in case of hypoglycemia or an insulin reaction.

3. Mathematical Model

The model is determined by the following considerations:

(1). The food that we take gives glucose to the digestive system and from there it goes to blood. The glucose from blood can also go to the digestive system when blood circulates through it.

(2). Surplus glucose from blood is stored in liver as glycogen and when, blood needs glucose, it can be released from the liver and given to the blood compartment.

(3). Pancreas gives insulin to blood. This insulin is necessary for metabolizing glucose for useful work in tissues.

(4). Glucose can be directly injected into the digestive system or blood. Insulin can also be injected into the blood stream.

Let c_g and c_i be the excess of concentration of glucose and insulin, respectively, at time t over their equilibrium values. Then

$$\frac{dc_g}{dt} = -m_1 c_g - m_2 c_i + G(t)$$

$$\frac{dc_i}{dt} = -m_3 c_i - m_4 c_g + I(t)$$

where $m_1, m_2, m_3, m_4 > 0$. By eliminating step by step, c_g, c_i we get

$$\frac{d^2 c_g}{dt^2} + 2\alpha \frac{dc_g}{dt} + \omega_0^2 c_g = S_1(t)$$
$$\frac{d^2 c_i}{dt^2} + 2\alpha \frac{dc_i}{dt} + \omega_0^2 c_i = S_2(t)$$

where

$$2\alpha = m_1 + m_3$$
$$\omega_0^2 = m_1 m_3 + m_2 m_4$$
$$S_1(t) = m_3 G - m_2 I + \frac{dG}{dt}$$
$$S_2(t) = m_1 I + m_4 G + \frac{dI}{dt}$$

The solutions are given by

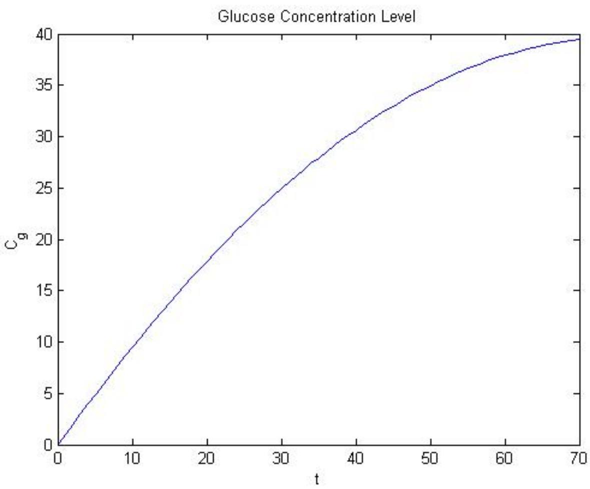
$$c_g = e^{-\alpha t}(A_1 \cos \omega t + A_2 \sin \omega t) + \frac{e^{-\alpha t}}{\omega} \left[\sin \omega t \int_0^t e^{\alpha t} \cos(\omega t) S_1(t) dt - \cos \omega t \int_0^t e^{\alpha t} \sin(\omega t) S_!(t) dt \right]$$
$$c_i = e^{-\alpha t}(A_3 \cos \omega t + A_4 \sin \omega t) + \frac{e^{-\alpha t}}{\omega} \left[\sin \omega t \int_0^t e^{-\alpha t} \cos(\omega t) S_2(t) dt - \cos \omega t \int_0^t e^{-\alpha t} \sin(\omega t) S_2(t) dt \right]$$

where $\omega^2 = \omega_0^2 - \alpha^2$. If $t = 0$, then $c_g = 0$, so that $A_1 = 0$. And on integrating, $A_2 = 0$. This leads to the solution

$$c_g = \frac{1}{\omega} \left(e^{-\alpha t} \sin \omega t \right)$$

Hence the excess glucose concentration is $c_g = \frac{1}{\omega} \left(e^{-\alpha t} \sin \omega t \right)$. In the case of a diabetic patient, it takes a longer time to recover to equilibrium state. If $\alpha = 0.0048$ and $\omega_0^2 = 0.0003$ then $\omega = \sqrt{\omega_0^2 - \alpha^2} = 0.0166$. The following table represents the variation of glucose concentration which is shown in the graph below:

No	t	$e^{-\alpha t}$	$\sin \omega t$	$c_g = \frac{1}{\omega} \left(e^{-\alpha t} \sin \omega t \right)$
1.	0	1	0	0
2.	4	0.9810	0.0664	3.924
3.	8	0.9623	0.1324	7.6752
4.	12	0.9440	0.1979	11.254
5.	16	0.9261	0.2625	14.645



Where t represents the time, c_g represents the glucose concentration level. As the excess concentration of glucose follows a exponential distribution, the amount of insulin to be injected also follows exponential distribution. Since the number of times, the insulin injection to be injected depends on the level of glucose, the function I(t) may be considered as

$$I(t) = \frac{\rho}{\bar{t} - t_0} e^{-\frac{\rho t}{\bar{t} - t_0}}, t \geq t_0$$

where ρ is the quantity of injection, $\bar{t}$ is the time lag to maximum, t_0-time of injection. To find the Reliability,

$$R(t) = e^{-\lambda t} = e^{-\frac{\rho t}{\bar{t} - t_0}}$$

Probability of failure $F(t) = 1 - R(t) = 1 - e^{-\frac{\rho t}{\bar{t} - t_0}}$. Mean time to Failure is given by

$$\begin{aligned} MTTF &= \int_0^\infty R(t)dt \\ &= \int_0^\infty e^{-\frac{\rho t}{\bar{t} - t_0}} dt \\ &= \frac{\bar{t} - t_0}{\rho t} \\ &= \frac{1}{\lambda} \end{aligned}$$

Hazard Rate:

$$\lambda(t) = \frac{-1}{R(t)} \frac{dR(t)}{dt} = \frac{\rho}{\bar{t} - t_0} = \frac{1}{MTTF}.$$

4. Conclusion

In this Paper, we deal with finding solutions to determine the Glucose and Insulin concentration and based on these solutions we study the variation of Glucose and Insulin concentration in the case of diabetic patients. Also, we determine the Reliability, Mean Time to Failure and Hazard Rate which helps the Health Care Professionals to take certain decisions while treating the diabetic patients.

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