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Reliability and Availability of Systems in Real Life

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Abstract: In this Paper, we focus our attention to deal with Systems involving Mixed Series and Parallel Configurations and the System Reliability of some Repairable Models involving Series and Parallel subunits existing in real life are studied with the aid of Markov method. We discuss two models in our study namely PP-model and PSP-model. Also we calculate the Availability of the System. We apply this technique to Railway Tracks involving Series and Parallel connections. We determine their Availability and the knowledge of which helps in the proper maintenance of the Tracks in the Railway Department.

Keywords: Reliability, Availability, Series Configuration, Parallel Configuration, Mixed Configurations and Markov Method.

1. Introduction

Reliability Analysis is based on repairs and failures of a System. The theory of Reliability is a science which studies the laws of occurrence of failures in technical equipments and complex Systems. The System is broken down to subsystems and elements whose individual Reliability factors can be estimated or determined. Depending on the manner in which these subsystems and elements are connected to constitute the given System, we calculate the System Reliability. The calculation of the Reliability with System exhibiting dependent failures and involving repairs or stand by operations is, in general complicated and several approaches have been suggested to carry out the computations. A technique that has much appeal and works well when failure hazards and repair hazards are constant requires the use of Markov models [5].

In Probability theory, a Markov model is a Stochastic model used to model randomly changing System, where it is assumed that future state depends only on the current state not on the events that occurred before. It is a Stochastic model which models temporal or sequential data. It provides a way to model the dependence of current information with previous information. The term Markov model originally referred exclusively to mathematical model in which future state of a system depends only on its current state and not on its past history. This memoryless characteristic called the Markovian property implies that transition from one state to another occur at constant rates. Markov models is a modelling technique that is widely used in the Reliability Analysis of Complex system. It is very flexible in the type of System and System behaviour it can model. This modelling is very helpful in most of the Real life situations. The model is quite helpful to model operation system with dependent failure and repair models. In fact it is widely used to perform Reliability and Availability analysis of responsible System with constant failure and repair rates. From time to time, the Markov models is also used to perform Human Reliability Analysis [6].

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The rest of the paper is organized as follows, in Section 2, we present the basic definitions and we discuss about Markov model. In Section 3, we deal with two different models. In Section 4 we apply these two models with problems connected to railway tracks where the tracks have been laid with Series and Parallel Connections in between. Finally Conclusion is drawn in section 5.

2. Definitions

Reliability is the probability of a device or system element performing its intended function under stated conditions without failure for given period of time.

Definition 2.1. *Reliability is defined by*

$$\begin{aligned} R(t) &= 1 - F(t) \\ &= 1 - \int_{-\infty}^{t} f(x)dx \end{aligned}$$

or

$$R(t) = \int_{t}^{\infty} f(x)dx$$

or

$$R(t) = e^{-\int_{0}^{t} \lambda(t)dt}$$

where $R(t)$ is the Reliability at time t.

Definition 2.2 (Availability)**.** *Probability that a repairable system or system is operational at a given point in time under a given set of environmental conditions. Availability depends on Reliability and maintainability.*

Definition 2.3 (Series Configuration)**.** *It is the simplest and probably the most commonly occurring or assumed configurations in reliability evaluation of engineering systems. The success of the system depends on the success of all its elements. If any one of the elements fails, the system fails. Figure 1 shows the block diagram of a series system.*

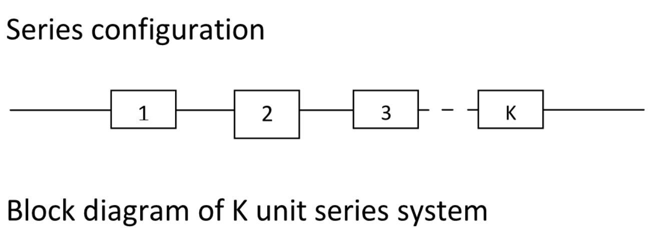

Figure 1.

In series system reliability is expressed by

$$R(S) = \prod_{i=1}^{n} P(X_i) \tag{1}$$

Definition 2.4 (Parallel Configuration)**.** *In this case, all units are active and at least one unit must perform successfully for the system success. Figure 2 shows the block diagram of a parallel system, each block represents a unit.*

In parallel system reliability is expressed by

$$R(S) = 1 - \prod_{i=1}^{n}(1 - P(X_i)) \tag{2}$$

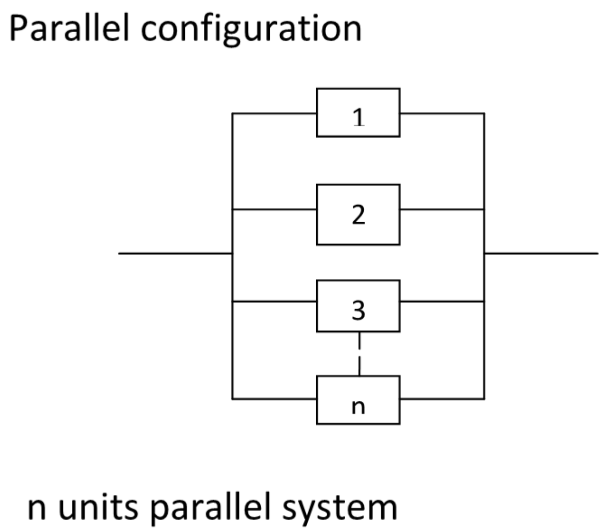

Figure 2.

Definition 2.5 (General Series-Parallel Configuration)**.** *The System consists of stage 1, stage 2,. . . , stage k connected in series. Each stage contains a number of redundant elements, stage 1 consisting of n_1 redundant elements connected in parallel. The reliability of the system is the product of the reliabilities of each stage. Stage i with n_i elements will have the reliability The Reliability of the General Series Parallel System is given by*

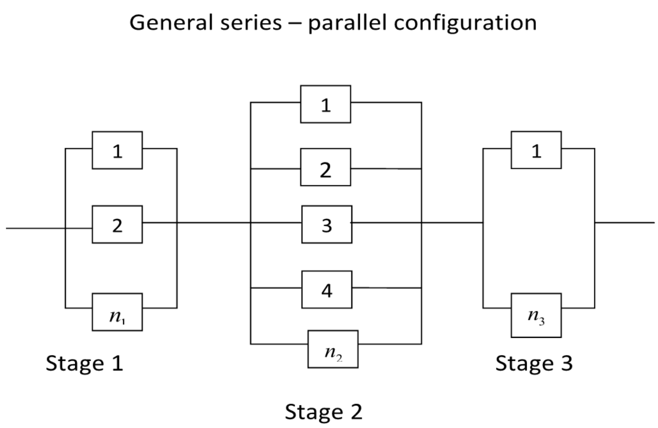

Figure 3.

$$R(S) = \prod_{i=1}^{k} \left[\left[1 - \prod_{j=1}^{n_k} (1 - P(X_{ij})) \right] \right] \tag{3}$$

3. Markov Models in Mixed Configuration System

Markov is widely used method to evaluate Reliability of Engineering system. The method is particularly useful to handle repairable systems with dependent failure and repair models. The method is subject to the following three assumptions.

- The probability of transition from one system state to another in the finite time interval Δt and $\alpha\Delta t$, where α is the transition rate (i.e., failure rate or repair rate)from one state to another.
- All occurrences are independent of one another,

- The transitional probability of two or more occurrences in time interval Δt from one system state to another is negligible (e.g $(\alpha\Delta t)(\alpha\Delta t) \to 0$).

For any given system, a Markov model consists of list of the possible states of that system, the possible transition paths between those states, and the rate parameters of those transitions. In Reliability Analysis the transitions usually consists of failures and repairs. When representing a Markov model graphically, each state is usually depicted as a "bubble" with arrows denoting the transition paths between states as depicted in the figure below for a single component that has just two states :normal and failed.

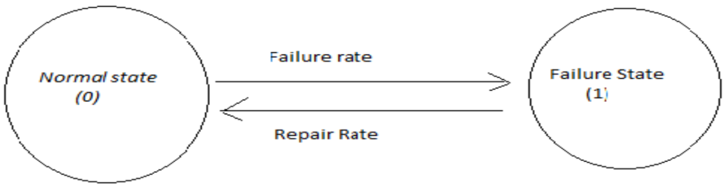

Figure 4. **Markov Model of a simple system**

$$P_0(t+\Delta t) = P_0(t)[1-\lambda_x \Delta t] + P_1(t)\mu_x \Delta t \tag{4}$$

$$P_1(t+\Delta t) = P_0(t)\lambda_x \Delta t + P_1(t)[1-\mu_x \Delta t] \tag{5}$$

The above equations can be reformulated as

$$\frac{dP_0(t)}{dt} = -P_0(t)\lambda_x + P_1(t)\mu_x \tag{6}$$

$$\frac{dP_1(t)}{dt} = -P_1(t)\mu_x + P_0(t)\lambda_x \tag{7}$$

Taking the limit as Δtapproaches to zero , assuming the initial conditions $P_0(0) = 1$; $P_1(0)$=0 and solving equations (3) and (4) we get

$$P_0(t) = \frac{\mu_x}{(\lambda_x+\mu_x)} - \frac{\lambda_x}{(\lambda_x+\mu_x)} e^{-(\lambda_x+\mu_x)t} \tag{8}$$

$$P_1(t) = \frac{\lambda_x}{(\lambda_x+\mu_x)} - \frac{\lambda_x}{(\lambda_x+\mu_x)} e^{-(\lambda_x+\mu_x)t} \tag{9}$$

where $P_0(t)$ and $P_1(t)$ denote the probabilities of the system in the normal and failed state.

4. Models of Mixed Configurations

In this section we present two different models of mixed configurations and analyze its Reliability through combinatorial rules of probability approach. Also we determine the Availability of the system with known failure and repair rates for the base events with the aid of Markov method.

4.1. Model 1: PP model

Consider a system with 2 Units connected in Parallel, each Unit has 2 subunits connected in Parallel. All the units involved in this system are independent and the base event probabilities are known. Using equation 1 we get the Reliability of a Parallel-Parallel (PP) Configuration System is given by

$$R(S) = 1 - \prod_{i=1}^{2}\left[1-\left[1-\prod_{j=1}^{2}(1-P(X_{ij}))\right]\right] \tag{10}$$

where

$$P_{ij} = \left(\frac{\mu_{ij}}{\lambda_{ij}+\mu_{ij}} + \frac{\lambda_{ij}}{\lambda_{ij}+\mu_{ij}} e^{-(\lambda_{ij}+\mu_{ij})t}\right) \tag{11}$$

In this formula we get

$$R(t) = (P(X_{11}) + P(X_{12}) - P(X_{11})P(X_{12}))(1 - P(X_{21}) - P(X_{22}) + P(X_{21})P(X_{22})) + (P(X_{21}) + P(X_{22}) - P(X_{21})P(X_{22})) \tag{12}$$

Availability of the system as

$$t \to \infty A(S)_{t\to\infty} = \frac{1}{(\lambda_{11}+\mu_{11})(\lambda_{12}+\mu_{12})(\lambda_{21}+\mu_{21})(\lambda_{22}+\mu_{22})} \left\{ \begin{array}{l} \lambda_{11}\mu_{12}\lambda_{21}\lambda_{22} + (\lambda_{12}+\mu_{12}) \\ \left(\begin{array}{l} \mu_{11}\lambda_{21}\lambda_{22} + \mu_{21}\lambda_{11}\lambda_{22} \\ +\mu_{11}\mu_{21}\lambda_{22} + 2\mu_{11}\mu_{21}\mu_{22} \\ +2\lambda_{11}\mu_{21}\mu_{22} \end{array} \right) \end{array} \right\} \tag{13}$$

4.2. Model 2: PSP Model

Consider a Parallel-Series-Parallel configuration model where the system has 2 units connected in Parallel, each of the 2 units has 2 subunits connected in Series. Each of the Series subunits has 2 units connected in Parallel. All the units involved in this system are independent and the base event probabilities are known. The Reliability of a Parallel -Series - Parallel (PSP) Configuration System is given by

$$R(t) = 1 - \prod_{i=1}^{2}\left[\left(1 - \left[\prod_{j=1}^{2}\left(\left(1 - \prod_{k=1}^{2}(1 - P_{ijk})\right)\right)\right]\right)\right] \tag{14}$$

where

$$P_{ijk} = \left(\frac{\mu_{ijk}}{\lambda_{ijk}+\mu_{ijk}} + \frac{\lambda_{ijk}}{\lambda_{ijk}+\mu_{ijk}} e^{-(\lambda_{ijk}+\mu_{ijk})t}\right) \tag{15}$$

In this formula we get

$$R(t) = \sum_{i=1}^{2}\left[\prod_{j=1}^{2}(P(X_{ij1}) + P(X_{ij2}) - P(X_{ij1})P(X_{ij2}))\right] - \prod_{i=1}^{2}\left[\prod_{j=1}^{2}(P(X_{ij1}) + P(X_{ij2}) - P(X_{ij1})P(X_{ij2}))\right] \tag{16}$$

Availability of the system as $t \to \infty$

$$A(S)_{t\to\infty} = \left[\frac{1}{\prod_{i=1}^{2}\left[\prod_{j=1}^{2}(\lambda_{ij1}+\mu_{ij1})(\lambda_{ij2}+\mu_{ij2})\right]} \left\{ \begin{array}{c} \sum_{i=1}^{2}\left[\prod_{j=1}^{2} \begin{array}{l} (\lambda_{ij1}+\mu_{ij1})(\lambda_{ij2}+\mu_{ij2}) \\ (\mu_{ij1}\lambda_{ij2}+\mu_{ij1}\mu_{ij2}+\lambda_{ij1}\mu_{ij2}) \end{array}\right] \\ - \\ \prod_{i=1}^{2}\left[\prod_{j=1}^{2}(\mu_{ij1}\lambda_{ij2}+\mu_{ij1}\mu_{ij2}+\lambda_{ij1}\mu_{ij2})\right] \end{array} \right\}\right] \tag{17}$$

5. Numerical Examples

Example 5.1 (PP Model)**.** *A train starts at a particular place and it can choose either track 1 or 2 depending on the signal. In track 1, it has two crossings 11 & 12. It can travel in either of the crossings based on the signal. Similarly, the track 2 has two crossings 21 & 22 and it can divert in any of the tracks. The train travels from city A to city B. The probability of the train reaching the destination with this path configuration is based on the failure rate and repair rate in each of the tracks.*

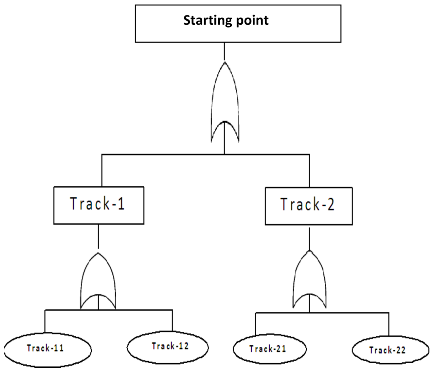

Figure 5.

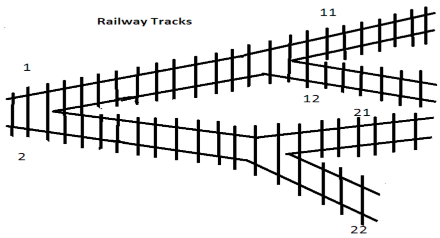

Figure 6.

Consider the value of the failure and repair rates for the base events as $\lambda_{11} = 0.3$; $\lambda_{12} = 0.2$; $\lambda_{21} = 0.4$; $\lambda_{22} = 0.1$; $\mu_{11} = 0.5$; $\mu_{12} = 0.3$; $\mu_{21} = 0.1$; $\mu_{22} = 0.4$. By equation (13), we get the Availability of the System as $A(S)_{t\to\infty} = 0.496$.

Example 5.2 (PSP model)**.** *A train starts at a particular place it can choose either of the tracks 1 or 2 depending on signal. While it choose track 1, the track 1 has two crossings 11 & 12 and the track 2 has two crossings 21 & 22 in which it can travel, further the tracks 11 & 12 has 2 tracks each, that is track 11 can travel in either of the crossings has 111 or 112 based on the signal. Similarly , track 12 has 121 & 122 in which it can travel in either of the tracks. Similarly track 21 has two crossings 211 & 212 and track 22 has two crossings 221 & 222 in which it can travel in either of the tracks. The train travels from city A to city B. The probability of the train reaching the destination with this path configuration is based on the failure rate and repair rate in each of the tracks.*

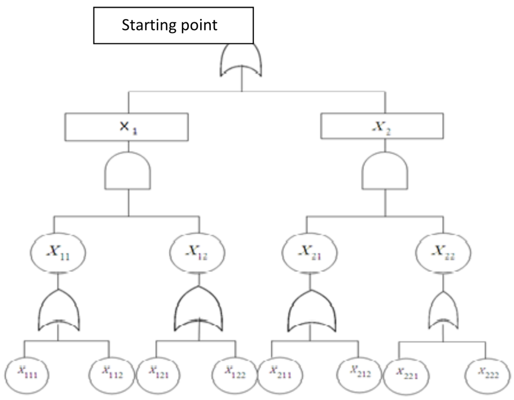

Figure 7.

Consider the values of the failure and repair rates for the base events as $\lambda_{111} = 0.2$; $\lambda_{112} = 0.1$; $\lambda_{121} = 0.4$; $\lambda_{122} = 0.3$;

$\lambda_{211} = 0.5$; $\lambda_{212} = 0.2$; $\lambda_{221} = 0.4$; $\lambda_{222} = 0.1$; $\mu_{111} = 0.4$; $\mu_{112} = 0.1$; $\mu_{121} = 0.3$; $\mu_{122} = 0.4$; $\mu_{211} = 0.1$; $\mu_{212} = 0.5$; $\mu_{221} = 0.3$; $\mu_{222} = 0.2$.

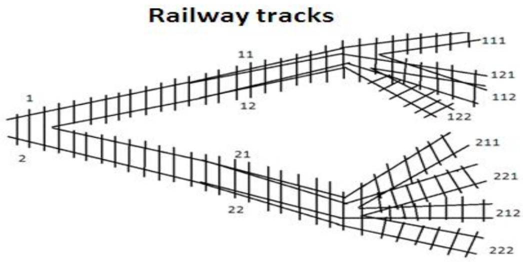

Figure 8.

By equation (17), we get the Availability of the System $A(S)_{t\to\infty} = 0.72$.

6. Conclusion

Markov modelling is a widely used technique in the study of Reliability analysis of the System. In this paper , we focussed our study to make of use of the Markov approach to determine the System Reliability and Availability, for models of mixed series and parallel configurations involving sub units with known failure and repair rates. We apply these models to problems connected to Railway tracks and determine its Availability for the proper maintenance of the tracks in the Railways.

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