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Neutrosohic Fuzzy Magdm Environment Based on Normal Distribution

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Abstract: The attribute weights calculated from completely unknown information using the normal distribution. The neutrosophic fuzzy ordered weighted averaging operator is utilized to aggregate all individual neutrosophic fuzzy decision matrices provided by the decision-makers into the collective neutrosophic fuzzy decision matrix, and then we use the obtained attribute weights from Poisson distribution and the neutrosophic fuzzy hybrid averaging operator to fuse the neutrosophic fuzzy information in the collective neutrosophic fuzzy decision matrix to get the overall neutrosophic fuzzy values of alternatives, and then rank the alternatives, and select the most desirable alternative.

Keywords: Ordered weighted averaging; Hybrid averaging operator; normal based weights.

1. Introduction

Group decision making problems can be roughly defined as decision situations where two or more decision makers or experts try to achieve a common solution to a decision problem, which consists of a set of possible solutions or alternatives [11, 12] in terms of intuitionistic fuzzy sets in Atanassov [1]. Experts must express their individual opinion on each of the alternatives. However, in many situations a problem arises when some experts consider that their individual opinions have not been sufficiently taken into account, and therefore they disagree with the solution achieved. This may lead to either a lack of implication in future group decision making problems or a behaviour against the solution obtained. For this reason, the need for making relevant decisions under consensus is becoming increasingly common in a variety of social situations. The Gaussian distribution method is utilized to support and handle some critical situations in MAGDM problems.

2. MAGDM with Correlation Coefficient of IFS & IVIFS Theory

By correlation analysis, the joint relationship of two variables can be examined with a measure of interdependence of the two variables [2]. Fuzzy correlation has captured the attention of researchers recently. Correlation coefficient of fuzzy sets, interval-valued fuzzy sets, intuitionistic fuzzy sets and interval-valued intuitionistic fuzzy sets are already in literature. Bustince and Burillo [4] discussed that vague sets are intuitionistic fuzzy sets, and they were applied in decision making problem. Chaudhuri and Bhattachary [5] studied on correlation between fuzzy sets. Chiang, and Lin [7] introduced partial correlation of fuzzy sets. Chiclana [8] studied few induced ordered weighted averaging operators for solving group decision-making problems based on fuzzy preference relations. Li [14] analysed multi-attribute decision making models and methods

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using intuitionistic fuzzy sets. Merigo and Gil-Lafuente [17] investigated OWA operators in generalized distances, and they [18] found the induced generalized OWA operator in decision making. Wei [21] discussed Grey relational analysis model for dynamic hybrid multiple attribute decision making. Yager [23] introduced centred OWA operators in decision making. Various attempts were made by researchers in the recent days to define the correlation coefficient of intuitionistic fuzzy sets and interval-valued intuitionistic fuzzy sets. As vague sets deal with truth membership value, false membership value and the vague degree, and have more ability to deal with uncertain information than traditional fuzzy sets [15, 16] many researchers pay attention on vague set theory.

Hung and Wu [10] introduced the concepts of positively and negatively correlated results based on the concept of centroid for intuitionistic fuzzy sets lying in the interval $[-1, 1]$. It is well known that the conventional correlation analysis using probabilities and statistics was inadequate to handle uncertainty of failure data and modeling. The method to measure the correlation between two variables involving fuzziness is a challenge to classical statistical theory. Gernstenkorn and Manko [6], Hong and Hwang [9], and Yu [24] define a different correlation formula to measure the interrelation of intuitionistic fuzzy sets. In their definition, the correlation coefficient lies between 0 and 1, differing from the conventional range of $[-1, 1]$. The work of Wang and Li [14] suffers from the same problem of the correlation lying between 0 and 1, in studying the correlation coefficient of interval valued fuzzy sets. Chiang and Lin [7, 8] took random samples from fuzzy sets, treating membership grades to be crisp observations, to calculate the correlation lying in the interval $[-1, 1]$, where the sense of fuzziness is lost. Park [20] also worked on the correlation coefficient of interval valued intuitionistic fuzzy sets applied in multiple-attribute group decision making problems. Kao and Liu [13] introduced fuzzy measures for the correlation coefficient of fuzzy numbers lying in the interval $[-1, 1]$. Buckley [3] using the approach of fuzzy probabilities, provided a tool which could be utilized for the fuzzy correlation coefficient applications. Consensus processes imply that experts achieve an agreement about the problem before making the decision, thus yielding a more accepted solution by the organization, society or themselves. Different consensus approaches have been proposed ranging from rigid methods to flexible approaches [11, 12]. In these approaches, it is crucial to establish a consensus measure to calculate the level of agreement. Consensus measures are indicators to evaluate how far a group of experts' opinions are from unanimity. Mohanty and Bhasker [19] have applied the concepts of Linguistic Quantifiers in the product classifications by the customer preference in Internet-Business. In this chapter, the method to derive the weights of the experts using Gaussian Distribution based method proposed by Xu [22] is analysed.

3. Solving MAGDM Problems with Gaussian Distribution Function

Let us consider a situation where there is an unfair argument among the experts in fixing the weights in a decision making problem. In that case we need to relieve the influence of unfair arguments on the decision variables. Xu [22] introduced a procedure for generating the OWA weights based on the use of the Gaussian distribution. They are referred as Gaussian weights which are given as follows:

Definition 3.1. *Consider a Gaussian distribution $G(\mu_n, \sigma_n)$, where μ_n is the mean of the collection and σ_n is the deviation of the collection, and given by:*

$$\begin{aligned} \mu_n &= \frac{1}{n}\sum_{j=1}^{n} j = \frac{n+1}{2} \\ \sigma_n &= \sqrt{\frac{1}{n}\sum_{j=1}^{n}(j-\mu_n)^2}. \end{aligned} \tag{1}$$

Let $G(j) = \frac{1}{\sqrt{2\pi}\sigma_n} e^{-(j-\mu_n)^2/2\sigma_n^2}$.

Definition 3.2. *Then the associated OWA weights are defined as:*

$$w_j = \frac{G_j}{\sum\limits_{j=1}^{n} G(j)} = \frac{e^{-(j-\mu_n)^2/2\sigma_n{}^2}}{\sum\limits_{j=1}^{n} e^{-(j-\mu_n)^2/2\sigma_n{}^2}} \tag{2}$$

where $w_j \in [0,1]$ *and* $\sum\limits_j w_j = 1$. *It can be noted that the closer* j *is to* $\mu_n = \frac{n+1}{2}$, *the larger* w_j. *Furthermore, if* n *is odd, the maximal value of* w_j *occurs for* $j = \frac{n+1}{2}$. *If* n *is even, the maximal value of* w_j *occurs for* $j = \frac{n}{2}$ *and* $j = \frac{n}{2}+1$. *It can also be shown that the weighting vector generated using this approach is symmetric, i.e.,* $w_j = w_{n+1-j}$.

Procedures 3.3. *To derive a weight vector* w *by normal distribution method for* $n = 1, 2, \ldots, 5$;

$$\mu_n = \frac{1+n}{2}$$

$$\sigma_n = \sqrt{\frac{1}{n}\sum_i^n (i-\mu_n)^2}$$

$$\sigma_1 = 0$$

$$\sigma_2 = 0.6123$$

$$\sigma_3 = \sqrt{\frac{1}{3}\sum_i^3 (i-\mu_3)^2} = 0.8164$$

$$\sigma_4 = \sqrt{\frac{1}{4}\sum_i^4 (i-\mu_4)^2} = 1.1180$$

$$\sigma_5 = \sqrt{\frac{1}{5}\sum_i^5 (i-\mu_n)^2} = 1.4142$$

Definition 3.4 (OWA Weight)**.**

$$w_j = \frac{e^{-\left[\frac{(j-\mu_n)^2}{2\sigma_n^2}\right]}}{\sum\limits_{i=1}^{n} e^{-\left[\frac{(i-\mu_n)^2}{2\sigma_n^2}\right]}}; \quad i,j = 1,\ldots,5$$

$$w_1 = \frac{e^{-\left[\frac{(1-\mu_5)^2}{2\sigma_5^2}\right]}}{e^{-\left[\frac{(1-\mu_5)^2}{2\sigma_5^2}\right]} + e^{-\left[\frac{(2-\mu_5)^2}{2\sigma_5^2}\right]} + e^{-\left[\frac{(3-\mu_5)^2}{2\sigma_5^2}\right]} + e^{-\left[\frac{(4-\mu_5)^2}{2\sigma_5^2}\right]} + e^{-\left[\frac{(5-\mu_5)^2}{2\sigma_5^2}\right]}}$$

$$= 0.1116$$

$$w_2 = \frac{e^{-\left[\frac{(2-\mu_5)^2}{2\sigma_5^2}\right]}}{e^{-\left[\frac{(1-\mu_5)^2}{2\sigma_5^2}\right]} + e^{-\left[\frac{(2-\mu_5)^2}{2\sigma_5^2}\right]} + e^{-\left[\frac{(3-\mu_5)^2}{2\sigma_5^2}\right]} + e^{-\left[\frac{(4-\mu_5)^2}{2\sigma_5^2}\right]} + e^{-\left[\frac{(5-\mu_5)^2}{2\sigma_5^2}\right]}}$$

$$= 0.2364$$

$$w_3 = \frac{e^{-\left[\frac{(3-\mu_5)^2}{2\sigma_5^2}\right]}}{e^{-\left[\frac{(1-\mu_5)^2}{2\sigma_5^2}\right]} + e^{-\left[\frac{(2-\mu_5)^2}{2\sigma_5^2}\right]} + e^{-\left[\frac{(3-\mu_5)^2}{2\sigma_5^2}\right]} + e^{-\left[\frac{(4-\mu_5)^2}{2\sigma_5^2}\right]} + e^{-\left[\frac{(5-\mu_5)^2}{2\sigma_5^2}\right]}}$$

$$= 0.3036$$

$$w_4 = \frac{e^{-\left[\frac{(4-\mu_5)^2}{2\sigma_5^2}\right]}}{e^{-\left[\frac{(1-\mu_5)^2}{2\sigma_5^2}\right]} + e^{-\left[\frac{(2-\mu_5)^2}{2\sigma_5^2}\right]} + e^{-\left[\frac{(3-\mu_5)^2}{2\sigma_5^2}\right]} + e^{-\left[\frac{(4-\mu_5)^2}{2\sigma_5^2}\right]} + e^{-\left[\frac{(5-\mu_5)^2}{2\sigma_5^2}\right]}}$$

$$= 0.2364$$

$$w_5 = \frac{e^{-\left[\frac{(5-\mu_5)^2}{2\sigma_5^2}\right]}}{e^{-\left[\frac{(1-\mu_5)^2}{2\sigma_5^2}\right]} + e^{-\left[\frac{(2-\mu_5)^2}{2\sigma_5^2}\right]} + e^{-\left[\frac{(3-\mu_5)^2}{2\sigma_5^2}\right]} + e^{-\left[\frac{(4-\mu_5)^2}{2\sigma_5^2}\right]} + e^{-\left[\frac{(5-\mu_5)^2}{2\sigma_5^2}\right]}}$$
$$= 0.1116$$

4. An Algorithm I for Getting Weights to Decision Matrices

Definition 4.1. *A neutrosophic fuzzy set A on the universe of discourse X characterized by a truth membership function $TA(x)$, an indeterminacy function $IA(x)$ and a falsity membership function $FA(x)$ is defined as $A = \{\langle x, TA(x), IA(x), FA(x)\rangle : x \in X\}$, where $TA, IA, FA : X \to [0,1]$ and $0 \le TA(x) \le 1$; $0 \le IA(x) \le 1$; $0 \le FA(x) \le 1$, for all $x \in X$.*

Steps for an algorithm:

Step 1: Utilize the NFOWA operator to aggregate all individual neutrosophic fuzzy decision matrices $R^{(k)} = \langle (T_{ij}(k), I_{ij}(k), F_{ij}(k)\rangle = (r_{ij}^{(k)})$ (k varies from 1, 2,3, and 4) into a collective neutrosophic fuzzy decision matrix $R = (r_{ij})_{m\times n}$.

Step 2: Calculate the weight information using the Gaussian distribution function given in equation (1) and (2).

Step 3: Use the NFHA operator to get the overall values rj of the alternatives $\mathcal{O}_j$ $(j = 1, 2, \ldots, n)$.

Step: 4 Using $r^* = (1, 0, 0) = (TA^*, IA^*, FA^*)$, find

$$d(r^*, rj) = \sqrt{(T_A^* - T_{j_A})^2 + (I_A^* - I_{j_A})^2 + (F_A^* - F_{j_A})^2}$$

to calculate the distances between informational neutrosophic values $r_j = (T_{j_A}, I_{j_A}, F_{j_A})$ $(j = 1, 2, ..., n)$.

Step 5: Rank the alternatives based on distances.

Step 6: Select the best alternative.

5. Numerical Illustration

Step 1: Assume that the information in decision making are in neutrosophic fuzzy matrices as follows:

$$R^1 = \begin{bmatrix} \langle 0.25, 0.54, 0.8\rangle & \langle 0.3, 0.4, 0.9\rangle & \langle 0.7, 0.35, 0.5\rangle & \langle 0.9, 0.2, 0.8\rangle \\ \langle 0.6, 0.5, 0.5\rangle & \langle 0.6, 0.2, 0.3\rangle & \langle 0.2, 0.4, 0.9\rangle & \langle 0.6, 0.23, 0.7\rangle \\ \langle 0.3, 0.45, 0.9\rangle & \langle 0.7, 0.1, 0.4\rangle & \langle 0.6, 0.5, 0.5\rangle & \langle 0.4, 0.2, 0.9\rangle \\ \langle 0.45, 0.38, 0.27\rangle & \langle 0.37, 0.68, 0.16\rangle & \langle 0.6, 0.25, 0.3\rangle & \langle 0.1, 0.4, 0.8\rangle \end{bmatrix}$$

$$R^2 = \begin{bmatrix} \langle 0.1, 0.3, 0.7\rangle & \langle 0.6, 0.6, 0.5\rangle & \langle 0.4, 0.2, 0.1\rangle & \langle 0.3, 0.7, 0.6\rangle \\ \langle 0.3, 0.55, 0.37\rangle & \langle 0.75, 0.42, 0.1\rangle & \langle 0.32, 0.67, 0.56\rangle & \langle 0.35, 0.56, 0.72\rangle \\ \langle 0.5, 0.4, 0.32\rangle & \langle 0.65, 0.25, 0.32\rangle & \langle 0.6, 0.3, 0.1\rangle & \langle 0.75, 0.25, 0.55\rangle \\ \langle 0.27, 0.9, 0.81\rangle & \langle 0.31, 0.4, 0.6\rangle & \langle 0.75, 0.65, 0.55\rangle & \langle 0.3, 0.7, 0.9\rangle \end{bmatrix}$$

$$R^3 = \begin{bmatrix} \langle 0.32, 0.47, 0.6\rangle & \langle 0.9, 0.1, 0.3\rangle & \langle 0.6, 0.4, 0.5\rangle & \langle 0.3, 0.5, 0.7\rangle \\ \langle 0.12, 0.32, 0.52\rangle & \langle 0.17, 0.81, 0.9\rangle & \langle 0.5, 0.3, 0.1\rangle & \langle 0.45, 0.65, 0.27\rangle \\ \langle 0.50, 0.6, 0.23\rangle & \langle 0.56, 0.52, 0.23\rangle & \langle 0.3, 0.6, 0.1\rangle & \langle 0.57, 0.52, 0.55\rangle \\ \langle 0.54, 0.83, 0.72\rangle & \langle 0.73, 0.86, 0.61\rangle & \langle 0.5, 0.52, 0.4\rangle & \langle 0.6, 0.4, 0.2\rangle \end{bmatrix}$$

$$R^4 = \begin{bmatrix} \langle 0.7, 0.3, 0.1\rangle & \langle 0.5, 0.4, 0.4\rangle & \langle 0.2, 0.1, 0.6\rangle & \langle 0.7, 0.9, 0.6\rangle \\ \langle 0.3, 0.56, 0.73\rangle & \langle 0.57, 0.24, 0.1\rangle & \langle 0.23, 0.76, 0.65\rangle & \langle 0.53, 0.65, 0.27\rangle \\ \langle 0.32, 0.32, 0.6\rangle & \langle 0.56, 0.52, 0.32\rangle & \langle 0.1, 0.3, 0.9\rangle & \langle 0.57, 0.52, 0.55\rangle \\ \langle 0.72, 0.5, 0.18\rangle & \langle 0.13, 0.6, 0.4\rangle & \langle 0.55, 0.56, 0.78\rangle & \langle 0.7, 0.1, 0.6\rangle \end{bmatrix}$$

$$R^5 = \begin{bmatrix} \langle 0.52, 0.45, 0.1\rangle & \langle 0.57, 0.37, 0.1\rangle & \langle 0.76, 0.65, 0.23\rangle & \langle 0.57, 0.52, 0.55\rangle \\ \langle 0.3, 0.6, 0.7\rangle & \langle 0.7, 0.4, 0.1\rangle & \langle 0.3, 0.7, 0.6\rangle & \langle 0.5, 0.4, 0.6\rangle \\ \langle 0.2, 0.3, 0.2\rangle & \langle 0.6, 0.2, 0.5\rangle & \langle 0.1, 0.6, 0.65\rangle & \langle 0.3, 0.9, 0.7\rangle \\ \langle 0.27, 0.5, 0.81\rangle & \langle 0.75, 0.25, 0.32\rangle & \langle 0.32, 0.67, 0.56\rangle & \langle 0.35, 0.56, 0.72\rangle \end{bmatrix}$$

Step 2: To derive a weight vector w by Normal Distribution Method

$$\mu_n = \frac{1+n}{2}\sigma_n = \sqrt{\frac{1}{n}\sum_i^n (i-\mu_n)^2}$$

OWA Weight

$$w_j = \frac{e^{-\left[\frac{(j-\mu_n)^2}{2\sigma_n^2}\right]}}{\sum_{i=1}^{n} e^{-\left[\frac{(i-\mu_n)^2}{2\sigma_n^2}\right]}}; \quad i, j = 1, \ldots, 5$$

$$w = (0.1116, 0.2364, 0.3036, 0.2364, 0.1116)^T$$

Step 3: The reduced fuzzy neutrosophic matric is

$$R = \begin{bmatrix} \langle 0.4176, 0.5976, 0.4588\rangle & \langle 0.7029, 0.6201, 0.5160\rangle & \langle 0.5254, 0.6717, 0.5721\rangle & \langle 0.5632, 0.3178, 0.3439\rangle \\ \langle 0.2950, 0.5068, 0.4260\rangle & \langle 0.5598, 0.4585, 0.4491\rangle & \langle 0.3483, 0.4397, 0.4345\rangle & \langle 0.4735, 0.4286, 0.4929\rangle \\ \langle 0.4115, 0.5507, 0.5122\rangle & \langle 0.6047, 0.6059, 0.6730\rangle & \langle 0.2972, 0.5345, 0.4512\rangle & \langle 0.5853, 0.4400, 0.4346\rangle \\ \langle 0.5098, 0.2524, 0.3512\rangle & \langle 0.5154, 0.3350, 0.5037\rangle & \langle 0.5820, 0.04400,\ 0.4346\rangle & \langle 0.5069, 0.5416, 0.3165\rangle \end{bmatrix}$$

Step 4: Using the weights $w = \{0.2717, 0.2608, 0.2254, 0.2421\}$ obtained from Poisson distribution. New reduced row matrix is

$$R = \begin{bmatrix} \langle 0.4115, 0.5054, 0.5805\rangle & \langle 0.6048, 0.4811, 0.4611\rangle & \langle 0.4538, 0.4676, 0.5207\rangle & \langle 0.05332, 0.5579, 0.6153\rangle \end{bmatrix}$$

Step 5: $d = \sqrt{\frac{1}{2}\left[\sum\left[(1-T)^2 + (0-I)^2 + (0-F)^2\right]\right]}$.

$$d(r, r_1) = 0.6851 = A_1$$

$$d(r, r_2) = 0.5478 = A_2$$

$$d(r, r_3) = 0.6455 = A_3$$

$$d(r, r_4) = 0.6737 = A_4.$$

Step 6: $A_1 > A_4 > A_3 > A_2$.

Step 7: A_1 is best alternative.

6. Conclusion

We have investigated the MAGDM problems under neutrosophic fuzzy environment, and proposed an approach to handling the situations where the attribute values are characterized by NFSs, and the information about attribute weights completely unknown. The proposed approach first fuses all individual neutrosophic fuzzy decision matrices into the collective neutrosophic fuzzy decision matrix by using the NFOWA operator. Then we have used the obtained attribute weights and the NFHA operator to get the overall neutrosophic fuzzy values of alternatives. The proposed approach in this work not only can comfort the influence of unjust arguments on the decision results, but also avoid losing or distorting the original decision information in the process of aggregation. Thus, the proposed approach provides us an effective and practical way to deal with multi-person multi-attribute decision making problems, where the attribute values are characterized by NFSs and the information about attribute weights is partially known. The suitable alternative is selected through the algorithm from the given neutrosophic information in which the unknown weights are derived based upon normal distribution.

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