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Transient MHD Free Convective Fluid Flow Past a Moving Accelerated Vertical Porous Plate in the Presence of Viscous Dissipation

K. Rajeshwar Reddy[1,*] and M. Chenna Krishna Reddy[2]

1 Department of Mathematics, Malla Reddy College of Engg & Tech, Medchal, Hyderabad, Telangana, India.

2 Department of Mathematics, Osmania University, Hyderabad, Telangana, India.

Abstract: A numerical study is presented on the effect Transient MHD free convective fluid flow past a moving accelerated vertical porous plate in the presence of viscous dissipation has been studied. The dimensionless governing coupled non-linear boundary layer partial differential equations are solved by an efficient finite element method. The variations of the fluid velocity, temperature and concentration fields with the help of different flow parameters are presented graphically. And also,the effects these flow parameters on Skin-friction, Rate of heat and mass transfer are discussed through tabular forms. This model finds applications in geophysics, astrophysics and also in the design of high temperature industrial processing systems.

Keywords: Transient, MHD, Free convection, Viscous dissipation and FEM.

1. Introduction

The most common type of body force, which acts on a fluid, is due to gravity, so that the body force can be defined as in magnitude and direction by the acceleration due to gravity. Sometimes, electromagnetic effects are important. The electric and magnetic fields themselves must obey a set of physical laws, which are expressed by Maxwell's equations. The solution of such problems requires the simultaneous solution of the equations of fluid mechanics and electromagnetism. One special case of this type of coupling is known as magneto hydrodynamic. Coupled heat and mass transfer phenomenon in porous media is gaining attention due to its interesting applications. The flow phenomenon in this case is relatively complex than that in pure thermal/solutal convection process. Processes involving heat and mass transfer in porous media are often encountered in the chemical industry, in reservoir engineering in connection with thermal recovery process, in the study of dynamics of hot and salty springs of a sea. Underground spreading of chemical waste and other pollutants, grain storage, evaporation cooling, and solidification are a few other application areas where combined thermosolutal convection in porous media are observed. However, the exhaustive volume of work devoted to this area is amply documented by the most recent books by Ingham and Pop [1], Nield and Bejan [2], Pop and Ingham [3], Vafai [4] studied the problem of transient flow of a fluid past a moving semi-infinite vertical porous plate. However, many problem areas which are important in applications, as well as in theory still persist.

* *E-mail: kattarajeshwarreddy@gmail.com*

Unsteady free convective flow on taking into account the mass transfer phenomenon past an infinite vertical porous plate with constant suction was studied by Soundalgekar and Wavre [5], Kafousias [6] have studied the effects of free convective currents on the flow field of an incompressible viscous fluid past an impulsively started infinite vertical porous plate with constant suction. However,this analysis is not applicable for other fluids whose Prandtl number is different from unity.Soundalgekar and Ganesan [7] have analyzed transient free convective flow past a semi-infinite vertical flat plate, taking into account mass transfer by an implicit finite difference method of Crank Nicolson type. Free convection at a vertical plate with transpiration has investigated by Kolar and Sastri [8], Yih [9] have analyzed the effect of transpiration on coupled heat andmass transfer in mixed convection over a vertical plate embedded in a saturated porous medium. Elbashbeshy [10] has investigated the mixed convection along a vertical plate embedded in non-darcian porous medium with suction and injection. Chin [11] has studied the effectof variable viscosity on mixed convection boundary layer flow over a vertical surface embedded in a porous medium. MHD steady free convection flow from vertical surface in porous mediumhas been studied by Reddy [12]. Thermal radiation in fluid dynamics has become a significant branch of the engineering sciences and is an essential aspect of various scenarios in mechanical,aerospace, chemical, environmental, solar power and hazards engineering. For some industrial applications such as glass production and furnace design and in space technology applications such as cosmical flight aerodynamics rocket, propulsion systems, plasma physics and spacecraft re-entry aerothermodynamics which operate at higher temperatures, radiation effects can be significant. In view of this, Hossain and Takhar [13] have analyzed the effect of radiation on mixed convection along a vertical plate with uniform surface temperature. Pal and Talukdar [14] have studied the buoyancy and chemical reaction effects on MHD mixed convection heat and mass transfer in a porous medium with thermal radiation and ohmic heating.

Israel-Cookey [15] studied the effects of viscous dissipation and radiation on unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium with time dependent suction. The effects of radiation on a steady combined free-forced convective and mass transfer flow of a viscous incompressible electrically conducting and radiating fluid over an isothermal semi-infinite vertical porous flat plate embedded in a porous medium studied by Bala Anki Reddy and Bhaskar Reddy [16]. The governing non-linear partial differential equations and their boundary conditions are reduced into a system of ordinary differential equations by a similarity transformation. This system is solved numerically using Runge-Kutta fourth order method along with shooting technique. Influence of thermal radiation on transient magneto hydrodynamic coutte flow through a porous medium by using finite difference method discussed by Baoku [17]. Radiation and mass transfer effects on an unsteady magneto hydrodynamic convective and dissipative fluid flow past a vertical porous plate has been analyzed by Gnaneswara Reddy and Bhaskar Reddy [18]. The governing equations of motion, energy and species are transformed into ordinary differential equations using time dependent similarity parameter. Greif [19] showed that, for an optically thin limit, the fluid does not absorb its own emitted radiation, this means that there is no self-absorption, but the fluid does absorb radiation emitted by the boundaries. Mohammed Ibrahim and Bhaskar Reddy [20] analyzed the radiation effects on the heat and mass transfer characteristics of a viscous incompressible electrically conducting fluid near an isothermal vertical stretching sheet, in the presence of viscous dissipation and heat generation. The governing equations are transformed by using similarity transformation and the resultant dimensionless equations are solved numerically using the Runge-Kutta fourth order method with shooting technique.

Vasu [21] studied the radiation and mass transfer effects on transient free convection flow of a dissipative fluid past semi-infinite vertical plate with uniform heat and mass flux. Ramana Reddy [22] have been studied Mixed convective MHD flow and mass transfer past an accelerated infinite vertical porous plate. Anand Rao [23] have investigated Radiation effects on an unsteady MHD vertical porous plate in the presence of homogeneous chemical reaction. Siva Reddy Sheri and Prasanthi Modugula [24] have found thermal-diffusion and diffusion-thermo effects on MHD flow through porous medium past an exponentially accelerated inclined plate with variable temperature. Siva Reddy Sheri and Prashanthi Modugula [25] have studied Heat and mass transfer effects on unsteady MHD flow over an inclined porous plate embedded in porous medium with Soret-Dufour and chemical Reaction. Siva Reddy Sheri [26] have found Heat and mass transfer effects on MHD natural convection Flow past an infinite inclined plate with ramped temperature

The object of the present paper is to study the transient MHD free convective fluid flow past a moving accelerated vertical porous plate in the presence of viscous dissipation. The problem is governed by the system of coupled non-linear partial differential equations whose exact solutions are difficult to obtain, if possible. So, finite element method has been adopted for its solution, which is more economical from computational point of view.

2. Mathematical Formulation

Consider the effect of thermal radiation on an unsteady mixed convective mass flow of a viscous incompressible electrically conducting fluid past an accelerating vertical infinite porous flat plate with viscous dissipation. We made the following assumptions.

(1). In Cartesian coordinate system, letx'-axis is taken to be along the plate and the y'-axis normal to the plate. Since the plate is considered infinite in x'-direction, hence all physical quantities will be independent of x'-direction.

(2). The wall is maintained at constant temperature (T'_w) and concentration (C'_w) higher than the ambient temperature (T'_∞) and concentration (C'_∞) respectively.

(3). A uniform magnetic field of magnitude B_o is applied normal to the plate. The transverse applied magnetic field and magnetic Reynold's number are assumed to be very small, so that the induced magnetic field is negligible.

(4). The homogeneous chemical reaction is of first order with rate constant $\bar{K}$ between the diffusing species and the fluid is neglected.

(5). It is assumed that there is no applied voltage which implies the absence of an electric field.

(6). It is assumed that the plate is accelerating with a velocity $u = U_o$ in its own plane for $t' \geq 0$.

(7). The fluid has constant kinematic viscosity and constant thermal conductivity and the Boussinesq's approximation have been adopted for the flow.

The magneto hydrodynamic unsteady mixed convective boundary layer equations under the Boussinesq's approximations are:

Continuity Equation:

$$\frac{\partial v'}{\partial y'} = 0 \Rightarrow v' = -v'_o \quad (Constant) \tag{1}$$

Momentum Equation:

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = g\beta\left(T' - T'_\infty\right) - \nu\frac{u'}{k'} + g\beta^*\left(C' - C'_\infty\right) + v\frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_o^2}{\rho}u' \tag{2}$$

Energy Equation:

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \frac{\kappa}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{v}{c_p}\left(\frac{\partial u'}{\partial y'}\right)^2 - \frac{\partial q_r}{\partial y'} \tag{3}$$

Concentration Equation:

$$\frac{\partial C'}{\partial t'} + v'\frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial y'^2} \tag{4}$$

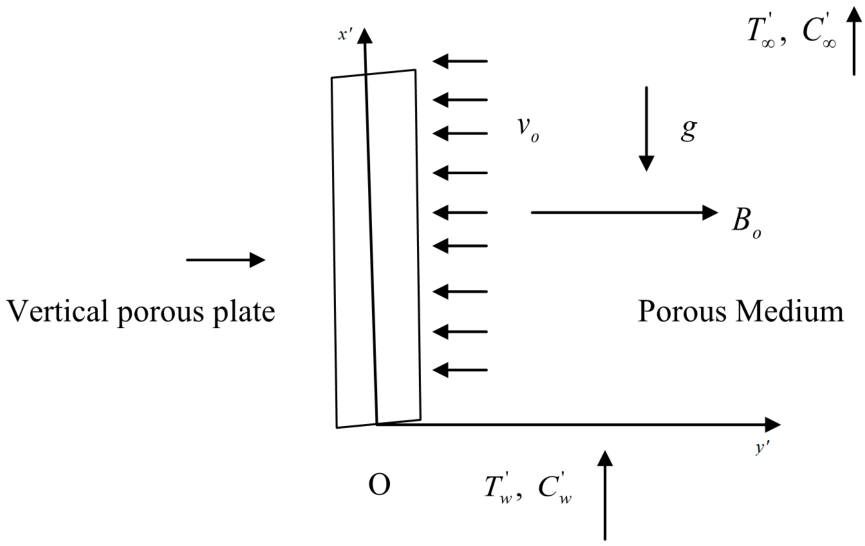

Figure 1. **Physical sketch and geometry of the problem**

The boundary conditions of the problem are:

$$\left.\begin{array}{l} t' \leq 0: \{\ u' = 0,\ v' = 0,\ T' = 0,\ C' = 0\ for\ all\ y' \\ t' \geq 0: \left\{\begin{array}{l} u' = U_O,\ v' = -v'_o,\ T' = T'_w,\ C' = C'_w\ at\ y' = 0 \\ u' \to 0,\ T' \to T'_\infty,\ C' \to C'_\infty\ \ as\ y' \to \infty \end{array}\right\} \end{array}\right\} \tag{5}$$

The radiative heat flux term is simplified by making use of the Rossel and approximation as

$$q_r = -\frac{4\bar{\sigma}}{3k^*}\frac{\partial T'^4}{\partial y'} \tag{6}$$

Here $\bar{\sigma}$ is Stefan-Boltzmann constant and k^* is the mean absorption coefficient. It is assumed that the temperature differences within the flow are sufficiently small so that T'^4 can be expressed as a linear function of T' after using Taylor's series to expand T'^4 about the free stream temperature T'_h and neglecting higher-order terms. This results in the following approximation:

$$T'^4 \cong 4T'^3_h T' - 3T'^4_h \tag{7}$$

Using equations (6) and (7) in the last term of equation (3), we obtain:

$$\frac{\partial q_r}{\partial y} = -\frac{16\bar{\sigma}T'^3_h}{3k^*}\frac{\partial^2 T'}{\partial y'^2} \tag{8}$$

Introducing (8) in the equation (3), the energy equation becomes:

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \frac{\kappa}{\rho\, c_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{v}{c_p}\left(\frac{\partial u'}{\partial y'}\right)^2 + \frac{16\bar{\sigma}T'^3_h}{3k^*}\frac{\partial^2 T'}{\partial y'^2} \tag{9}$$

Introducing the following non-dimensional variables and parameters,

$$\left.\begin{array}{l} y = \frac{y' v'_o}{v},\ t = \frac{t' v'^2_o}{4v},\ \omega = \frac{4v\omega'}{v'^2_o},\ u = \frac{u'}{U_o},\ M = \left(\frac{\sigma B_o^2}{\rho}\right)\frac{v}{v'^2_o},\ K = \frac{k'\ v'^2_o}{\nu^2},\ Sc = \frac{\nu}{D}, \\ \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty},\ C = \frac{C' - C'_\infty}{C'_w - C'_\infty},\ \Pr = \frac{v}{k},\ Gr = \frac{vg\beta\left(T'_w - T'_\infty\right)}{U_o v'^3_o},\ Gc = \frac{g\beta^* v\left(C'_w - C'_\infty\right)}{U_o v'^3_o}, \\ Ec = \frac{v'^2_o}{c_p\left(T'_w - T'_\infty\right)},\ R = \frac{\kappa\, k^*}{4\sigma T'^3_h} \end{array}\right\} \tag{10}$$

Substituting (10) in equations (2), (9) and (4) under boundary conditions (5), we get:

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = (Gr)\,\theta + (Gc)\,C + \frac{\partial^2 u}{\partial y^2} - \left(M + \frac{1}{K}\right)u \tag{11}$$

$$\frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial y} = \frac{1}{\mathrm{Pr}}\left(1 + \frac{4}{3R}\right)\frac{\partial^2 \theta}{\partial y^2} + (Ec)\left(\frac{\partial u}{\partial y}\right)^2 \tag{12}$$

$$\frac{\partial C}{\partial t} - \frac{\partial C}{\partial y} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \tag{13}$$

The corresponding boundary conditions are:

$$\left.\begin{aligned} &u = 1,\ \ \theta = 1,\ \ C = \ 1\ at\ y = 0 \\ &u \to 0,\ \ \theta \to 0,\ \ C \to 0\ \ as\ y \to \infty \end{aligned}\right\} \tag{14}$$

The skin-friction, Nusselt number and Sherwood number are important physical parameters for this type of boundary layer flow. The skin-friction at the plate, which in the non-dimensional form is given by

$$\tau = \frac{\tau_w'}{\rho U_o v} = \left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{15}$$

The rate of heat transfer coefficient, which in the non-dimensional form in terms of the Nusselt number is given by

$$Nu = -x\frac{\left(\frac{\partial T'}{\partial y'}\right)_{y'=0}}{T_w' - T_\infty'} \Rightarrow Nu\ Re_x^{-1} = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} \tag{16}$$

The rate of mass transfer coefficient, which in the non-dimensional form in terms of the Sherwood number, is given by

$$Sh = -x\frac{\left(\frac{\partial C'}{\partial y'}\right)_{y'=0}}{C_w' - C_\infty'} \Rightarrow Sh\ Re_x^{-1} = -\left(\frac{\partial C}{\partial y}\right)_{y=0} \tag{17}$$

Where $Re = \frac{U_o x}{\nu}$ is the local Reynolds number. The mathematical formulation of the problem is now completed. Equations (11), (12) and (13) present a coupled nonlinear system of partial differential equations and are to be solved by using initial and boundary conditions(14). However, exact solutions are difficult, whenever possible. Hence, these equations are solved by the Finite element method.

3. Numerical Solution by FEM

An excellent description of Galerkin finite element method is presented in the text books Bathe [27] and Reddy [28]. By applying Galerkin finite element method for equation (4) over the element (e), $(y_j \le y \le y_k)$ is:

$$\int_{y_j}^{y_k} \left\{ N^T \left[\frac{\partial^2 u^{(e)}}{\partial y^2} - \frac{\partial u^{(e)}}{\partial t} + \frac{\partial u^{(e)}}{\partial y} - Au^{(e)} + P \right] \right\} dy = 0 \tag{18}$$

Where $P = (Gr)\theta_i^j + (Gc)C_i^j$, $A = M + \frac{1}{K}$. Integrating the first term in equation (18) by parts one obtains

$$N^{(e)T}\left\{\frac{\partial u^{(e)}}{\partial y}\right\}_{y_j}^{y_k} - \int_{y_j}^{y_k}\left\{\frac{\partial N^{(e)T}}{\partial y}\frac{\partial u^{(e)}}{\partial y} + N^{(e)T}\left(\frac{\partial u^{(e)}}{\partial t} - \frac{\partial u^{(e)}}{\partial y} + Au^{(e)} - P\right)\right\}dy = 0 \tag{19}$$

Neglecting the first term in equation (19), one gets:

$$\int_{y_j}^{y_k}\left\{\frac{\partial N^{(e)T}}{\partial y}\frac{\partial u^{(e)}}{\partial y} + N^{(e)T}\left(\frac{\partial u^{(e)}}{\partial t} - \frac{\partial u^{(e)}}{\partial y} + Au^{(e)} - P\right)\right\}dy = 0$$

Let $u^{(e)} = N^{(e)}\phi^{(e)}$ be the linear piecewise approximation solution over the element (e), $(y_j \le y \le y_k)$, where $N^{(e)} = [N_j \;\; N_k]$, $\phi^{(e)} = [u_j \;\; u_k]^T$ and $N_j = \frac{y_k - y}{y_k - y_j}$, $N_k = \frac{y - y_j}{y_k - y_j}$ are the basis functions. One obtains:

$$\int_{y_j}^{y_k} \left\{ \begin{bmatrix} N_j' N_j' & N_j' N_k' \\ N_j' N_k' & N_k' N_k' \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} \right\} dy + \int_{y_j}^{y_k} \left\{ \begin{bmatrix} N_j N_j & N_j N_k \\ N_j N_k & N_k N_k \end{bmatrix} \begin{bmatrix} \dot{u}_j \\ \dot{u}_k \end{bmatrix} \right\} dy - \int_{y_j}^{y_k} \left\{ \begin{bmatrix} N_j N_j' & N_j N_k' \\ N_j' N_k & N_k' N_k \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} \right\} dy$$
$$+ A \int_{y_j}^{y_k} \left\{ \begin{bmatrix} N_j N_j & N_j N_k \\ N_j N_k & N_k N_k \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} \right\} dy = P \int_{y_j}^{y_k} \begin{bmatrix} N_j \\ N_k \end{bmatrix} dy$$

Simplifying we get

$$\frac{1}{l^{(e)2}} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} + \frac{1}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} \dot{u}_j \\ \dot{u}_k \end{bmatrix} - \frac{1}{2l^{(e)}} \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} + \frac{A}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} = \frac{P}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Where prime and dot denotes differentiation w.r.t y and time t respectively. Assembling the element equations for two consecutive elements $(y_{i-1} \le y \le y_i)$ and $(y_i \le y \le y_{i+1})$ following is obtained:

$$\frac{1}{l^{(e)2}} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix} + \frac{1}{6} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} \dot{u}_{i-1} \\ \dot{u}_i \\ \dot{u}_{i+1} \end{bmatrix} - \frac{1}{2l^{(e)}} \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix}$$
$$+ \frac{A}{6} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix} = \frac{P}{2} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \tag{20}$$

Now put row corresponding to the node i to zero, from equation (20) the difference schemes with $l^{(e)} = h$ is:

$$\frac{1}{h^2}\left[-u_{i-1} + 2u_i - u_{i+1}\right] + \frac{1}{6}\left[\dot{u}_{i-1} + 4\dot{u}_i + \dot{u}_{i+1}\right] - \frac{1}{2h}\left[-u_{i-1} + u_{i+1}\right] + \frac{A}{6}\left[u_{i-1} + 4u_i + u_{i+1}\right] = P \tag{21}$$

Applying the trapezoidal rule, following system of equations in Crank Nicholson method is obtained:

$$A_1 u_{i-1}^{n+1} + A_2 u_i^{n+1} + A_3 u_{i+1}^{n+1} = A_4 u_{i-1}^n + A_5 u_i^n + A_6 u_{i+1}^n + P^* \tag{22}$$

Now from equations (12) and (13) following equations are obtained:

$$B_1 \theta_{i-1}^{n+1} + B_2 \theta_i^{n+1} + B_3 \theta_{i+1}^{n+1} = B_4 \theta_{i-1}^n + B_5 \theta_i^n + B_6 \theta_{i+1}^n + Q^* \tag{23}$$

$$D_1 C_{i-1}^{n+1} + D_2 C_i^{n+1} + D_3 C_{i+1}^{n+1} = D_4 C_{i-1}^n + D_5 C_i^n + D_6 C_{i+1}^n \tag{24}$$

Where $A_1 = 2 + Ak + 3rh - 6r$, $A_2 = 8 + 4Ak + 12r$, $A_3 = 2 + Ak - 3rh - 6r$, $A_4 = 2 - Ak - 3rh + 6r$, $A_5 = 8 - 4Ak - 12r$, $A_6 = 2 - Ak + 3rh + 6r$, $B_1 = 2(\mathrm{Pr}) + 3rh(\mathrm{Pr}) - 6R_1 r$, $B_2 = 8(\mathrm{Pr}) + 12R_1 r$, $B_3 = 2(\mathrm{Pr}) - 3rh(\mathrm{Pr}) - 6R_1 r$, $B_4 = 2(\mathrm{Pr}) - 3rh(\mathrm{Pr}) + 6R_1 r$, $B_5 = 8(\mathrm{Pr}) - 12R_1 r$, $B_6 = 2(\mathrm{Pr}) + 3rh(\mathrm{Pr}) + 6R_1 r$, $D_1 = 2(Sc) + 3rh(Sc) - 6r$, $D_2 = 8(Sc) + 12r$, $D_3 = 2(Sc) - 3rh(Sc) - 6r$, $D_4 = 2(Sc) - 3rh(Sc) + 6r$, $D_5 = 8(Sc) - 12r$, $D_6 = 2(Sc) + 3rh(Sc) + 6r$, $P^* = 12Phk = 12hk(Gr)\theta_i^j + 12hk(Gc)C_i^j$,$Q^* = 12kQ = 12k(\mathrm{Pr})\,(Ec)\left(\frac{\partial u_i^j}{\partial y}\right)^2$, $R_1 = 1 + \frac{4}{3R}$. Here $r = \frac{k}{h^2}$ and h, k are mesh sizes along

$y-$direction and time-direction respectively. Index i refers to space and j refers to the time. In the equations (22), (23) and (24) taking $i = 1(1)n$ and using boundary conditions (14), then the following system of equations are obtained:

$$A_i X_i = B_i;\ i = 1(1)n \tag{25}$$

Where A_i's are matrices of order n and X_i, B_i's are column matrices having n-components. The solutions of above system of equations are obtained by using Thomas algorithm for velocity, temperature and concentration. Also, numerical solutions for these equations are obtained by C-programme. In order to prove the convergence and stability of Galerkin finite element method, the same C-programme was run with smaller values of h and k and no significant change was observed in the values of u, θ and C. Hence the Galerkin finite element method is stable and convergent.

4. Results and Discussions

The formulation of the problem that accounts for the transient MHD free convective fluid flow past a moving accelerated vertical porous plate in the presence of viscous dissipation is performed in the preceding sections. The governing equations of the flow field are solved numerically by using a finite element method. The above presented equations enable us to carry out numerical computations. The following parameter values are adopted for computations unless otherwise indicated in the figures and table: $Gr = 1.0$, $Gc = 1.0$, $M = 1.0$, $K = 1.0$, $\Pr = 0.71$, $R = 1.0$, $E_c = 0.001$, $Sc = 0.22$. The boundary conditions for $\eta \to \infty$ are replaced by those at $\eta_{\max}$ where the value of $\eta_{\max}$ is sufficiently large, so that the velocity at $\eta = \eta_{\max}$ is equal to the relevant free stream velocity. We choose $\eta_{\max} = 4$. To assess the accuracy of the present method, comparisons between the present results and previously published data Ramana Reddy [22]. Figures (2) and (3) exhibit the effect of thermal Grashof number and solutal Grashof numbers on the velocity profile with other parameters are fixed. The Grashof number signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. As expected, it is observed that there is a rise in the velocity due to the enhancement of thermal buoyancy force. Also, as Gr increases, the peak values of the velocity increases rapidly near the porous plate and then decays smoothly to the free stream velocity. The solutal Grashof number defines the ratio of the species buoyancy force to the viscous hydrodynamic force. As expected, the fluid velocity increases and the peak value is more distinctive due to increase in the species buoyancy force. The velocity distribution attains a distinctive maximum value in the vicinity of the plate and then decreases properly to approach the free stream value. It is noticed that the velocity increases with increasing values of the solutal Grashof number.

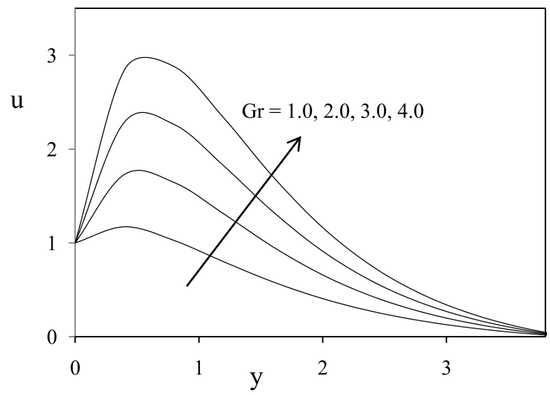

Figure 2. Velocity profiles for different values of thermal Grashof number Gr

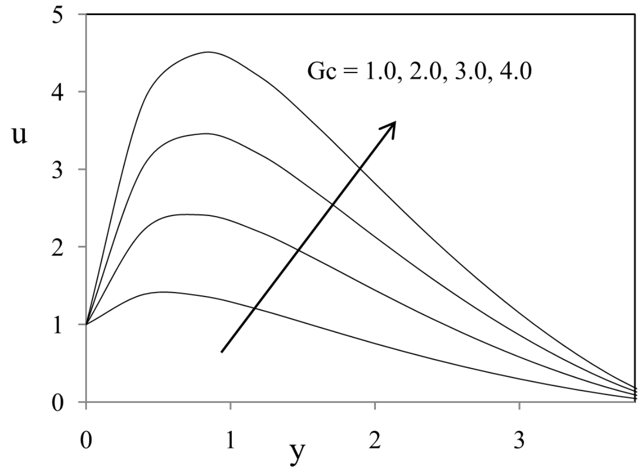

Figure 3. **Velocity profiles for different values of Solutal Grashof number Gc**

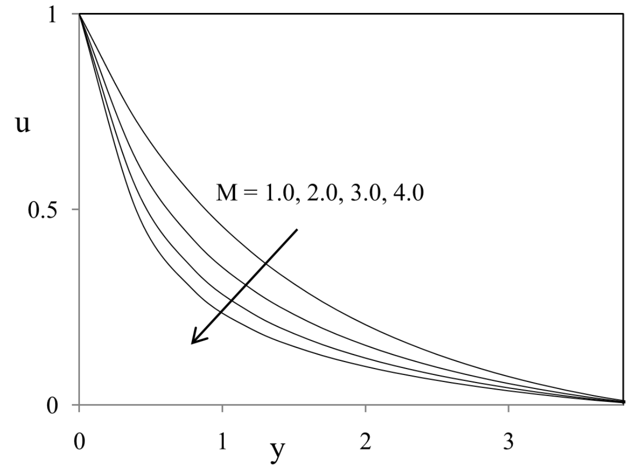

Figure 4. **Velocity profiles for different values of Hartmann number M**

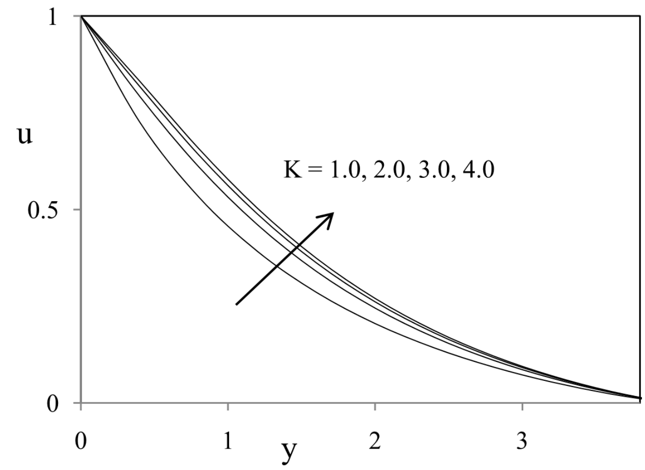

Figure 5. **Velocity profiles for different values of Permeability parameter K**

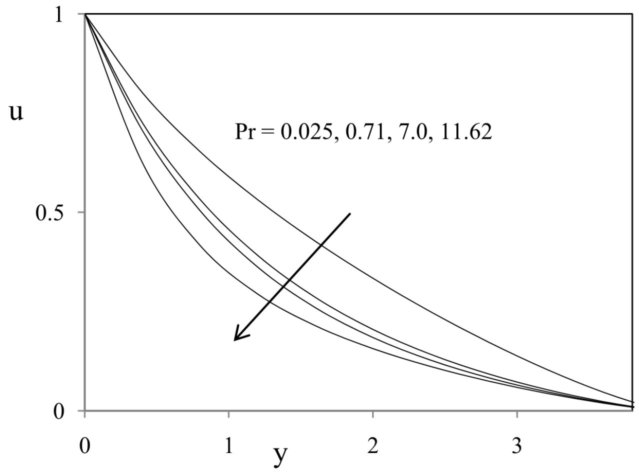

Figure 6. **Velocity profiles for different values of Prandtl number Pr**

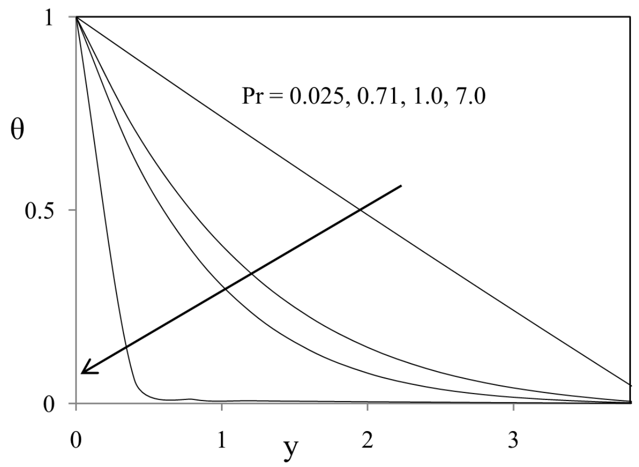

Figure 7. **Temperature profiles for different values of Prandtl number Pr**

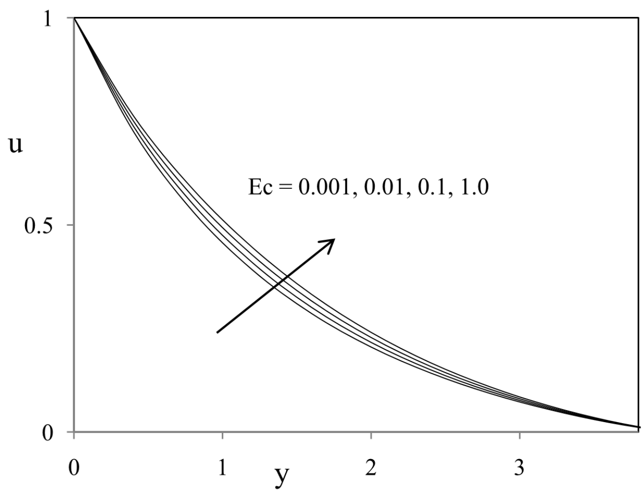

Figure 8. **Velocity profiles for different values of Eckert number Ec**

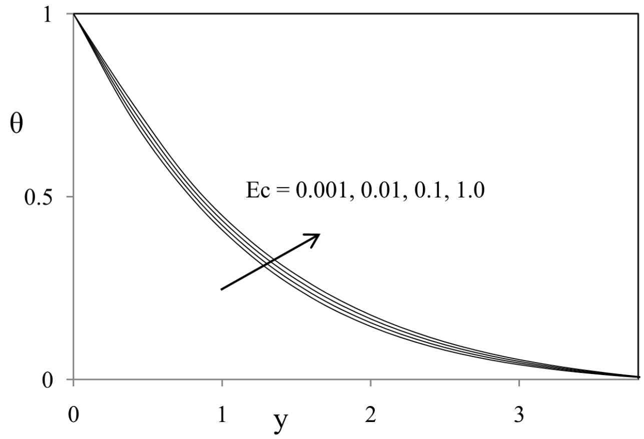

Figure 9. **Temperature profiles for different values of Eckert number Ec**

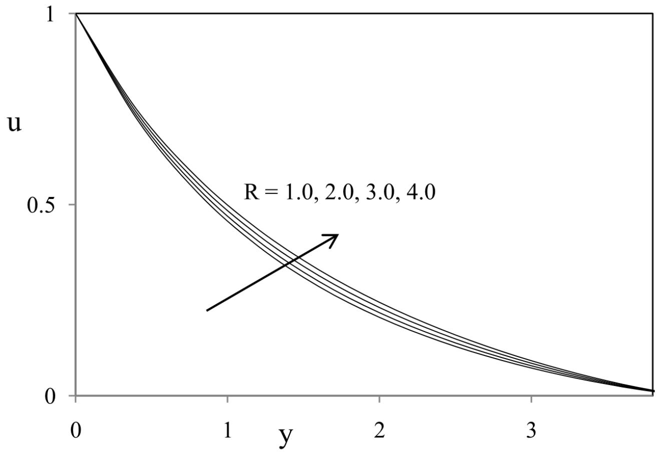

Figure 10. **Velocity profiles for different values of thermal radiation parameter R**

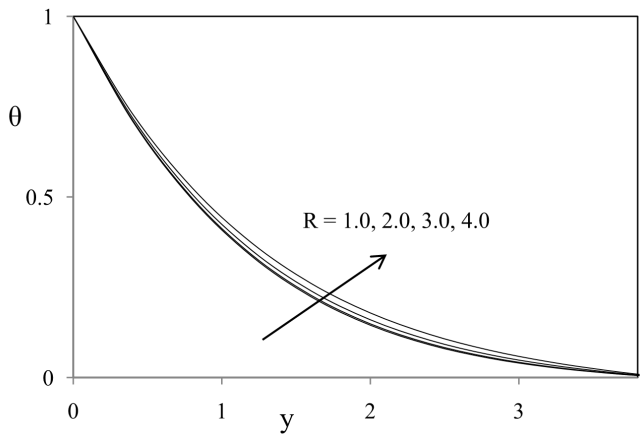

Figure 11. **Temperature profiles for different values of thermal radiation parameter R**

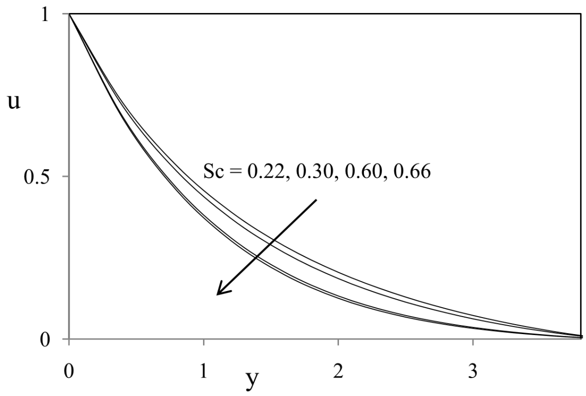

Figure 12. **Velocity profiles for different values of Schmidt number Sc**

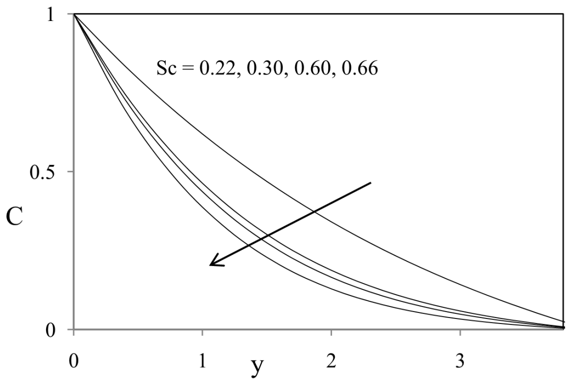

Figure 13. **Concentration profiles for different values of Schmidt number Sc**

The effect of the Hartmann number (M) is shown in figure (4). It is observed that the velocity of the fluid decreases with the increase of the magnetic field number values. The decrease in the velocity as the Hartmann number (M) increases is because the presence of a magnetic field in an electrically conducting fluid introduces a force called the Lorentz force, which acts against the flow if the magnetic field is applied in the normal direction, as in the present study. This resistive force slows down the fluid velocity component as shown in figure (4). The effect of Permeability parameter(K)is presented in the figure (5). From this figure we observe that, the velocity is increases with increasing values of K.

Figures 6 and 7 illustrate the velocity and temperature profiles for different values of the Prandtl number Pr. The Prandtl number defines the ratio of momentum diffusivity to thermal diffusivity. The numerical results show that the effect of increasing values of Prandtl number results in a decreasing velocity (Figure 6). From Figure 7, it is observed that an increase in the Prandtl number results a decrease of the thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that smaller values of Pr are equivalent to increasing the thermal conductivities, and therefore heat is able to diffuse away from the heated plate more rapidly than for higher values of Pr. Hence in the case of smaller Prandtl numbers as the boundary layer is thicker and the rate of heat transfer is reduced. The influence of the viscous dissipation parameter i.e., the Eckert number (Ec) on the velocity and temperature are shown in figures (8) and (9) respectively. The Eckert number (Ec) expresses the relationship between the kinetic energy in the flow and the enthalpy. It embodies the conversion of kinetic energy into internal energy by work done against the viscous fluid stresses. Greater viscous dissipative heat causes a rise in the temperature as well as the velocity. This behavior is evident from figures (8) and (9).

This behavior is evident from figures (10) and (11). The effect of the thermal radiation parameter on the velocity and temperature profiles in the boundary layer are illustrated in figures (10) and (11) respectively. Increasing the thermal radiation parameter produces significant increase in the thermal condition of the fluid and its thermal boundary layer. This increase in the fluid temperature induces more flow in the boundary layer causing the velocity of the fluid there to decrease. The effect of Schmidt number Sc on the velocity and concentration are shown in figures (12) and (13). As the Schmidt number increases, the velocity and concentration decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. Reductions in the velocity and concentration distributions are accompanied by simultaneous reductions in the velocity and concentration boundary layers.

We observe from this Table 1, the skin-friction rises under the effects of Grashof number, Modified Grashof number, Eckert number, Thermal radiation parameter and Permeability parameter. And also falls under the effects of Prandtl number, Schmidt number Hartmann number. The profiles for Nusselt number (Nu) due to temperature profile under the effect of Prandtl number, Eckert number and Thermal radiation parameter are presented in the Table 2. From this table we observe that, the Nusselt number due to temperature profile rises under the effect of Eckert number and Thermal radiation parameter. And temperature falls under the effects of Prandtl number. The profiles for Sherwood number (Sh) due to concentration profiles under the effect of Schmidt number is presented in the Table 3. We see from this table the Sherwood number due to concentration profile falls under the effect of Schmidt number. Table 4 shows the comparison between values of skin-friction coefficient τ for different values of Gr, Gc, Sc, Pr, K and M. In fact, this results show a close agreement, hence an encouragement for further study of the effects of other varies of parameters.

Gr	Gc	Pr	Sc	M	K	Ec	R	τ
1.0	1.0	0.71	0.22	1.0	1.0	0.001	1.0	3.2954
2.0	1.0	0.71	0.22	1.0	1.0	0.001	1.0	4.4918
1.0	2.0	0.71	0.22	1.0	1.0	0.001	1.0	5.3946
1.0	1.0	7.00	0.22	1.0	1.0	0.001	1.0	2.1650
1.0	1.0	0.71	0.30	1.0	1.0	0.001	1.0	3.1168
1.0	1.0	0.71	0.22	2.0	1.0	0.001	1.0	2.6004
1.0	1.0	0.71	0.22	1.0	2.0	0.001	1.0	3.7905
1.0	1.0	0.71	0.22	1.0	1.0	0.100	1.0	3.2986
1.0	1.0	0.71	0.22	1.0	1.0	0.001	2.0	3.4094

Table 1. **Skin-friction results (τ) for the values of Gr, Gc, Pr, Sc, M, K, Ec and R**

Pr	Ec	R	Nu
0.71	0.001	1.0	4.4972
7.00	0.001	1.0	1.0897
0.71	0.100	1.0	4.6270
0.71	0.001	2.0	4.6087

Table 2. **Rate of heat transfer (Nu) values for different values of Pr, Ec and R**

Sc	Sh
0.22	6.9193
0.30	6.5249

Table 3. **Rate of mass transfer (Sh) values for different values of Sc**

Gr	Gc	Pr	Sc	M	K	Present Results τ	Ramana Reddy [22] τ^*
1.0	1.0	0.71	0.22	1.0	1.0	2.5701	2.5705
2.0	1.0	0.71	0.22	1.0	1.0	2.9978	2.9984
1.0	2.0	0.71	0.22	1.0	1.0	3.6158	3.6191
1.0	1.0	7.00	0.22	1.0	1.0	2.1161	2.1150
1.0	1.0	0.71	0.30	1.0	1.0	1.9721	1.9716
1.0	1.0	0.71	0.22	2.0	1.0	1.5871	1.5889
1.0	1.0	0.71	0.22	1.0	2.0	3.0023	3.0018

Table 4. **Comparison of present Skin-friction results (τ)**

5. Conclusions

We summarize below the following results of physical interest on the velocity, temperature and concentration distributions of the flow field and also on the skin-friction, rate of heat and mass transfer at the wall.

(1). A growing Hartmann number or Prandtl number or Schmidt number retards the velocity of the flow field at all points.

(2). The effects of increasing Grashof number or Modified Grashof number or Permeability parameter or Eckert number or Permeability parameter or Thermal radiation parameter are to accelerate velocity of the flow field at all points.

(3). A growing Prandtl number decreases temperature of the flow field at all points and increases with increasing of Eckert number or Thermal radiation parameter.

(4). The Schmidt number decreases the concentration of the flow field at all points.

(5). A growing Hartmann number or Prandtl number or Schmidt number decreases the skin-friction while increasing Grashof number or Modified Grashof number or Permeability parameter or Eckert number or Thermal radiation parameter increases the skin-friction.

(6). The rate of heat transfer is decreasing with increasing of Prandtl number and increases with increasing of Eckert number and Thermal radiation parameter.

(7). The rate of mass transfer is decreasing with increasing of Schmidt number.

(8). On comparing the skin-friction (τ) results with the skin-friction (τ^*) results of Ramana Reddy [22] it can be seen that they agree very well.

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