Int. J. Math. And Appl., 6(1–E)(2018), 1235–1237
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of *Mathematics And its Applications*

# Geometric Approach of Topological Line of Investigation on Discrete Structures

**S. N. Banasode[1], Jayashree S. Radder[2,*] and Seema B. Patil[3]**

1 Department of Mathematics, R.L Science Institute, Belagavi, Karnataka, India.

2 Department of Mathematics, Tontadarya College of Engineering, Gadag, Karnataka, India.

3 Research Scholar, Rani Channamma University, Belagavi, Karnataka, India.

**Abstract:** In this paper we are confining to surfaces to atmost 3-dimensional smooth manifolds and assuming that X is our 2-dimensional surface and considering its discrete version, basically for topology. We have given a proposition connecting to metric space.

**Keywords:** Homeomorphism, discrete structures, metric space.


## 1. Introduction

Mathematical description of the problems under transportation naturally connects the topology (for networks and their large scale structures) of the underlined manifold estimates quantification for the feasibility of solutions associated with the evolution of the system is geometric in nature and i.e., how one would be interested in the geometry and models formed to the effective study of the transportation problems. From the beginning of $20^{\text{th}}$ century the classification problems in topology, concerning mainly about distinguishing spaces either is same or different engaged people and enabled them to see tremendous success .For example the classification of 2-dimensional surfaces, simple closed and orientable homeomorphic to a sphere $S^2$ (i.e., $X \simeq S^2$). By the end of 40's, the classification of 2-dimensional manifolds was complete and up to homeomorphisms they are either a sphere (a compact closed simple surface with zero handles) or a torus, i.e., a sphere with one handle etc and n torus or sphere with n-handles. Then, investigations for the classifications of higher dimensional manifolds began on similar arguments but soon it was realized that, the problem seemed quite difficult and muddled with the topological constraints. In fact, it was Henri Poincare in early 1900 suggested this problem in the form of conjecture. It states that, if X is a compact 3-dimension orientable manifold with its fundamental group trivial then X is homeomorphic to 3-sphere (i.e., $S^3$).

## 2. Main Result

Considering, $\mathbb{Z}_2^n : \mathbb{Z}_2 \times \mathbb{Z}_2 \times \cdots \times \mathbb{Z}_2$ and $G^n : G \times G \times \cdots \times G$, over some field. i.e., X is such a 3-dimensional manifold (G is isomorphic on field $\mathbb{Z}_2$). Poincare's claim was $X \simeq S^3$. This conjecture which remained opened unsolved for at most 100

* *E-mail: jayasr28@gmail.com*

years since its announcement was settled recently, by G. Perleman and was awarded fields medal, Clay Math Foundation Millenium prize money but refused by him to accept both the prizes. Brought to fore the kind of geometry that went into the problem initially thought to be a topological problem. It was through geometry this topological classification of 3-manifolds forced us to investigate 3-manifolds and to understand them for their topology and also geometry. What we intend to do here is the following, firstly assuming that X is our 2-dimensional surface and consider its discrete version, basically for topology. If X is a topological space, it may be discrete or continuous. Because, it is the topology that describes the nature of space.

If the space is discrete then we are done, otherwise, we are required to discretize the space. There are several ways of discretizing by the space, we refer the following relevant literature, [2, 3, 4] etc discrete structures, so we consider the discrete set and topologize the structure retaining discreteness. One such discrete group like $\langle \mathbb{Z}, + \rangle$ which is additive abelian group and infinite. Consider its Cartesian product (always a nice way of getting new sets) i.e., $\mathbb{Z} \times \mathbb{Z} \times \cdots \times \mathbb{Z}$ n times in short denoted as $\mathbb{Z}^n$ then one can consider the group $\langle \mathbb{Z}^n, + \rangle$ where addition is component of n-tuple integers. $\{(m : m^1, m^2, m^3 \dots m^n) \ \ where \ \ m^i \in \mathbb{Z}, \ I = 1, 2, \dots, n\}$ then $m, l \in \mathbb{Z}^n$ then their addition $m + l : m_1 + l_1, m_2 + l_2, \dots, m_n + l_n$ each $m_i + l_i$, $I = 1, 2, \dots, n \in \mathbb{Z}$ and therefore belongs to $\mathbb{Z}^n$. Objective is to metrize a discrete structure.

**Proposition 2.1.** *If X is a discrete set then we can put a metric on X and make X a metric space.*

Another setting is the following one. Fix a positive integer m and define following group namely, the addition modulo in group and this group is abelian $\langle \mathbb{Z}_m, + \rangle$. Now there are two situation, m is composite or m is prime. If m prime then m has only 2 divisors m has 1 and $m = p$ then p is prime. Then we should be looking for a vector space, arising from n-copies of $\mathbb{Z}_m$, i.e., $\mathbb{Z}_m^n = \mathbb{Z}_m \times \mathbb{Z}_m \times \cdots \times \mathbb{Z}_m$, over a field. Since $\mathbb{Z}_m$ is a field $\Leftrightarrow$ m is prime. Therefore we can construct a vector space $\mathbb{Z}_m^n$ over $\mathbb{Z}_m \Rightarrow m = p$, for prime $p = 2$.

Classically, if we were to have a space X which is a continue, for example $X = \mathbb{R}$, the real line itself then the standard (or usual) metric on $\mathbb{R}$, makes $\mathbb{R}$ a metric space and $\mathbb{R} \times \mathbb{R} \times \cdots \times \mathbb{R}$ n copies a metric space $(\mathbb{R}^n, d\ )$, d the product metric on $\mathbb{R}^n$.

$$(\mathbb{Z}, d_{\mathbb{Z}}) \dots \mathbb{Z} \subset (\mathbb{R}, d)$$

$$(\mathbb{Z}^2, d_{\mathbb{Z}^2}) \dots \mathbb{Z}^2 \subset (\mathbb{R}^2, d)$$

$$\dots\dots$$

$$(\mathbb{Z}^n, d_{\mathbb{Z}^n}) \dots \mathbb{Z}^n \subset (\mathbb{R}^n, d)$$

Having noticed in the earlier section, the example of discrete structure arising from $\mathbb{R}$, $\mathbb{R}^2$, $\mathbb{R}^n$ as subsets of integers $\mathbb{Z}$, $\mathbb{Z}^2$, $\mathbb{Z}^n$. Here in this section w shall simply define an abstract discrete structure well developed and studied mathematical objects namely the graphs.

# 3. Conclusion

We assumed that X is our 2-dimensional surface and considered its discrete version, basically for topology and having noticed in the earlier section, the example of discrete structure arising from $\mathbb{R}$, $\mathbb{R}^2$, $\mathbb{R}^n$ as subsets of integers $\mathbb{Z}$, $\mathbb{Z}^2$, $\mathbb{Z}^n$. Objective was to metrize a discrete structure. Here we can also imply and define an abstract discrete structure well developed and studied mathematical objects namely the graphs.

## References

[1] Kenneth M.Golden, *Climate Change and Mathematics transport*, American Mathematical Society, 56(5)(2009).

[2] W.Fulton, *Algebraic Topology : A First course*, Springer-Verlag, New York, (1995).

[3] J.Stillwell, *Geometry of Surfaces: Universitext*, Springer-Verlag, New York, (1992).

[4] Kenneth H. Rosin, *Handbook of Discrete and Combinatorial Mathematics*, Taylor & Francis, (2016).