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New Separation Axioms in Soft Bitopological Space

Research Article

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Abstract: The present paper introduces a new class of separation axioms called $(1,2)^*$-soft b-separation axioms using $(1,2)^*$-soft b-open set. Also the properties of $(1,2)^*$-soft bT_i-spaces (i =0,1,2) are soft bitopological properties under the bijection and irresolute open soft mapping. Further, we show that the properties of $(1,2)^*$-soft bT_i-spaces (i =0,1,2) are hereditary properties.

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1. Introduction

In real life situation, the problems in economics, engineering, social sciences, medical sciences etc.do not always involve crisp data. So, we cannot successfully use the traditional classical methods because of various types of uncertainties presented in these problems. To exceed these uncertainties, some kinds of these theories were given like theory of fuzzy sets, rough set which we can use as mathematical tools for dealings with uncertainties. But all these theories have their owm difficulities. The reason for these difficulities Molodtsov [6] initiated the concept of soft set theory as a new mathematical tools for dealings with uncertainties which is free from the above difficulities. Molodtsov successfully applied soft set theory in several direction, such as smoothness of functions, game theory, operations research, Riemann integration, Perrron integration, probablity, theory of measurement and so on.

In 1963, J.C. Kelly [5], first initiated the concept of bitopological spaces. After then many authors studied some of basic concepts and properties of bitopological space. In 1996, Andrijevic [1] introduced a new class of open sets in a topological space called b- open sets. Recently, in 2011, Shabir and Naz [7] initiated the study of the soft topological spaces. They defined soft topology as a collection of soft sets over X. Also they defined basic notations of soft topological spaces such as soft open and soft closed sets, soft subspace, soft closure, soft interior, soft separation axioms and established their several properties. Metin Akdag and Alkan Ozkan [11] are defined soft b- open sets and soft b- continuous map studied their properties. In the year 2014, Basavaraj M.Ittanagi [2] initiated the concept of soft bitopological spaces which are defined over an initial universe with a fixed set of parameters.

In the present paper, we introduce a new class of separation axioms called $(1,2)^*$-soft b-separation axioms using $(1,2)^*$-soft b-open set. In particular we study the properties of the $(1,2)^*$-soft bT_0 spaces, $(1,2)^*$-soft bT_1-spaces and $(1,2)^*$-soft b-Haustroff spaces. we give the characterizations of these spaces.

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2. Preliminaries

Throughout this paper, X is an initial universe, E is the set of parameters, $P(X)$ is the power set of X and A is a nonempty subset of E.

Definition 2.1 ([7]). *A soft set F_A on the universe X is defined by the set of ordered pairs $F_A = \{(x, f_A(x)) : x \in E\}$, where $f_A : E \to P(X)$ such that $f_A(x) = \phi$ if $x \in A$. Here f_A is called approximate function of the soft set F_A. The value of $f_A(x)$ may be arbitrary, some of them may be empty , some may have non empty intersection. The set of all soft sets over X will be denoted by $S(X)$.*

Definition 2.2 ([7]). *For two soft sets F_A, G_B over a common universe X, we say that F_A is a soft subset of G_B if*

(1). $A \subseteq B$ and

(2). For all $e \in A$, $F(e)$ and $G(e)$ are identical approximations

We write $F_A \tilde{\subset} G_B$. F_A is said to be a soft super set of G_B if G_B is a soft subset of F_A. We denoted it by $F_A \tilde{\supset} G_B$.

Definition 2.3 ([7]). *Two soft sets F_A and G_B over the common universe X are said to be soft equal if F_A is a soft subset of G_B and G_B is a soft subset of F_A.*

Definition 2.4 ([7]). *The soft union of two soft sets of F_A and G_B over the common universe X is the soft set H_C, where $C = A \cup B$ and for all $e \in C$,*

$$H(e) = \begin{cases} F(e), & \text{if } e \in A \setminus B \\ G(e), & \text{if } e \in B \setminus A \\ F(e) \cup G(e), & \text{if } e \in A \cap B \end{cases}$$

Definition 2.5 ([7]). *The soft intersection H_C of two soft sets F_A and G_B over a common universe X, denoted by $F_A \tilde{\cap} G_B$, is defined as $C = A \cap B$ and $H(e) = F(e) \cap G(e)$, for all $e \in C$.*

Definition 2.6 ([7]). *A soft set F_E over X is said to be a null soft set or empty soft set denoted by ϕ if for all $e \in E$, $F(e) = \phi$. It means that there is no element in X related to the parameter $e \in E$. Therefore, we can't display such elements in the soft sets, as it is meaningless to consider such parameters.*

Definition 2.7 ([7]). *A soft set F_E over X is said to be an absolute soft set denoted by $\tilde{X}$ or $F_{\tilde{E}}$ if for all $e \in E$, $F(e) = X$. Clearly $\tilde{X}^{\tilde{C}} = \phi$ and $\phi^{\tilde{C}} = \tilde{X}$.*

Definition 2.8 ([7]). *Let F_E be a soft set over X and Y be a non empty subset of X. Then the soft set of F_E over Y denoted by $({}^Y F_E)$ is defined as follows: ${}^Y F(\alpha) = Y \tilde{\cap} F(\alpha)$, for all $\alpha \tilde{\in} E$. In other words, $({}^Y F_E) = Y \tilde{\cap} F_E$.*

Definition 2.9 ([4]). *Let $F_E \tilde{\in} S(X)$. We say that $x_e = (e, \{x\})$ is a soft point of F_E if $e \in E$ and $x \in F(e)$.*

Definition 2.10 ([4]). *The soft point x_e said to be belonging to the soft set F_E, denoted by $x_e \tilde{\in} F_E$.*

Definition 2.11 ([3]). *Let $F_A \tilde{\in} S(X)$. The soft power set of F_A is defined by $\tilde{P}(A) = \{F_{A_i} : F_{A_i} \tilde{\subseteq} F_A, i \in I \subseteq N\}$ and its cardinality is defined by $\left|\tilde{P}(A)\right| = 2^{\sum_{x \in E} |f_A(x)|}$, where $|f_A(x)|$ is the cardinality of $f_A(x)$.*

Example 2.12. *[3] Let $X = \{x_1, x_2\}$, $E = \{e_1, e_2\}$ then $\tilde{X} = \{(e_1, \{x_1, x_2\}), (e_2, \{x_1, x_2\})\}$. The possible soft subsets are $F_{E_1} = \{(e_1, \{x_1\})\}$, $F_{E_2} = \{(e_1, \{x_2\})\}$, $F_{E_3} = \{(e_1, \{x_1, x_2\})\}$, $F_{E_4} = \{(e_2, \{x_1\})\}$, $F_{E_5} = \{(e_2, \{x_2\})\}$, $F_{E_6} = \{(e_2, \{x_1, x_2\})\}$, $F_{E_7} = \{(e_1, \{x_1\}), (e_2, \{x_1\})\}$, $F_{E_8} = \{(e_1, \{x_1\}), (e_2, \{x_2\})\}$, $F_{E_9} = \{(e_1, \{x_1\}), (e_2, \{x_1, x_2\})\}$, $F_{E_{10}} = \{(e_1, \{x_2\}), (e_2, \{x_1\})\}$, $F_{E_{11}} = \{(e_1, \{x_2\}), (e_2, \{x_2\})\}$, $F_{E_{12}} = \{(e_1, \{x_2\}), (e_2, \{x_1, x_2\})\}$, $F_{E_{13}} = \{(e_1, \{x_1, x_2\}), (e_2, \{x_1\})\}$, $F_{E_{14}} = \{(e_1, \{x_1, x_2\}), (e_2, \{x_2\})\}$, $F_{E_{15}} = \phi$, $F_{E_{16}} = \tilde{X}$.*

Definition 2.13 ([7]). *Let $\widetilde{\tau}$ be the collection of soft sets over X, then $\widetilde{\tau}$ is said to be a Soft Topology on $\widetilde{X}$ if*

(1). ϕ, $\widetilde{X}$ belongs to $\widetilde{\tau}$.

(2). The soft union of any number of soft sets in $\widetilde{\tau}$ belongs to $\widetilde{\tau}$.

(3). The soft intersection of any two soft sets in $\widetilde{\tau}$ belongs to $\widetilde{\tau}$.

The triplet $(\widetilde{X}, \widetilde{\tau}, E)$ is called a Soft Topological Space over X.

Definition 2.14 ([11]). *Let $\widetilde{X}$ be a non-empty soft set on the universe X with a parameter set E and $\widetilde{\tau}_1$, $\widetilde{\tau}_2$ are two different soft topologies on $\widetilde{X}$. Then $(\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ is called a soft bitopological space.*

Definition 2.15 ([11]). *Let $(\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ be a soft bitopological space and $F_A \widetilde{\subseteq} \widetilde{X}$. Then F_A is called $\widetilde{\tau_{1,2}}$-open if $F_A = F_B \widetilde{\cup} F_C$, where $F_B \widetilde{\in} \widetilde{\tau}_1$ and $F_C \widetilde{\in} \widetilde{\tau}_2$. The soft complement of $\widetilde{\tau_{1,2}}$-open set is called $\widetilde{\tau}_{1,2}$-closed.*

Definition 2.16 ([9]). *Let $\widetilde{X}$ be a soft bitopological space and $F_A \widetilde{\subseteq} \widetilde{X}$. Then F_A is called $(1,2)^*$-soft b-open set (briefly $(1,2)^*$-sb-open) if $F_A \widetilde{\subseteq} \widetilde{\tau}_{1,2}$-int($\widetilde{\tau}_{1,2}$-cl($F_A$)) $\widetilde{\cup}$ $\widetilde{\tau}_{1,2}$-cl($\widetilde{\tau}_{1,2}$-int(F_A)).*

Definition 2.17 ([9]). *Let $\widetilde{X}$ be a soft bitopological space and F_A be a soft set over $\widetilde{X}$.*

(1). $(1,2)^*$-soft b-closure (briefly $(1,2)^*$-sbcl(F_A)) of a set F_A in $\widetilde{X}$ is defined by $(1,2)^*$-sbcl(F_A) $= \widetilde{\cap} \left\{F_E \widetilde{\supseteq} F_A : F_E \text{ is } a(1,2)^* - soft\ b - closed \text{ set in} \widetilde{X}\right\}$.

(2). $(1,2)^*$-soft b-interior (briefly $(1,2)^*$-sbint(F_A)) of a set F_A in $\widetilde{X}$ is defined by $(1,2)^*$-sbint(F_A) $= \widetilde{\cup} \left\{F_B \widetilde{\subseteq} F_A : F_B \text{ is } a(1,2)^* - soft\ b - open \text{ set in} \widetilde{X}\right\}$.

Definition 2.18 ([7]). *Let $(\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ be a soft bitopological space over X and Y be non empty subset of X. Then $\widetilde{\tau}_{1Y} = \{(^Y F_E) : F_E \widetilde{\in} \widetilde{\tau}_1\}$ and $\widetilde{\tau}_{2Y} = \{(^Y G_E) : G_E \widetilde{\in} \widetilde{\tau}_2\}$ are said to be the relative soft topologies on $\widetilde{Y}$. Then $\left\{\widetilde{Y}, \widetilde{\tau}_{1Y}, \widetilde{\tau}_{2Y}, E\right\}$ is called the relative soft bitopological space of $(\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$.*

Definition 2.19 ([10]). *A soft mapping $\widetilde{f} : (\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E) \longrightarrow (\widetilde{Y}, \widetilde{\sigma_1}, \widetilde{\sigma_2}, E)$ is said to be $(1,2)^*$-soft b-continuous (briefly $(1,2)^*$-sb-continuous) if the inverse image of each $\widetilde{\sigma}_{1,2}$-open set of $\widetilde{Y}$ is $(1,2)^*$-sb-open set in $\widetilde{X}$.*

Definition 2.20 ([10]). *A soft mapping $\widetilde{f} : (\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E) \longrightarrow (\widetilde{Y}, \widetilde{\sigma_1}, \widetilde{\sigma_2}, E)$ is said to be $(1,2)^*$-soft b-irresolute (briefly $(1,2)^*$-sb-irresolute) if $\widetilde{f}^{-1}(F_A)$ is a $(1,2)^*$-sb-closed set in $\widetilde{X}$, for every $(1,2)^*$-sb-closed set F_A in $\widetilde{Y}$.*

Definition 2.21. *Let $(\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ be a soft bitopological space over X and $F_E \widetilde{\in} S(X).x_e \widetilde{\in} \widetilde{X}$ is said to be a $(1,2)^*$- soft b- limit point ($(1,2)^*$-sb-limit point) of F_E if every $(1,2)^*$-soft b-neighbourhood containing x_e contains a soft point of F_E other than x_e.*

Definition 2.22. *The collection of all $(1,2)^*$- soft b- limit points of F_E is called the $(1,2)^*$- soft b-derived set of F_E and is denoted by $(1,2)^*$- sbD(F_E).*

Definition 2.23 ([10]). *A soft mapping $\widetilde{f} : (\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E) \longrightarrow (\widetilde{Y}, \widetilde{\sigma_1}, \widetilde{\sigma_2}, E)$ is said to be $(1,2)^*$-soft b-open map (briefly $(1,2)^*$-sb-open) if the image of every $\tilde{\tau}_{1,2}$-open set of $\widetilde{X}$ is $(1,2)^*$-sb-open set in $\widetilde{Y}$.*

Definition 2.24 ([2]). *Let $(\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ be a soft bitopological space over X. Then the $(1,2)^*$-soft T_i axioms (where $i = 0, 1, 2$) are as follows.*

*$(1,2)^*$-**Soft** T_0 **axiom :** If for every x_e, $y_e \widetilde{\in} \widetilde{X}$ with $x_e \neq y_e$, there exist $\widetilde{\tau_{1,2}}$-open sets F_{E_1} and F_{E_2} such that either $x_e \widetilde{\in} F_{E_1}$ but $y_e \widetilde{\notin} F_{E_1}$ or $y_e \widetilde{\in} F_{E_2}$ but $x_e \widetilde{\notin} F_{E_2}$.*

$(1,2)^*$-***Soft*** T_1 ***axiom :*** *If for every* $x_e,\ y_e \widetilde{\in} \widetilde{X}$ *with* $x_e \neq y_e$, *there exist* $\widetilde{\tau_{1,2}}$*-open sets* F_{E_1} *and* F_{E_2} *such that* $x_e \widetilde{\in} F_{E_1}$ *but* $y_e \widetilde{\notin} F_{E_1}$ *and* $y_e \widetilde{\in} F_{E_2}$ *but* $x_e \widetilde{\notin} F_{E_2}$.

$(1,2)^*$-***Soft*** T_2 ***axiom :*** *If for every* $x_e,\ y_e \widetilde{\in} \widetilde{X}$ *with* $x_e \neq y_e$, *there exist* $\widetilde{\tau_{1,2}}$*- open sets* F_{E_1} *and* F_{E_2} *such that* $x_e \widetilde{\in} F_{E_1},\ y_e \widetilde{\in} F_{E_2}$ *and* $F_{E_1} \widetilde{\cap} F_{E_2} = \phi$.

3. $(1,2)^*$-soft b-Separation Axioms

In this section, we introduce and study the new concepts of $(1,2)^*$-soft b-separation axioms and investigated basic properties of these concepts in soft bitopological spaces.

Definition 3.1. *Let* $(\widetilde{X}, \widetilde{\tau_1}, \widetilde{\tau_2}, E)$ *be a soft bitopological space over* X *and for every soft points* $x_e, y_e \widetilde{\in} \widetilde{X}$ *with* $x_e \neq y_e$.*Then the soft bitopological space* $(\widetilde{X}, \widetilde{\tau_1}, \widetilde{\tau_2}, E)$ *is said to be* $(1,2)^*$*-soft* bT_0*-space* ($(1,2)^*$*-sb*T_0*-space*) *if there exists* $(1,2)^*$*-soft b-open sets* F_{E_1} *and* F_{E_2} *such that either* $x_e \widetilde{\in} F_{E_1}$ *but* $y_e \widetilde{\notin} F_{E_1}$ *or* $y_e \widetilde{\in} F_{E_2}$ *but* $x_e \widetilde{\notin} F_{E_2}$.

Example 3.2. *Let* $X = \{x, y\},\ E = \{e_1, e_2\},\ \widetilde{X} = \{(e_1, \{x, y\}), (e_2, \{x, y\})\}$. *The possible soft subsets are considered as in Example 2.12. Define* $\widetilde{\tau_1} = \{\widetilde{X}, \phi, F_{E_1}, F_{E_7}\}$ *and* $\widetilde{\tau_2} = \{\widetilde{X}, \phi, F_{E_3}\}$. *Then* $\widetilde{\tau_{1,2}}$*- open sets are* $(\widetilde{X}, F_{E_1}, F_{E_3}, F_{E_7}, F_{E_{13}}\}$ *and the collection of all* $(1,2)^*$*-soft b-open set is* $(1,2)^* - SbO(\widetilde{X}) = \{\widetilde{X}, \phi, F_{E_1}, F_{E_3}, F_{E_7}, F_{E_8}, F_{E_9}, F_{E_{13}}, F_{E_{14}}\}$. *Then* $(\widetilde{X}, \widetilde{\tau_1}, \widetilde{\tau_2}, E)$ *is a* $(1,2)^*$*-soft* bT_0*-space over* X.

Remark 3.1. *Every* $(1,2)^*$*-soft* bT_0*-space is soft bitopological space. But the following example shows that every soft bitopological space need not be* $(1,2)^*$*-soft* bT_0*-space .*

Example 3.3. *Consider the soft indiscrete bitopological space* $(\widetilde{X}, \widetilde{\tau_1}, \widetilde{\tau_2}, E)$ *over* X. *The only* $(1,2)^*$*-soft b-open sets are* ϕ *and* $\widetilde{X}$. *Now, the* $(1,2)^*$*-soft b-open set* $\widetilde{X}$ *contains* x_e *but it also contains* y_e. *Thus, there is no* $(1,2)^*$*-soft b-open set which contains* x_e *but does not contain* y_e. *Hence, it is not a* $(1,2)^*$*-soft* bT_0*-space.*

Proposition 3.4. *Let* $(\widetilde{X}, \widetilde{\tau_1}, \widetilde{\tau_2}, E)$ *be a soft bitopological space over* X *and* $x_e, y_e \widetilde{\in} \widetilde{X}$ *with* $x_e \neq y_e$, *then there exists* $(1,2)^*$*-soft b-open sets* F_{E_1} *and* F_{E_2} *such that either* $x_e \widetilde{\in} F_{E_1}$ *and* $y_e \widetilde{\in} F_{E_1}^C$ *or* $y_e \widetilde{\in} F_{E_2}$ *and* $x_e \widetilde{\in} F_{E_2}^C$. *Then, the soft bitopological space* $(\widetilde{X}, \widetilde{\tau_1}, \widetilde{\tau_2}, E)$ *is a* $(1,2)^*$*-soft* bT_0*-space.*

Proof. Let $x_e, y_e \widetilde{\in} \widetilde{X}$ with $x_e \neq y_e$ and let F_{E_1} and F_{E_2} be $(1,2)^*$-soft b-open sets such that either $x_e \widetilde{\in} F_{E_1}$ and $y_e \widetilde{\in} F_{E_1}^{\widetilde{C}}$ or $y_e \widetilde{\in} F_{E_2}$ and $x_e \widetilde{\in} F_{E_2}^{\widetilde{C}}$. If $x_e \widetilde{\in} F_{E_1}$ and $y_e \widetilde{\in} F_{E_1}^{\widetilde{C}}$, then $y_e \widetilde{\in} (F(e))^{\widetilde{C}}$ for all $e \widetilde{\in} E$.Therefore $y_e \widetilde{\notin} F_{E_1}$. Similarly, if $y_e \widetilde{\in} F_{E_2}$ and $x_e \widetilde{\in} F_{E_2}^{\widetilde{C}}$ then $x_e \widetilde{\notin} F_{E_2}$. Hence $(\widetilde{X}, \widetilde{\tau_1}, \widetilde{\tau_2}, E)$ is a $(1,2)^*$-soft bT_0-space. □

A characterization for $(1,2)^*$-soft bT_0-space is following.

Theorem 3.5. *A soft bitopological space* $(\widetilde{X}, \widetilde{\tau_1}, \widetilde{\tau_2}, E)$ *is* $(1,2)^*$*-soft* bT_0*-space over* X *if and only if* $(1,2)^*$*-sbcl* $\{x_e\} \neq (1,2)^*$*-sbcl*$\{y_e\}$ *for every pair of distinct soft point* $x_e,\ y_e$ *of* $\widetilde{X}$ *.*

Proof. Let $x_e,\ y_e \widetilde{\in} \widetilde{X}$ with $x_e \neq y_e$. Since $\widetilde{X}$ is $(1,2)^*$-soft bT_0-space, then there exists $(1,2)^*$-soft b-open sets F_E and G_E such that either $x_e \widetilde{\in} F_E$ but $y_e \widetilde{\notin} F_E$ or $y_e \widetilde{\in} G_E$ but $x_e \widetilde{\notin} G_E$. Since $\widetilde{X} \widetilde{\setminus} F_E$ is a $(1,2)^*$-soft b-closed set which does not contain x_e but y_e . By definition, $(1,2)^*$-sbcl(y_e) is the intersection of all $(1,2)^*$-soft b-closed set containing y_e . Therefore $(1,2)^*$-sbcl(y_e) $\widetilde{\subset} \widetilde{X} \widetilde{\setminus} F_E$. Hence $x_e \widetilde{\notin} \widetilde{X} \widetilde{\setminus} F_E$ implies that $x_e \widetilde{\notin} (1,2)^*$-sbcl$(y_e)$. Thus $x_e \widetilde{\in} (1,2)^*$-sbcl$(x_e)$ but $x_e \widetilde{\notin} (1,2)^*$-sbcl$(y_e)$. Hence $(1,2)^*$-sbcl $\{x_e\} \neq (1,2)^*$-sbcl$\{y_e\}$.

Conversely, assume that $x_e, y_e \widetilde{\in} \widetilde{X}$ with $x_e \neq y_e$ and $(1,2)^*$-sbcl $\{x_e\} \neq (1,2)^*$-sbcl$\{y_e\}$. Then by assumption, there exists atleast one soft point $z_e \widetilde{\in} \widetilde{X}$ such that $z_e \widetilde{\in} (1,2)^*$-sbcl $(\{x_e\})$ but $z_e \widetilde{\notin} (1,2)^*$-sbcl $(\{y_e\})$. Now we claim that $x_e \widetilde{\notin} (1,2)^*$-sbcl $(\{y_e\})$. Suppose not,$x_e \widetilde{\in} (1,2)^*$-sbcl $(\{y_e\})$ then $\{x_e\} \widetilde{\subset} (1,2)^*$-sbcl $(\{y_e\})$ which implies that $(1,2)^*$-sbcl $(\{x_e\}) \widetilde{\subset} (1,2)^*$-sbcl

$(\{y_e\})$. Hence $z_e \widetilde{\in}(1,2)^*$-sbcl $(\{x_e\})$ implies $z_e \widetilde{\in}(1,2)^*$-sbcl $(\{y_e\})$. This contradicts the fact that $z_e \widetilde{\notin}(1,2)^*$-sbcl $(\{y_e\})$. Therefore $x_e \widetilde{\notin}(1,2)^*$-sbcl $(\{y_e\})$. Now $x_e \widetilde{\in} [(1,2)^* - sbcl(\{y_e\})]^{\widetilde{C}}$ is a $(1,2)^*$-soft b-open set. Thus $[(1,2)^* - sbcl(\{y_e\})]^{\widetilde{C}}$ is a $(1,2)^*$-soft b-open set containing x_e but not y_e. Hence $(\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ is $(1,2)^*$-soft bT_0-space over X. □

Theorem 3.6. *A soft subspace of a $(1,2)^*$-soft bT_0-space is $(1,2)^*$-soft bT_0-space.*

Proof. Let $(\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ be a $(1,2)^*$-soft bT_0-space over X and $(\widetilde{Y}, \widetilde{\tau_{1Y}}, \widetilde{\tau_{2Y}}, E)$ be soft subspace of $(\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ over Y. Let $x_e,\ y_e \widetilde{\in} \widetilde{Y}$ such that $x_e \neq y_e$ and since $\widetilde{Y} \widetilde{\subseteq} \widetilde{X},\ x_e,\ y_e \widetilde{\in} \widetilde{X}$. Since $(\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ is a $(1,2)^*$-soft bT_0-space over X, there exists $(1,2)^*$-soft b-open sets F_{E_1} and F_{E_2} such that either $x_e \widetilde{\in} F_{E_1}$ but $y_e \widetilde{\notin} F_{E_1}$ or $y_e \widetilde{\in} F_{E_2}$ but $x_e \widetilde{\notin} F_{E_2}$. Now $x_e \widetilde{\in} \widetilde{Y} \widetilde{\cap} F_{E_1} = {}^Y F_{E_1}$ which is a $(1,2)^*$-soft b-open set in $(\widetilde{Y}, \widetilde{\tau_{1Y}}, \widetilde{\tau_{2Y}}, E)$. Consider $y_e \widetilde{\notin} F_{E_1}$, this implies that $y_e \widetilde{\notin} F(e)$ for some $e \widetilde{\in} E$. Therefore $y_e \widetilde{\notin} \widetilde{Y} \widetilde{\cap} F_{E_1} = {}^Y F_{E_1}$. Similarly if $y_e \widetilde{\in} F_{E_2}$ and $x_e \widetilde{\notin} F_{E_2}$, then $y_e \widetilde{\in} {}^Y F_{E_2}$ and $x_e \widetilde{\notin} {}^Y F_{E_2}$ .Thus $(\widetilde{Y}, \widetilde{\tau_{1Y}}, \widetilde{\tau_{2Y}}, E)$ is also a $(1,2)^*$-soft bT_0-space. □

Theorem 3.7. *Let $\widetilde{f} : (\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E) \longrightarrow (\widetilde{Y}, \widetilde{\sigma_1}, \widetilde{\sigma_2}, E)$ a bijective $(1,2)^*$-soft b-open mapping and if $\widetilde{X}$ is a $(1,2)^*$-soft T_0-space, then $\widetilde{Y}$ is a $(1,2)^*$-soft bT_0-space.*

Proof. Let y_{e_1}, y_{e_2} be two distinct soft points of $\widetilde{Y}$. Since $\widetilde{f}$ is bijective, there exists $x_{e_1}, x_{e_2} \widetilde{\in} \widetilde{X}$ such that $\widetilde{f}(x_{e_1}) = y_{e_1}$ and $\widetilde{f}(x_{e_2}) = y_{e_2}$. Since $\widetilde{X}$ is $(1,2)^*$-soft T_0-space, then there exists $\widetilde{\tau_{1,2}}$-open sets $G_{E_1}$and G_{E_2} of $\widetilde{X}$ such that $x_{e_1} \widetilde{\in} G_{E_1}$ but $x_{e_2} \widetilde{\notin} G_{E_1}$ or $x_{e_2} \widetilde{\in} G_{E_2}$ but $x_{e_1} \widetilde{\notin} G_{E_2}$. But $\widetilde{f}$ is a $(1,2)^*$-soft b-open mapping,then $\widetilde{f}(F_{A_1}), \widetilde{f}(F_{A_2})$ are $(1,2)^*$-soft b-open sets in $\widetilde{Y}$ with $y_{e_1} \widetilde{\in} \widetilde{f}(F_{A_1})$ but $y_{e_2} \widetilde{\notin} \widetilde{f}(F_{A_1})$ or $y_{e_2} \widetilde{\in} \widetilde{f}(F_{A_2})$ but $y_{e_1} \widetilde{\notin} \widetilde{f}(F_{A_2})$. Therefore $\widetilde{Y}$ is a $(1,2)^*$-soft bT_0-space. □

Theorem 3.8. *Let $\widetilde{f} : (\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E) \longrightarrow (\widetilde{Y}, \widetilde{\sigma_1}, \widetilde{\sigma_2}, E)$ a injective $(1,2)^*$-soft b- irresolute mapping and if $\widetilde{Y}$ is a $(1,2)^*$-soft bT_0-space, then $\widetilde{X}$ is a $(1,2)^*$-soft bT_0-space.*

Proof. Let $x_e, y_e \widetilde{\in} \widetilde{X}$ with $x_e \neq y_e$. Since $\widetilde{f}$ is injective and $\widetilde{Y}$ is $(1,2)^*$-soft bT_0-space, then there exists $(1,2)^*$-soft b-open sets F_{E_1} and F_{E_2} such that either $f(x_e) \widetilde{\in} F_{E_1}$ but $f(y_e) \widetilde{\notin} F_{E_1}$or $f(y_e) \widetilde{\in} F_{E_2}$ but $f(x_e) \widetilde{\notin} F_{E_2}$ with $f(x_e) \neq f(y_e)$. Since $\widetilde{f}$ is $(1,2)^*$-soft b- irresolute mapping, $f^{-1}(F_{E_1})$ and $f^{-1}(F_{E_2})$ are in $(1,2)^*$-soft b-open sets in $\widetilde{X}$ such that $x_e \widetilde{\in} f^{-1}(F_{E_1})$ but $y_e \widetilde{\notin} f^{-1}(F_{E_1})$ or $y_e \widetilde{\in} f^{-1}(F_{E_2})$ but $x_e \widetilde{\notin} f^{-1}(F_{E_2})$. Thus $\widetilde{X}$ is a $(1,2)^*$-soft bT_0-space. □

Definition 3.9. *Let $(\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ be a soft bitopological space over X and for every soft points $x_e,\ y_e \widetilde{\in} \widetilde{X}$ with $x_e \neq y_e$. Then the soft bitopological space $(\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ is said to be $(1,2)^*$-soft bT_1-space ($(1,2)^*$-sbT_1-space) if there exists $(1,2)^*$-soft b-open sets F_{E_1} and F_{E_2} such that either $x_e \widetilde{\in} F_{E_1}$ but $y_e \widetilde{\notin} F_{E_1}$ and $y_e \widetilde{\in} F_{E_2}$ but $x_e \widetilde{\notin} F_{E_2}$.*

Example 3.10. *Let $X = \{x,y,z\}, E = \{e_1\}$ the soft subsets of X is $SS_E(X)$ and $|S(X)| = 8$. They are $\widetilde{X},\ \phi,\ G_{E_1} = \{(e_1,\{x\})\},\ G_{E_2} = \{(e_1,\{y\})\}, G_{E_3} = \{(e_1,\{z\})\}, G_{E_4} = \{(e_1,\{x,y\})\},\ G_{E_5} = \{(e_1,\{x,z\})\},\ G_{E_6} = \{(e_1,\{y,x\})\}$. Define $\widetilde{\tau}_1 = \{\widetilde{X}, \phi, G_{E_4}\}$ and $\widetilde{\tau}_2 = \{\widetilde{X}, \phi, G_{E_6}\}$. Then $\widetilde{\tau_{1,2}}$-open sets are $\{\widetilde{X}, \phi, G_{E_4}, G_{E_6}\}$. Then $(\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ is a soft bitopological space. The collection of $(1,2)^*$-soft b-open sets are $(1,2)^*$-$SbO(\widetilde{X}) = \{\widetilde{X}, \phi, G_{E_2}, G_{E_4}, G_{E_5}, G_{E_6}\}$. and $(1,2)^*$-soft b-closed sets are $(1,2)^*$-$SbC(\widetilde{X}) = \{\widetilde{X}, \phi, G_{E_5}, G_{E_3}, G_{E_2}, G_{E_1}\}$. Then this soft bitopological space is $(1,2)^*$-soft bT_1-space .*

Proposition 3.11. *Let $(\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ be a soft bitopological space over X and $x_e, y_e \widetilde{\in} \widetilde{X}$ such that $x_e \neq y_e$. If there exists $(1,2)^*$-soft b-open sets F_{E_1} and F_{E_2} such that $x_e \widetilde{\in} F_{E_1}$ but $y_e \widetilde{\in} F^C_{E_1}$ and $y_e \widetilde{\in} F_{E_2}$ but $x_e \widetilde{\in} F^C_{E_2}$. Then, the soft bitopological space $(\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ is a $(1,2)^*$-soft bT_1-space.*

Proof. It is similar to the proof of proposition 3.4 □

The following theorem is a characterization for $(1,2)^*$-soft bT_1-space.

Theorem 3.12. *Let $(\widetilde{X},\widetilde{\tau}_1,\widetilde{\tau}_2,E)$ be a $(1,2)^*$-soft bT_1-space over X if and only if for each $x_e \widetilde{\in} \widetilde{X}$, every soft singleton$\{x_e\}$ over X is $(1,2)^*$-soft b-closed set.*

Proof. Suppose that $(\widetilde{X},\widetilde{\tau}_1,\widetilde{\tau}_2,E)$ is a $(1,2)^*$-soft bT$_1$-space over X and $x_e \widetilde{\in} \widetilde{X}$. Now we have to prove that the soft singleton set $\{x_e\}$ over X is $(1,2)^*$-soft b-closed set. Suppose $\{x_e\}$ is not $(1,2)^*$-soft b-closed . Then $(1,2)^*$-sbcl$(\{x_e\}) \neq \{x_e\}$. So there exists $y_e \neq x_e$, $y_e \widetilde{\in}(1,2)^*$-sbcl$(\{x_e\})$. This contradicts the fact that $(\widetilde{X},\widetilde{\tau}_1,\widetilde{\tau}_2,E)$ be a $(1,2)^*$-soft bT$_1$-space. Therefore, soft singleton$\{x_e\}$ over X is $(1,2)^*$-soft b-closed set.
Conversely, suppose the soft singleton $\{x_e\}$ is $(1,2)^*$-soft b-closed for every $x_e \widetilde{\in} \widetilde{X}$. Since $\{x_e\}$ is $(1,2)^*$-soft b-closed, $\{x_e\}^C$ is $(1,2)^*$-soft b-open set in $\widetilde{X}$. Let $x_e, y_e \widetilde{\in} \widetilde{X}$ and $x_e \neq y_e$ such that $\{x_e\}$ and$\{y_e\}$ are $(1,2)^*$-soft b-closed sets, then $\{x_e\}^C$ and $\{y_e\}^C$ are $(1,2)^*$-soft b-open sets. Therefore $y_e \widetilde{\in} \{x_e\}^C$ but$x_e \widetilde{\notin} \{x_e\}^C$ and $x_e \widetilde{\in} \{y_e\}^C$ but $y_e \widetilde{\notin} \{y_e\}^C$.Thus $(\widetilde{X},\widetilde{\tau}_1,\widetilde{\tau}_2,E)$ is a $(1,2)^*$-soft bT$_1$-space over X. □

Theorem 3.13. *A soft subspace of a $(1,2)^*$-soft bT_1-space is $(1,2)^*$-soft bT_1-space.*

Proof. Let $(\widetilde{X},\widetilde{\tau}_1,\widetilde{\tau}_2,E)$ be a $(1,2)^*$-soft bT$_1$-space over X and $(\widetilde{Y},\widetilde{\tau_{1Y}},\widetilde{\tau_{2Y}},E)$ be soft subspace of $(\widetilde{X},\widetilde{\tau}_1,\widetilde{\tau}_2,E)$ over Y. Let $x_e,\ y_e \widetilde{\in} \widetilde{Y}$ such that $x_e \neq y_e$. Since $\widetilde{Y} \widetilde{\subseteq} \widetilde{X}, x_e, y_e \widetilde{\in} \widetilde{X}$ and $x_e \neq y_e$. Since $(\widetilde{X},\widetilde{\tau}_1,\widetilde{\tau}_2,E)$ is a $(1,2)^*$-soft bT$_1$-space over X, there exists $(1,2)^*$-soft b-open sets F_{E_1} and F_{E_2} in $\widetilde{X}$ such that $x_e \widetilde{\in}\ F_{E_1}$ but $y_e \widetilde{\notin} F_{E_1}$ and $y_e \widetilde{\in} F_{E_2}$ but $x_e \widetilde{\notin} F_{E_2}$. Hence $x_e \widetilde{\in} \widetilde{Y} \widetilde{\cap}\ F_{E_1} = {}^{Y}F_{E_1}$ which is a $(1,2)^*$-soft b-open set in $(\widetilde{Y},\widetilde{\tau_{1Y}},\widetilde{\tau_{2Y}},E)$. Since $y_e \widetilde{\notin} F_{E_1}, y_e \widetilde{\notin} \widetilde{Y} \widetilde{\cap}\ F_{E_1} = {}^{Y}F_{E_1}$. Similarly if $y_e \widetilde{\in}\ F_{E_2}$ and $x_e \widetilde{\notin} F_{E_2}$, then $y_e \widetilde{\in}\ {}^{Y}\ F_{E_2}$ but $x_e \widetilde{\notin}\ {}^{Y}\ F_{E_2}$. Thus $(\widetilde{Y},\widetilde{\tau_{1Y}},\widetilde{\tau_{2Y}},E)$ is also a $(1,2)^*$-soft bT$_1$-space. □

Proposition 3.14. *Every $(1,2)^*$-soft bT_1- space is $(1,2)^*$-soft bT_0- space.*

Proof. Let $(\widetilde{X},\widetilde{\tau}_1,\widetilde{\tau}_2,E)$ be a $(1,2)^*$-soft bT$_1$-space. Then for every $x_e,\ y_e \widetilde{\in} \widetilde{X}$ with $x_e \neq y_e$, there exist $(1,2)^*$-soft b-open sets F_{E_1} and F_{E_2} such that $x_e \widetilde{\in} F_{E_1}$ but $y_e \widetilde{\notin} F_{E_1}$ and $y_e \widetilde{\in} F_{E_2}$ but $x_e \widetilde{\notin} F_{E_2}$. Therefore $(\widetilde{X},\widetilde{\tau}_1,\widetilde{\tau}_2,E)$ is $(1,2)^*$-soft bT$_0$- space. □

The converse of the above proposition need not be true.

Example 3.15. *Let $X = \{x,y\}$, $E = \{e_1,e_2\}$, $\widetilde{X} = \{(e_1,\{x,y\}),(e_2,\{x,y\})\}$. The possible soft subsets are considered as in Example 2.12.*
Define $\widetilde{\tau}_1 = \{\widetilde{X},\phi,F_{E_1},F_{E_7}\}$ and $\widetilde{\tau}_2 = \{\widetilde{X},\phi,F_{E_3}\}$. Then $\widetilde{\tau_{1,2}}$-soft open sets are $(\widetilde{X},F_{E_1},F_{E_3},F_{E_7},F_{E_{13}}\}$ and the collection of all $(1,2)^*$-soft b-open sets are
$(1,2)^* - SbO(\widetilde{X}) = \{\widetilde{X},\phi,F_{E_1},F_{E_3},F_{E_7},F_{E_8},F_{E_9},F_{E_{13}},F_{E_{14}}\}$. and $(1,2)^*$-soft b-closed sets are $(1,2)^* - SbC(\widetilde{X}) = \{\widetilde{X},\phi,F_{E_12},F_{E_6},F_{E_11},F_{E_10},F_{E_2},F_{E_5},F_{E_4}\}$. Then $(\widetilde{X},\widetilde{\tau}_1,\widetilde{\tau}_2,E)$ is a $(1,2)^*$-soft bT_0-space over X but not $(1,2)^*$-soft bT_1-space over X. Since the soft singleton set F_{E_1} is not $(1,2)^*$-soft b-closed set.

Theorem 3.16. *If every finite soft subset of a soft bitopological space $(\widetilde{X},\widetilde{\tau}_1,\widetilde{\tau}_2,E)$ is $(1,2)^*$-soft b-closed set ,then $(\widetilde{X},\widetilde{\tau}_1,\widetilde{\tau}_2,E)$ is $(1,2)^*$-soft bT_1-space.*

Proof. Let $x_e,\ y_e \widetilde{\in} \widetilde{X}$ with $x_e \neq y_e$. Then by hypothesis , $\{x_e\}$ and $\{y_e\}$ are $(1,2)^*$-soft b-closed sets which implies that $\{x_e\}^C$ and $\{y_e\}^C$ are $(1,2)^*$-soft b-open sets such that $x_e \widetilde{\in} \{y_e\}^C$ and$y_e \widetilde{\in} \{x_e\}^C$. Therefore $(\widetilde{X},\widetilde{\tau}_1,\widetilde{\tau}_2,E)$ is $(1,2)^*$-soft bT$_1$-space. □

Theorem 3.17. *Let $\widetilde{f} : (\widetilde{X},\widetilde{\tau}_1,\widetilde{\tau}_2,E) \longrightarrow (\widetilde{Y},\widetilde{\sigma_1},\widetilde{\sigma_2},E)$ a bijective $(1,2)^*$-soft b-open mapping and if $\widetilde{X}$ is a $(1,2)^*$-softT_1-space, then $\widetilde{Y}$ is a $(1,2)^*$-soft bT_1-space.*

Proof. Let y_{e_1}, y_{e_2} be two distinct soft points of $\widetilde{Y}$. Since $\widetilde{f}$ is bijective, there exists x_{e_1},x_{e_2} $\widetilde{\in}$ $\widetilde{X}$ such that $\widetilde{f}(x_{e_1}) = y_{e_1}$ and $\widetilde{f}(x_{e_2}) = y_{e_2}$. Since $\widetilde{X}$ is $(1,2)^*$-softT_1-space, then there exists $\widetilde{\tau_{1,2}}$-open sets G_{E_1} and G_{E_2} of $\widetilde{X}$ such that x_{e_1} $\widetilde{\in}$ G_{E_1} but $x_{e_2}$$\widetilde{\notin}$ G_{E_1} and x_{e_2} $\widetilde{\in}$ G_{E_2} but x_{e_1} $\widetilde{\notin}$ G_{E_2}. But $\widetilde{f}$ is a $(1,2)^*$-soft b-open mapping ,then $\widetilde{f}(F_{A_1}), \widetilde{f}(F_{A_2})$ are $(1,2)^*$-soft b-open sets in $\widetilde{Y}$ with y_{e_1} $\widetilde{\in}$ $\widetilde{f}(F_{A_1})$ but y_{e_2} $\widetilde{\notin}$ $\widetilde{f}(F_{A_1})$ and y_{e_2} $\widetilde{\in}$ $\widetilde{f}(F_{A_2})$ but y_{e_1} $\widetilde{\notin}$ $\widetilde{f}(F_{A_2})$ .Therefore $\widetilde{Y}$ is a $(1,2)^*$-soft bT$_1$-space. □

Theorem 3.18. *Let $\widetilde{f} : (\widetilde{X}, \widetilde{\tau_1}, \widetilde{\tau_2}, E) \longrightarrow (\widetilde{Y}, \widetilde{\sigma_1}, \widetilde{\sigma_2}, E)$ a injective $(1,2)^*$-soft b- irresolute mapping and if $\widetilde{Y}$ is a $(1,2)^*$-soft bT_1-space, then $\widetilde{X}$ is a $(1,2)^*$-soft bT_1-space.*

Proof. The proof of the theorem is similar to the Theorem 3.8. □

Definition 3.19. *Let $(\widetilde{X}, \widetilde{\tau_1}, \widetilde{\tau_2}, E)$ be a soft bitopological space over X and for every soft points x_e, y_e $\widetilde{\in}$ $\widetilde{X}$ with $x_e \neq y_e$. Then the soft bitopological space $(\widetilde{X}, \widetilde{\tau_1}, \widetilde{\tau_2}, E)$ is said to be $(1,2)^*$-soft bT_2-space ($(1,2)^*$-sbT_2-space) or$(1,2)^*$-soft b- Housdroff space if there exists $(1,2)^*$-soft b-open sets F_{E_1} and F_{E_2} such that x_e $\widetilde{\in}$ F_{E_1}, y_e $\widetilde{\in}$ F_{E_2} and $F_{E_1} \widetilde{\cap} F_{E_2} = \phi$.*

Example 3.20. *Consider a $(1,2)^*$-soft discrete bitopological space$(\widetilde{X}, \widetilde{\tau_1}, \widetilde{\tau_2}, E)$. Let x_e, y_e be two distinct soft points of $\widetilde{X}$. And $\{x_e\}, \{y_e\}$ are $(1,2)^*$-soft b-open sets of x_e and y_e respectively such that $\{x_e\}$ $\widetilde{\cap}$ $\{y_e\} = \phi$. Hence $(\widetilde{X}, \widetilde{\tau_1}, \widetilde{\tau_2}, E)$ is a $(1,2)^*$-soft bT_2-space or$(1,2)^*$-soft b- Housdroff space.*

Theorem 3.21. *A soft subspace of a $(1,2)^*$-soft bT_2-space is $(1,2)^*$-soft bT_2-space.*

Proof. Let $(\widetilde{X}, \widetilde{\tau_1}, \widetilde{\tau_2}, E)$ be a $(1,2)^*$-soft bT$_2$-space over X and $(\widetilde{Y}, \widetilde{\tau_{1Y}}, \widetilde{\tau_{2Y}}, E)$ be soft subspace of $(\widetilde{X}, \widetilde{\tau_1}, \widetilde{\tau_2}, E)$ over Y. Let x_e, y_e $\widetilde{\in}$ $\widetilde{Y}$ such that $x_e \neq y_e$. Then x_e, y_e $\widetilde{\in}$ $\widetilde{X}$ and $x_e \neq y_e$ .Since $(\widetilde{X}, \widetilde{\tau_1}, \widetilde{\tau_2}, E)$ is a $(1,2)^*$-soft bT$_2$-space over X, there exists $(1,2)^*$-soft b-open sets F_{E_1} and F_{E_2} in $\widetilde{X}$ such that x_e $\widetilde{\in}$ $F_{E_1}$and $y_e$$\widetilde{\in}$ F_{E_2} and $F_{E_1} \widetilde{\cap} F_{E_2} = \phi$. It follows that x_e $\widetilde{\in}$ $F_{E_1}(e), y_e$ $\widetilde{\in}$ $F_{E_2}(e)$ and $F_{E_1}(e) \widetilde{\cap} F_{E_2}(e) = \phi$ for all $e \in E$. Thus x_e $\widetilde{\in}$ $\widetilde{Y} \widetilde{\cap} F_{E_1} = {}^{Y}F_{E_1}$, y_e $\widetilde{\in}$ $\widetilde{Y} \widetilde{\cap} F_{E_2} = {}^{Y}F_{E_2}$ and ${}^{Y}F_{E_1} \widetilde{\cap} {}^{Y}F_{E2} = \phi$, where, ${}^{Y}F_{E_1}$, ${}^{Y}F_{E_2}$ are $(1,2)^*$-soft b-open sets in $\widetilde{Y}$. Therefore $(\widetilde{Y}, \widetilde{\tau_{1Y}}, \widetilde{\tau_{2Y}}, E)$ is a $(1,2)^*$-soft bT$_2$-space. □

The characterization for $(1,2)^*$-soft b-Housdroff space is following.

Theorem 3.22. *A soft bitopological space $(\widetilde{X}, \widetilde{\tau_1}, \widetilde{\tau_2}, E)$ is a $(1,2)^*$-soft bT_2-space over X if and only if for distinct points x_e, y_e of $\widetilde{X}$, there exists a $(1,2)^*$-soft b-poen set F_A containing x_e but not y_e such that y_e $\widetilde{\notin}$ $(1,2)^*$-sbcl(F_A).*

Proof. Let x_e and y_e be two distinct soft points in $(1,2)^*$-soft bT$_2$-space $(\widetilde{X}, \widetilde{\tau_1}, \widetilde{\tau_2}, E)$. Then there exists disjoint $(1,2)^*$-soft b-open sets F_A and G_B such that x_e $\widetilde{\in}$ F_A and y_e $\widetilde{\in}$ G_B. This implies that x_e $\widetilde{\in}$ ${G_B}^{\widetilde{C}}$. So ${G_B}^{\widetilde{C}} = F_A$ is a $(1,2)^*$-soft b-closed set containing x_e but not y_e and $(1,2)^*$-sbcl$(F_A) = F_A$. Hence y_e $\widetilde{\notin}$ $(1,2)^*$-sbcl(F_A).

On the other hand, let x_e and y_e be two distinct soft points in $(\widetilde{X}, \widetilde{\tau_1}, \widetilde{\tau_2}, E)$. Then there exists a $(1,2)^*$-soft b-poen set F_A containing x_e but not y_e such that y_e $\widetilde{\notin}$ $(1,2)^*$-sbcl(F_A). This implies that y_e $\widetilde{\in}$ $[(1,2)^* - sbcl(\{F_A\})]^{\widetilde{C}}$. Hence F_A and $[(1,2)^* - sbcl(\{F_A\})]^{\widetilde{C}}$ are two disjoint $(1,2)^*$-soft b-open sets containing x_e and y_e respectively. Thus $(\widetilde{X}, \widetilde{\tau_1}, \widetilde{\tau_2}, E)$ is a $(1,2)^*$-soft bT$_2$-space over X □

Theorem 3.23. *Let$(\widetilde{X}, \widetilde{\tau_1}, \widetilde{\tau_2}, E)$ be a $(1,2)^*$-soft bT_2-space over X and x_e $\widetilde{\in}$ X. Then every soft singleton $\{x_e\}$ is $(1,2)^*$-soft b-closed.*

Proof. Let $(\widetilde{X}, \widetilde{\tau_1}, \widetilde{\tau_2}, E)$ be a $(1,2)^*$-soft bT$_2$-space over X. Let x_e, y_e $\widetilde{\in}$ $\widetilde{X}$ and $x_e \neq y_e$, then there exists $(1,2)^*$-soft b-open sets F_{E_1} and F_{E_2} such that x_e $\widetilde{\in}$ F_{E_1}, y_e $\widetilde{\in}$ F_{E_2} and $F_{E_1} \widetilde{\cap} F_{E_2} = \phi$. Since F_{E_2} is a $(1,2)^*$-soft b-open set containing y_e such that F_{E_2} does not contain x_e or F_{E_2} does not contain any other soft point of $\{x_e\}$. Hence a soft point y_e of $\widetilde{X}$

distinct from x_e cannot be a $(1,2)^*$-soft b-limit point of $\{x_e\}$. Hence $(1,2)^*$-soft b-derived set of x_e is $(1,2)^*$-sbD$\{x_e\} = \phi$ and since $(1,2)^*$-sbcl$\{x_e\} = \{x_e\} \tilde{\cup} (1,2)^*$-sbD$\{x_e\} = \{x_e\} \tilde{\cup} \phi = \{x_e\}$. Hence $\{x_e\}$ is $(1,2)^*$-soft b-closed. □

Proposition 3.24. *Every $(1,2)^*$-soft bT_2- space is $(1,2)^*$-soft bT_1- space.*

Proof. Let $(\widetilde{X}, \widetilde{\tau_1}, \widetilde{\tau_2}, E)$ be a $(1,2)^*$-soft bT_2-space. Then, for every $x_e,\ y_e \tilde{\in} \widetilde{X}$ and $x_e \neq y_e$, there exist $(1,2)^*$-soft b-open sets F_{E_1} and F_{E_2} of x_e and y_e such that $F_{E_1} \tilde{\cap} F_{E_2} = \phi$. $x_e \tilde{\in} F_{E_1} \Rightarrow x_e \tilde{\notin} F_{E_2}$ as $F_{E_1} \tilde{\cap} F_{E_2} = \phi$, similarly, $y_e \tilde{\in} F_{E_2}$. This implies that $y_e \tilde{\notin} F_{E_1}$. Hence, $x_e \tilde{\in} F_{E_1}$ but $y_e \tilde{\notin} F_{E_1}$ and $y_e \tilde{\in} F_{E_2}$ but $x_e \tilde{\notin} F_{E_2}$. Therefore, the soft bitopological space $(\widetilde{X}, \widetilde{\tau_1}, \widetilde{\tau_2}, E)$ is a $(1,2)^*$-soft bT_1-space. □

The converse of the above proposition is not true is shown in the following Example.

Example 3.25. *Let $X = \{x, y, z\}, E = \{e_1\}$ the soft subsets of X is given as in the Example 3.10. Define $\widetilde{\tau_1} = \{\widetilde{X}, \phi, G_{E_4}\}$ and $\widetilde{\tau_2} = \{\widetilde{X}, \phi, G_{E_6}\}$. Then $\widetilde{\tau_{1,2}}$-open sets are $\{\widetilde{X}, \phi, G_{E_4}, G_{E_6}\}$. Then $(\widetilde{X}, \widetilde{\tau_1}, \widetilde{\tau_2}, E)$ is a soft bitopological space. The collection of $(1,2)^*$-soft b-open sets are $(1,2)^*$-SbO$(\widetilde{X}) = \{\widetilde{X}, \phi, G_{E_2}, G_{E_4}, G_{E_5}, G_{E_6}\}$ and $(1,2)^*$-soft b-closed sets are $(1,2)^*$-SbC$(\widetilde{X}) = \{\widetilde{X}, \phi, G_{E_5}, G_{E_3}, G_{E_2}, G_{E_1}\}$. Then this soft bitopological space is $(1,2)^*$-soft bT_1-space. Since every soft singleton set is $(1,2)^*$-soft b-closed set.*
Consider the soft points $(e_1, \{x\}), (e_1, \{z\}) \tilde{\in} \widetilde{X}$ and $(e_1, \{x\}) \neq (e_1, \{y\})$; there does not exists disjoint $(1,2)^*$-soft b-open sets.Then $(\widetilde{X}, \widetilde{\tau_1}, \widetilde{\tau_2}, E)$ is not a $(1,2)^*$-soft bT_2-space.

Theorem 3.26. *Let $\widetilde{f} : (\widetilde{X}, \widetilde{\tau_1}, \widetilde{\tau_2}, E) \longrightarrow (\widetilde{Y}, \widetilde{\sigma_1}, \widetilde{\sigma_2}, E)$ be a bijective $(1,2)^*$-soft b-open mapping and if $\widetilde{X}$ is a $(1,2)^*$-softT_2-space, then $\widetilde{Y}$ is a $(1,2)^*$-soft bT_2-space.*

Theorem 3.27. *Let $\widetilde{f} : (\widetilde{X}, \widetilde{\tau_1}, \widetilde{\tau_2}, E) \longrightarrow (\widetilde{Y}, \widetilde{\sigma_1}, \widetilde{\sigma_2}, E)$ a injective $(1,2)^*$-soft b- irresolute mapping and if $\widetilde{Y}$ is a $(1,2)^*$-soft bT_2-space, then $\widetilde{X}$ is a $(1,2)^*$-soft bT_2-space.*

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