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On Mildly B-Normal Spaces and Some Functions

Research Article

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Abstract: In this paper, by using Bg-closed sets we obtain a characterization of mildly B-normal spaces and use it to improve the preservation theorems of mildly B-normal spaces.

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Keywords: Bg-closed sets, mildly B-normal spaces, characterization of mildly B-normal spaces.

1. Introduction

The notion of mildly normal spaces was introduced by Singal and Singal [14]. Palaniappan and Rao [12] have defined and investigated the notion of regular g-closed sets as a generalization of g-closed sets due to Levine [6]. In this paper, by using regular Bg-closed sets we obtain a characterization of mildly B-normal simply extended topological spaces.

2. Preliminaries

Throughout this paper, $(X, \tau(B_X))$, $(Y, \sigma(B_Y))$ *and* $(Z, \eta(B_Z))$(briefly X, Y and Z) will denote simply extended topological spaces.

Definition 2.1. *A subset A of a topological space X is said to be*

(1). regular open [5] if A = int(cl(A));

(2). regular g-closed (briefly rg-closed) [12] if $cl(A) \subset U$ *whenever* $A \subset U$ *and U is a regular open set in X.*

(3). generalized closed (briefly g-closed) [6] if $cl(A) \subset U$ *whenever* $A \subset U$ *and U is open in X.*

(4). rg-open (resp. g-open, regular closed) if the complement of A is rg-closed (resp. g-closed, regular open). The family of all regular open (resp. regular closed) sets of X is denoted by RO(X) (resp. RC(X)).

Definition 2.2 ([15])**.** *A topological space X is said to be mildly normal if for every pair of disjoint* $H, K \in RC(X)$*, there exist disjoint open sets U, V of X such that* $H \subset U$ *and* $K \subset V$*.*

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Definition 2.3 ([12]). *A subset A of X is said to be quasi H-closed relative to X , if for every cover $\{V_\alpha : \alpha \in \nabla\}$ of A by open sets of X, there exists a finite subset ∇_0 of ∇ such that $A \subset \cup\{cl(V_\alpha) : \alpha \in \nabla_0\}$.*

Definition 2.4 ([5]). *A subset a of a space X is said to be α-regular if for each point of $x \in A$ and each open set U of X containing x, there exists an open set G of X such that $x \in G \subset cl(G) \subset U$.*

Definition 2.5 ([13]). *A subset a of a topological space X is said to be α-paracompact if every cover of A by open sets of X is defined by a cover of A which consists of open sets of X and is locally finite in X.*

Definition 2.6 ([14]). *A topological space X is said to be mildly-normal if for every pair of disjoint H, $K \in RC(X)$, there exist disjoint open sets U, V of X such that $H \subset U$ and $K \subset V$.*

Definition 2.7 ([10]). *A function $f : X \to Y$ is said to be almost g-continuous (resp. almost rg-continuous) if $f^{-1}(R)$ is g-closed (resp. rg-closed) in X, for every $R \in RC(Y)$.*

Definition 2.8. *A function $f : X \to Y$ is said to be*

(1). g-continuous [3] (resp. rg-continuous [12]) if $f^{-1}(F)$ is g-closed (resp. rg-closed) in X for every closed set F of Y;

(2). R-map [4], rc-continuous [4] or regular irresolute [12] (resp. almost continuous [14]) if $f^{-1}(V) \in RO(X)$ (resp. $\tau(X)$) for every $V \in RO(Y)$;

(3). completely continuous [1] or regular continuous [12] if $f^{-1}(V) \in RO(X)$ for every open set V of Y.

Definition 2.9 ([10]). *A topological space X is said to be regular-$T_{1/2}$ if every rg-closed set of X is regular closed.*

Definition 2.10 ([12]). *A function $f : X \to Y$ is said to be rg-irresolute if $f^{-1}(F)$ is rg-closed in X for every rg-closed set F of Y.*

Definition 2.11. *A function $f : X \to Y$ is said to be*

(1). regular closed [12] (resp. g-closed [8], rg-closed [10]) if f(F) is regular closed (resp. g-closed, rg-closed [10]) in Y for every closed set F of X;

(2). rc-preserving [10] (resp. almost closed [14], almost g-closed [10], almost rg-closed [10]) if f(F) is regular closed (resp. closed, g-closed, rg-closed) in Y for every $F \in RC(X)$.

Remark 2.12 ([11]). *In among others, it is shown that a compact set of a regular space is rg-closed.*

Definition 2.13 ([7]). *Levine in 1964 defined $\tau(B) = \{O \cup (\acute{O} \cap B) : O, \acute{O} \in \tau\}$ and called it simple extension of τ by B, where $B \notin \tau$. The sets in $\tau(B)$ are called B-open sets. and the complement of B-open set is called B-closed.*

Definition 2.14 ([7]). *Let S be a subset of a simply extended topological space X. Then*

(1). The B-closure of S, denoted by Bcl(S), is defined as $\cap \{F : S \subseteq F \text{ and } F \text{ is B-closed}\}$;

(2). The B-interior of S, denoted by Bint(S), is defined as $\cup \{F : F \subseteq S \text{ and } F \text{ is B-open}\}$.

Definition 2.15. *A subset A of a simply extended topological space $(X, \tau(B_X))$ is called Bg-closed set [2] if $BCl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X. The complement of Bg-closed set is called Bg-open set.*

Definition 2.16 ([9]). *A function $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ is called B-continuous if $f^{-1}(V)$ is B-open in X, for every B-open set V of Y.*

3. Regular Bg-closed Sets

Definition 3.1. *A subset A is said to be regular B-open (resp. regular B-closed) if A=Bint(Bcl(A)) (resp. A= Bcl(Bint(A))). The family of regular B-open (resp. regular B-closed) sets of a simply extended topological space X is denoted by BRO(X) (resp. BRC(X)).*

Definition 3.2. *A subset A of a simply extended topological space X is said to be*

(1). regular Bg-closed (briefly rBg-closed) if $Bcl(A) \subset U$ *whenever* $A \subset U$ *and* $U \in BRO(X)$.

(2). B-generalized closed (briefly Bg-closed) if $Bcl(A) \subset U$ *whenever* $A \subset U$ *and U is B-open in X.*

(3). rBg-open (resp. Bg-open) if the complement of A is rBg-closed (resp.Bg-closed).

Result 3.3. *We have the following implications for properties of subsets:*

regular B-closed ⇒ B-closed ⇒ Bg-closed ⇒ rBg-closed.

Where none of these implications is reversible as shown by Examples (below).

Example 3.4. *Let* $X = \{a, b, c\}$, $\tau = \{X, \emptyset\}$ *and* $B = \{b, c\}$ *then* $\tau(B) = \{\phi, X, \{b, c\}\}$. *Then*

(1). $\{a,\ b\}$ *is Bg-closed but not B-closed.*

(2). $\{b\}$ *is Brg-closed but not Bg-closed.*

Example 3.5. *Let* $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{a, b\}\}$ *and* $B = \{b\}$ *then* $\tau(B) = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$. *Then* $\{c\}$ *is B-closed but not regular B-closed.*

4. Characterization of Mildly B-normal Spaces

Definition 4.1. *A simply extended topological space X is said to be mildly B-normal if for every pair of disjoint* $H, K \in BRC(X)$, *there exist disjoint B-open sets U, V of X such that* $H \subset U$ *and* $K \subset V$.

Lemma 4.2. *A subset A of a simply extended topological space X is rBg-open if and only if* $F \subset Bint(A)$ *whenever* $F \in BRC(X)$ *and* $F \subset A$.

Theorem 4.3. *The following are equivalent for a simply extended topological space X.*

(1). X is mildly B-normal;

(2). for any disjoint $H, K \in BRC(X)$, *there exist disjoint Bg-open sets U, V such that* $H \subset U$ *and* $K \subset V$;

(3). for any disjoint $H, K \in BRC(X)$, *there exist disjoint rBg-open sets U, V such that* $H \subset U$ *and* $K \subset V$;

(4). for any disjoint $H \in BRC(X)$ *and any* $V \in BRO(X)$ *containing H, there exists a rBg-open set U of X such that* $H \subset U \subset Bcl(U) \subset V$.

Proof. It is obvious that (1) implies (2) and (2) implies (3).

(3) ⇒ (4) Let $H \in BRC(X)$ and $H \subset V \in BRO(X)$. There exist disjoint rBg-open sets U, W such that $H \subset U$ and $X - V \subset W$. By Lemma 4.2, we have $X - V \subset$Bint(W) and $U \cap Bint(W) = \phi$. Therefore, we obtain $Bcl(U) \cap Bint(W) = \phi$ and hence $H \subset U \subset Bcl(U) \subset X - Bint(W) \subset V$.

(4) ⇒ (1) Let H, K be disjoint regular B-closed sets of X. Then $H \subset X - K \in$BRO(X) and there exists a rBg-open set G of X such that $H \subset G \subset Bcl(G) \subset X - K$. Put U = Bint(G) and $V = X - Bcl(G)$. Then U and V are disjoint B-open sets of X such that $H \subset U$ and $K \subset V$. Therefore, X is mildly B-normal. □

5. Some Functions

Definition 5.1. *A function $f : X \to Y$ is said to be almost Bg-continuous (resp. almost rBg-continuous) if $f^{-1}(R)$ is Bg-closed (resp. rBg-closed), for every $R \in BRC(Y)$.*

Definition 5.2. *A function $f : X \to Y$ is said to be*

(1). Bg-continuous (resp. rBg-continuous) if $f^{-1}(F)$ is Bg-closed (resp. rBg-closed) for every B-closed set F of Y;

(2). BR-map (resp. almost B-continuous) if $f^{-1}(V) \in BRO(X)$ (resp. $\tau(B)(X)$) for every $V \in BRO(Y)$;

(3). completely B-continuous if $f^{-1}(V) \in BRO(X)$ for every B-open set V of Y.

From the definitions stated above, we obtain the following diagram:

complete B-continuity	$\longrightarrow$	BR-map
$\downarrow$		$\downarrow$
B-continuity	$\longrightarrow$	almost B-continuity
$\downarrow$		$\downarrow$
Bg-continuity	$\longrightarrow$	almost Bg-continuity
$\downarrow$		$\downarrow$
rBg-continuity	$\longrightarrow$	almost rBg-continuity

Remark 5.3. *None of the implications in Diagram I is reversible as shown by the following Examples.*

Example 5.4.

(1). Let $X = Y = \{a,\ b,\ c\}$, $\tau = \{\phi,\ X\}$ and $B_X = \{a\}$ then $\tau(B_X) = \{\phi,\ X,\ \{a\}\}$. Let $\sigma = \{\phi,\ Y\}$ and $B_Y = \{a,\ b\}$ then $\sigma(B_Y) = \{\phi,\ Y,\ \{a,\ b\}\}$. Let $f : (X, \tau(B_X)) \to (Y, \sigma(B_X))$ be an identity map. Then f is BR-map (resp. almost B-continuous) but not completely B-continuous (resp. B-continuous).

(2). Let $X = Y = \{a,\ b,\ c\}$, $\tau = \{\phi,\ X,\ \{a\}\}$ and $B_X = \{a,\ b\}$ then $\tau(B_X) = \{\phi,\ X,\ \{a\},\ \{a,\ b\}\}$. Let $\sigma = \{\phi,\ Y\}$ and $B_Y = \{a\}$ then $\sigma(B_Y) = \{\phi,\ Y,\ \{a\}\}$. Let $f : (X, \tau(B_X)) \to (Y, \sigma(B_X))$ be an identity map. Then f is almost Bg-continuous but not Bg-continuous.

Example 5.5.

(1). Let $X = Y = \{a,\ b,\ c\}$, $\tau = \{\phi,\ X\}$ and $B_X = \{a\}$ then $\tau(B_X) = \{\phi,\ X,\ \{a\}\}$. Let $\sigma = \{\phi,\ Y\}$ and $B_Y = \{a\}$ then $\sigma(B_Y) = \{\phi,\ Y,\ \{a\}\}$. Let $f : (X, \tau(B_X)) \to (Y, \sigma(B_X))$ be an identity map. Then f is B-continuous but not completely B-continuous.

(2). Let $X = Y = \{a,\ b,\ c\}$, $\tau = \{\phi,\ X,\ \{a\},\ \{a,\ b\}\}$ and $B_X = \{b\}$ then $\tau(B_X) = \{\phi,\ X,\ \{a\},\ \{b\},\ \{a,\ b\}\}$. Let $\sigma = \{\phi,\ Y,\ \{a\},\ \{a,\ b\}\}$ and $B_Y = \{b\}$ then $\sigma(B_Y) = \{\phi,\ Y,\ \{a\},\ \{b\},\ \{a,\ b\}\}$. Let $f : (X, \tau(B_X)) \to (Y, \sigma(B_X))$ be an identity map. Then f is almost B-continuous but not BR-map.

Example 5.6. *Let $X = Y = \{a,\ b,\ c\}$, $\tau = \{\phi,\ X,\ \{a\}\}$ and $B_X = \{a,\ b\}$ then $\tau(B_X) = \{\phi,\ X,\ \{a\},\ \{a,\ b\}\}$. Let $\sigma = \{\phi,\ Y\}$ and $B_Y = \{b\}$ then $\sigma(B_Y) = \{\phi,\ Y,\ \{a\},\ \{b\},\ \{a,\ b\}\}$. Let $f : (X, \tau(B_X)) \to (Y, \sigma(B_X))$ be an identity map. Then f is Bg-continuous(resp. almost B-continuous) but not B-continuous(resp. almost Bg-continuous).*

Example 5.7. *Let $X = Y = \{a,\ b,\ c\}$, $\tau = \{\phi,\ X,\ \{a\}\}$ and $B_X = \{a,\ b\}$ then $\tau(B_X) = \{\phi,\ X,\ \{a\},\ \{a,\ b\}\}$. Let $\sigma = \{\phi,\ Y,\ \{a\}\}$ and $B_Y = \{b\}$ then $\sigma(B_Y) = \{\phi,\ Y,\ \{a\},\ \{a,\ c\}\}$. Let $f : (X, \tau(B_X)) \to (Y, \sigma(B_X))$ be an identity map. Then f is rBg-continuous but not Bg-continuous.*

Example 5.8. *Let $X = Y = \{a,\ b,\ c\}$, $\tau = \{\phi,\ X,\ \{a\}\}$ and $B_X = \{c\}$ then $\tau(B_X) = \{\phi,\ X,\ \{a\},\ \{c\},\ \{a,\ c\}\}$. Let $\sigma = \{\phi,\ Y,\ \{a\}\}$ and $B_Y = \{b\}$ then $\sigma(B_Y) = \{\phi,\ Y,\ \{a\},\ \{b\},\ \{a,\ b\}\}$. Let $f : (X, \tau(B_X)) \to (Y, \sigma(B_X))$ be an identity map. Then f is almost rBg-continuous but neither almost Bg-continuous nor rBg-continuous.*

Definition 5.9. *A simply extended topological space X is said to be regular B-$T_{1/2}$ if every rBg-closed set of X is regular B-closed.*

Proposition 5.10. *If a function $f : X \to Y$ is rBg-continuous and X is regular B-$T_{1/2}$, then f is completely B-continuous.*

Proof. Let F be any B-closed set of Y. Since f is rBg-continuous, $f^{-1}(F)$ is rBg-closed in X and hence $f^{-1}(F) \in BRC(X)$. Therefore, f is completely B-continuous. □

Definition 5.11. *A function $f : X \to Y$ is said to be rBg-irresolute if $f^{-1}(F)$ is rBg-closed in X for every rBg-closed set F of Y. Every rBg-irresolute function is rBg-continuous but not conversely as shown by the following Example.*

Example 5.12. *Let $X = Y = \{a,\ b,\ c\}$, $\tau = \{\phi,\ X,\ \{a\}\}$ and $B_X = \{a,\ b\}$ then $\tau(B_X) = \{\phi,\ X,\ \{a\},\ \{a,\ b\}\}$. Let $\sigma = \{\phi,\ Y\}$ and $B_Y = \{a\}$ then $\sigma(B_Y) = \{\phi,\ Y,\ \{a\}\}$. Let $f : (X, \tau(B_X)) \to (Y, \sigma(B_X))$ be an identity map. Then f is B-continuous and Bg-continuous but not rBg-irresolute.*

Corollary 5.13. *If $f : X \to Y$ is rBg-irresolute and X is regular B-$T_{1/2}$, then f is BR-map.*

Proof. This is an immediate consequence of Proposition 5.10. □

Definition 5.14. *A function $f : X \to Y$ is said to be*

(1). regular B-closed (resp. Bg-closed, rBg-closed) if f(F) is regular B-closed (resp. Bg-closed, rBg-closed) in Y for every B-closed set F of X;

(2). rBc-preserving (resp. almost B-closed, almost Bg-closed, almost rBg-closed) if f(F) is regular B-closed (resp. B-closed, Bg-closed, rBg-closed) in Y for every $F \in BRC(X)$.

From the definitions stated above, we obtain the following diagram:

$$\begin{array}{ccc}
\text{regular B-closed} & \longrightarrow & \text{rBc-preserving} \\
\downarrow & & \downarrow \\
\text{B-closed} & \longrightarrow & \text{almost B-closed} \\
\downarrow & & \downarrow \\
\text{Bg-closed} & \longrightarrow & \text{almost Bg-closed} \\
\downarrow & & \downarrow \\
\text{rBg-closed} & \longrightarrow & \text{almost rBg-closed}
\end{array}$$

Remark 5.15. *None of the implications in Diagram II is reversible.*

Example 5.16. *Let $X = Y = \{a,\ b,\ c\}$, $\tau = \{\phi,\ X,\ \{a\}\}$ and $B_X = \{b\}$ then $\tau(B_X) = \{\phi,\ X,\ \{a\},\ \{b\},\ \{a,\ b\}\}$. Let $\sigma = \{\phi,\ Y,\ \{a\}\}$ and $B_Y = \{b\}$ then $\sigma(B_Y) = \{\phi,\ Y,\ \{a\},\ \{b\},\ \{a,\ b\}\}$. Let $f : (X, \tau(B_X)) \to (Y, \sigma(B_X))$ be an identity map. Then f is*

(1). rBc-preserving but not regular B-closed.

(2). regular B-closed but not B-closed.

Example 5.17.

(1). Let $X = Y = \{a,\ b,\ c\}$, $\tau = \{\phi,\ X\}$ *and* $B_X = \{a\}$ *then* $\tau(B_X) = \{\phi,\ X,\ \{a\}\}$. *Let* $\sigma = \{\phi,\ Y\}$ *and* $B_Y = \{a,\ b\}$ *then* $\sigma(B_Y) = \{\phi,\ Y,\ \{a,\ b\}\}$. *Let* $f : (X, \tau(B_X)) \to (Y, \sigma(B_X))$ *be an identity map. Then f is B-closed but not almost B-closed.*

(2). Let $X = Y = \{a,\ b,\ c\}$, $\tau = \{\phi,\ X\}$ *and* $B_X = \{b,\ c\}$ *then* $\tau(B_X) = \{\phi,\ X,\ \{b,\ c\}\}$. *Let* $\sigma = \{\phi,\ Y\}$ *and* $B_Y = \{a,\ b\}$ *then* $\sigma(B_Y) = \{\phi,\ Y,\ \{a,\ b\}\}$. *Let* $f : (X, \tau(B_X)) \to (Y, \sigma(B_X))$ *be an identity map. Then f is Bg-closed but not B-closed.*

(3). Let $X = Y = \{a,\ b,\ c\}$, $\tau = \{\phi,\ X,\ \{a\}\}$ *and* $B_X = \{b\}$ *then* $\tau(B_X) = \{\phi,\ X,\ \{a\},\ \{b\},\ \{a,\ b\}\}$. *Let* $\sigma = \{\phi,\ Y,\ \{a\},\ \{a,\ b\},\ \{a,\ c\}\}$ *and* $B_Y = \{b\}$ *then* $\sigma(B_Y) = \{\phi,\ Y,\ \{a\},\ \{b\},\ \{a,\ b\},\ \{a,\ c\}\}$. *Let* $f : (X, \tau(B_X)) \to (Y, \sigma(B_X))$ *be an identity map. Then f is almost B-closed but not rBc-preserving.*

(4). Let $X = Y = \{a,\ b,\ c\}$, $\tau = \{\phi,\ X,\ \{a\}\}$ *and* $B_X = \{b\}$ *then* $\tau(B_X) = \{\phi,\ X,\ \{a\},\ \{b\},\ \{a,\ b\}\}$. *Let* $\sigma = \{\phi,\ Y,\ \{c\},\ \{b,\ c\}\}$ *and* $B_Y = \{b\}$ *then* $\sigma(B_Y) = \{\phi,\ Y,\ \{b\},\ \{c\},\ \{b,\ c\}\}$. *Let* $f : (X, \tau(B_X)) \to (Y, \sigma(B_X))$ *be an identity map. Then f is almost Bg-closed(resp. Bg-closed, Bg-closed) but not almost B-closed(resp. almost Bg-closed, rBg-closed).*

Proposition 5.18. *Let X and Y be simply extended topological spaces. Let* $f : X \to Y$ *be a function. Then*

(1). if f is rBg-continuous rBc-preserving, then it is rBg-irresolute;

(2). if f is an BR-map and rBg-closed, then f(A) is rBg-closed in Y for every rBg-closed set A of X.

Proof.

(1). Let A be any rBg-closed set of Y and $U \in BRO(X)$ containing $f^{-1}(A)$. Put $V = Y - f(X - U)$, then we have $A \subset V$, $f^{-1}(V) \subset U$ and $V \in BRO(Y)$ since f is rBc-preserving. Hence we obtain $Bcl(A) \subset V$ and hence $f^{-1}(Bcl(A)) \subset U$. By the rBg-continuity of f, we have $Bcl(f^{-1}(A)) \subset Bcl(f^{-1}(Bcl(A))) \subset U$. This shows that $f^{-1}(A)$ is rBg-closed in X. Therefore, f is rBg-irresolute.

(2). Let A be any rBg-closed set of X and $V \in BRO(X)$ containing $f(A)$. Since f is an BR-map, $f^{-1}(V) \in BRO(X)$ and $A \subset f^{-1}(V)$. Therefore, we have $Bcl(A) \subset f^{-1}(V)$ and hence $f(Bcl(A)) \subset V$. Since f is rBg-closed, f(Bcl(A)) is rBg-closed in Y and hence we obtain $Bcl(f(A)) \subset Bcl(f(Bcl(A))) \subset U$. This shows that $f(A)$ is rBg-closed in Y. □

Corollary 5.19. *Let X and Y be simply extended topological spaces. Let* $f : X \to Y$ *be a function. Then*

(1). if f is B-continuous regular B-closed, $f^{-1}(A)$ *is rBg-closed in X for every rBg-closed set A of Y;*

(2). if f is BR-map and B-closed, f(A) is rBg-closed in Y for every rBg-closed set A if X.

Proof. This is an immediate consequence of Proposition 5.18. □

Proposition 5.20. *Let X and Y be simply extended topological spaces. A surjection* $f : X \to Y$ *is almost rBg-closed (resp. almost Bg-closed) if and only if for each subset S of Y and each* $U \in BRO(X)$ *containing* $f^{-1}(S)$ *there exists an rBg-open (resp. Bg-open) set V of Y such that* $S \subset V$ *and* $f^{-1}(V) \subset U$.

Proof. We prove only the first case, the proof of the second being entirely analogous.

Necessity : Suppose that f is almost rBg-closed. Let S be a subset of Y and $U \in BRO(X)$ containing $f^{-1}(S)$. Put V=$Y - f(X - U)$, then V is an rBg-open set of Y such that $S \subset V$ and $f^{-1}(V) \subset U$.

Sufficiency : Let F be any regular B-closed set of X. Then f^{-1}(Y−f(F))⊂ $X - F$ and $X - F$ $\in BRO(X)$. There exists an rBg-open set V of Y such that $Y - f(F)$ ⊂V and $f^{-1}(V) \subset$ X−F. Therefore, we have f(F)⊃ $Y - V$ and $F \subset f^{-1}$ $(Y - V)$. Hence, we obtain f(F)=$Y - V$ and f(F) is rBg-closed in Y. This shows that f is almost rBg-closed. □

6. Preservation Theorems

In this section we investigate preservation theorems concerning mildly B-normal spaces

Theorem 6.1. *Let X and Y be simply extended topological spaces. If $f : X \to Y$ is an almost rBg-continuous rBc-preserving (resp. almost B-closed) injection and Y is mildly B-normal (resp. B-normal), then X is mildly B-normal.*

Proof. Let A and C be any disjoint regular B-closed sets of X. Since f is an rBc-preserving (resp. almost B-closed) injection, f(A) and f(C) are disjoint regular B-closed (resp. B-closed) sets of Y. By the mild B-normality (resp. B-normality) of Y, there exist disjoint B-open sets U and V of Y such that $f(A) \subset U$ and $f(C) \subset V$. Now, put G = Bint(Bcl(U)) and H = Bint(Bcl(V)), then G and H are disjoint regular B-open sets such that $f(A) \subset G$ and $f(C) \subset H$. Since f is almost rBg-continuous, $f^{-1}(G)$ and $f^{-1}(H)$ are disjoint rBg-open sets containing A and C, respectively. It follows from Theorem 4.3 that X is mildly B-normal. □

Theorem 6.2. *Let X and Y be simply extended topological spaces. If $f : X \to Y$ is a completely B-continuous amlost Bg-closed surjection and X is mildly B-normal, then Y is B-normal.*

Proof. Let A and C be any disjoint B-closed sets of Y. Then $f^{-1}(A)$ and $f^{-1}(C)$ are disjoint regular B-closed sets of X. Since X is mildly B-normal, there exist disjoint B-open sets U and V such that $f^{-1}(A) \subset U$ and $f^{-1}(C) \subset V$. Let G = Bint(Bcl(U)) and H = Bint(Bcl(V)), then G and H are disjoint regular B-open sets such that $f^{-1}(A) \subset G$ and $f^{-1}(C) \subset H$. By Proposition 5.20, there exists Bg-open sets K and L of Y such that $A \subset K$, $C \subset L$, $f^{-1}(K) \subset G$ and $f^{-1}(L) \subset H$. Since G and H are disjoint, so are K and L. Since K and L are Bg-open, we obtain $A \subset Bint(K)$, $C \subset Bint(L)$ and $Bint(K) \cap Bint(L) = \phi$. This shows that Y is B-normal. □

Corollary 6.3. *Let X and Y be simply extended topological spaces. If $f : X \to Y$ is a completely B-continuous B-closed surjection and X is mildly B-normal, then Y is B-normal.*

Theorem 6.4. *Let X and Y be simply extended topological spaces. Let $f : X \to Y$ be an BR-map (resp. almost B-continuous) and almost rBg-closed surjection. If X is mildly B-normal (resp. B-normal), then Y is mildly B-normal.*

Proof. Let A and C be any disjoint regular B-closed sets of Y. Then $f^{-1}(A)$ and $f^{-1}(C)$ are disjoint regular B-closed (resp. B-closed) sets of X. Since X is mildly B-normal (resp. B-normal), there exist disjoint B-open sets U and V of X such that $f^{-1}(A) \subset U$ and $f^{-1}(C) \subset V$. Put G = Bint(Bcl(U)) and H = Bint(Bcl(V)), then G and H are disjoint regular B-open sets of X such that $f^{-1}(A) \subset G$ and $f^{-1}(C) \subset H$. By Proposition 5.20, there exists rBg-open sets K and L of Y such that $A \subset K$, $C \subset L$, $f^{-1}(K) \subset G$ and $f^{-1}(L) \subset H$. Since G and H are disjoint, so are K and L. It follows from Theorem 4.3 that Y is mildly B-normal. □

Corollary 6.5. *Let X and Y be simply extended topological spaces. If $f : X \to Y$ is an almost B-continuous amlost B-closed surjection and X is B-normal, then Y is mildly B-normal.*

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