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An Intuitionistic Fuzzy Meet Semi L-Ideal

Research Article

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Abstract: Intuitionistic fuzzy meet semi L-ideal of intuitionistic fuzzy meet subsemilattice is introduced in this Paper. Intuitionistic fuzzy meet semi L-ideal is defined and examples are given. The properties of intuitionistic fuzzy meet semi L-ideal are discussed. Intuitionistic fuzzy level meet semi L-ideals are defined, examples and properties are established.

Keywords: L-ideal, Intuitionistic fuzzy meet semi L-ideal, Subsemilattice.

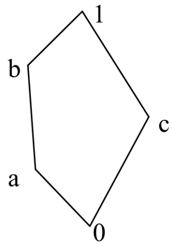
1. Introduction and Preliminaries

The concept of fuzzy sets was introduced in 1965 by L.A. Zadeh [14]. In [4], K.T. Atanassov introduced intuitionistic fuzzy set. J. Hashimoto [6] discussed the ideal theory for lattice. In [7] K.H. Him and Y.B.Jun gave some properties of intuitionistic fuzzy ideals of semigroups. B. Chellapa and B. Anandh in [5] introduced fuzzy join semi L-ideal. In [10] Liu Wang Jin discussed the operations on fuzzy ideals. K.V. Thomas and Latha S. Nair [13] introduced and studied intuitionistic fuzzy sublattice and ideals. In this present paper intuitionistic fuzzy meet semi L-ideal, intuitionistic fuzzy level meet semi L-ideal are discussed. Some theorems and results related to this topic are also established.

Definition 1.1. *An intuitionistic fuzzy semilattice $A = \langle \mu_A, \nu_A \rangle$ is called an intuitionistic fuzzy meet semi L-ideal if for all $x, y \in A$,*

1. $\mu(x \wedge y) \geq \max\{\mu(x), \mu(y)\}$
2. $\nu(x \wedge y) \leq \min\{\nu(x), \nu(y)\}$.

Example 1.2. *Let $A = \{0, a, b, c, 1\}$. Let $\mu : A \to [0,1]$ and $\nu : A \to [0,1]$ be a fuzzy meet subsemilattices in A defined by* $\langle \mu(0), \nu(0) \rangle = \langle 0.8, 0.2 \rangle$; $\langle \mu(a), \nu(a) \rangle = \langle 0.5, 0.5 \rangle$; $\langle \mu(b), \nu(b) \rangle = \langle 0.5, 0.3 \rangle$; $\langle \mu(c), \nu(c) \rangle = \langle 0.7, 0.2 \rangle$; $\langle \mu(1), \nu(1) \rangle = \langle 0.4, 0.5 \rangle$.



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Then A is an intuitionistic fuzzy meet semi L-ideal.

Definition 1.3. *Let A and B be any two intuitionistic fuzzy meet semi L-ideal of X. We define the following relations and operations:*

1. $A \subseteq B$ *iff* $\mu_A(x) \leq \mu_B(x)$ *and* $\nu_A(x) \geq \nu_B(x)$, *for all* $x \in X$.

2. $A = B$ *iff* $\mu_A(x) = \mu_B(x)$ *and* $\nu_A(x) = \nu_B(x)$, *for all* $x \in X$.

3. $\bar{A} = \{\langle x, \nu_A(x), \mu_A(x)\rangle / x \in X\}$.

4. $A \cap B = \{\langle x, \min\{\mu_A(x), \mu_B(x)\}, \max\{\nu_A(x), \nu_B(x)\}\rangle / x \in X\}$.

5. $A \cup B = \{\langle x, \max\{\mu_A(x), \mu_B(x)\}, \min\{\nu_A(x), \nu_B(x)\}\rangle / x \in X\}$.

6. $\Box A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle / x \in X\}$.

7. $\Diamond A = \{\langle x, \nu_A(x), 1 - \nu_A(x)\rangle / x \in X\}$.

2. Main Results

Theorem 2.1. *Two intuitionistic fuzzy meet semi L-ideals* $\langle A, \mu_1, \nu_1\rangle$, $\langle A, \nu_1, \nu_2\rangle$ *of a intuitionistic fuzzy meet semilattice A such that card* $Im\mu_1 < \infty$, *card* $Im\nu_1 < \infty$ *and card* $Im\mu_2 < \infty$, *card* $Im\nu_2 < \infty$ *are equal if and only if* $Im\mu_1 = Im\mu_2$ *and* $F\mu_1 = F\mu_2$; $Im\nu_1 = Im\nu_2$ *and* $F\nu_1 = F\nu_2$.

Proof. Let $\langle A, \mu_1, \nu_1\rangle$, $\langle A, \nu_1, \nu_2\rangle$ be two intuitionistic fuzzy meet semi L-ideal of a intuitionistic fuzzy meet semilattice A such that card $Im\mu_1 < \infty$, card $Im\nu_1 < \infty$ and card $Im\mu_2 < \infty$, card $Im\nu_2 < \infty$. Assume that μ_1 and μ_2 are equal, ν_1 and ν_2 are equal. (ie) $\mu_1(x) = \mu_2(x)$ and $\nu_1(x) = \nu_2(x)$, for all $x \in A$.

$$\mu_1(x) \in Im\mu_2 \text{ and } \mu_2(x) \in Im\mu_1 \tag{1}$$

But $\mu_2(x) \in Im\mu_2$. Therefore

$$Im\mu_2 \subseteq Im\mu_1 \tag{2}$$

Similarly it can be proved that

$$Im\mu_1 \subseteq Im\mu_2 \tag{3}$$

From equation (2) and (3) we get

$$Im\mu_1 = Im\mu_2 \tag{4}$$

Now $\nu_1(x) = \nu_2(x)$, $\nu_1(x) \in Im\nu_2$ and $\nu_2(x) \in Im\nu_1$. But $\nu_2(x) \in Im\nu_2$. Therefore

$$Im\nu_2 \subseteq Im\nu_1 \tag{5}$$

Similarly it can be proved that

$$Im\nu_1 \subseteq Im\nu_2 \tag{6}$$

From equation (2) and (3) we get

$$Im\nu_1 = Im\nu_2 \tag{7}$$

Let $\mu_{1t} \in F_{\mu 1}$ and $x \in \mu_t \Rightarrow \mu_1(x) \leq t,\, t \in Im\mu_1 \Rightarrow \mu_2(x) \leq t,\, t \in Im\mu_1$ (by (4)) $\Rightarrow x \in \mu_{2t} \Rightarrow \mu_{1t} \subseteq \mu_{2t}$. Similarly it can be proved that $\mu_{2t} \subseteq \mu_{1t}$. Hence $\mu_{1t} = \mu_{2t} \Rightarrow \mu_{2t} \in F_{\mu 1}$. But $\mu_{2t} \in F_{\mu 2}$

$$\Rightarrow F_{\mu 2} \subseteq F_{\mu 1} \tag{8}$$

Similarly

$$F_{\mu 1} \subseteq F_{\mu 2} \tag{9}$$

From (8) and (9), we get

$$F_{\mu 1} = F_{\mu 2} \tag{10}$$

Equation (4) and (10) completes $Im\mu_1 = Im\mu_2$ and $F_{\mu 1} = F_{\mu 2}$. Let $\nu_{1t} \in F_{\nu 1}$ and $x \in \nu_{1t} \Rightarrow \nu_1(x) \geq t, t \in Im\nu_1 \Rightarrow \nu_2(x) \geq t,\, t \in Im\nu_1$ (by (7)) $\Rightarrow x \in \nu_{2t} \Rightarrow \nu_{1t} \subseteq \nu_{2t}$. Similarly it can be proved that $\nu_{2t} \subseteq \nu_{1t}$. Hence $\nu_{1t} = \nu_{2t} \Rightarrow \nu_{2t} \in F_{\nu 1}$. But $\nu_{2t} \in F_{\nu 2}$

$$\Rightarrow F_{\nu 2} \subseteq F_{\nu 1} \tag{11}$$

Similarly

$$F_{\nu 1} \subseteq F_{\nu 2} \tag{12}$$

From (11) and (12), we get

$$F_{\nu 1} = F_{\nu 2} \tag{13}$$

Equation (7) and (13) completes $Im\nu_1 = Im\nu_2$ and $F_{\nu 1} = F_{\nu 2}$.

Conversely, assume that $Im\mu_1 = Im\mu_2$ and $F_{\mu 1} = F_{\mu 2}$ and $Im\nu_1 = Im\nu_2$ and $F_{\nu 1} = F_{\nu 2}$.

To prove that $\mu_1 = \mu_2$ and $\nu_1 = \nu_2$. Suppose $\mu_1 \neq \mu_2$ and $\nu_1 \neq \nu_2$. Then there exist $x \in A$ such that $\mu_1(x) \neq \mu_2(x)$ and $\nu_1(x) \neq \nu_2(x)$ then $Im\mu_1 \neq Im\mu_2$ and $Im\nu_1 \neq Im\nu_2$. (ie) $F_{\mu 1} \neq F_{\mu 2}$ and $F_{\nu 1} \neq F_{\nu 2}$. This is a contradiction. Hence $\mu_1(x) = \mu_2(x)$ and $\nu_1(x) = \nu_2(x)$ for all $x \in A$. Therefore $\mu_1 = \mu_2$ and $\nu_1 = \nu_2$. □

Theorem 2.2. *If F is any intuitionistic fuzzy meet semi L-ideal of a intuitionistic fuzzy meet semilattice A, $F \neq A$ then the intuitionistic fuzzy meet semi L-filter $\langle \mu, \nu \rangle$ of A is defined by*

$$\mu(x) = s \;\; if\, x \in F \qquad\qquad \nu(x) = 1 - s \;\; if\, x \in F$$
$$= t \;\; if\, x \in A - F \qquad\qquad = 1 - t \;\; if\, x \in A - F$$

where $t \in [0,1]$, $t < s$ is a intuitionistic fuzzy meet semi L-ideal of A.

Proof. Let $x, y \in A$. To prove that $\langle \mu, \nu \rangle$ is a intuitionistic fuzzy meet semi L-ideal of A. It is prove by considering exhaustive following three cases.

Case(1): $x, y \in F$, $\mu(x) = s$, $\mu(y) = s$, $\nu(x) = 1 - s$, $\nu(y) = 1 - s$. As $x, y \in F$, $x \wedge y \in F$, since F is a intuitionistic fuzzy meet semi L-ideal of A. Then $x \wedge y \in F$ which implies $\mu(x \wedge y) = s$ and $\max\{\mu(x), \mu(y)\} = s = \max\{s, s\}$. Therefore $\mu(x \wedge y) \geq \max\{\mu(x), \mu(y)\}$. As $x, y \in F$, $x \wedge y \in F$, since F is a intuitionistic fuzzy meet semi L-ideal of A. Then $x \wedge y \in F$, $\nu(x \wedge y) = 1 - s$ and $\min\{\nu(x), \nu(y)\} = \{1 - s, 1 - s\} = 1 - s$. Therefore $\nu(x \wedge y) \leq \min\{\nu(x), \nu(y)\}$. Hence $\langle \mu, \nu \rangle$ is an intuitionistic fuzzy meet semi L-ideal of A.

Case (2): Let $x \in F$, $y \in A \sim F$, $\mu(x) = s$, $\mu(y) = t$. As $x \in F$, $y \in A \sim F$, $x \wedge y \in F$, which implies that $\mu(x \wedge y) = s$ and $\max\{\mu(x), \mu(y)\} = \max\{s, t\} = s$. Therefore $\mu(x \wedge y) \geq \max\{\mu(x), \mu(y)\}$. Let $x \in F$, $y \in A \sim F$, $\nu(x) = 1 - s$, $\nu(y) = 1 - t$.

As $x \in F$, $y \in A \sim F$, $x \wedge y \in F$, which implies that $\nu(x \wedge y) = 1 - s$ and $\min\{\nu(x), \nu(y)\} = \min\{1 - s, 1 - t\} = 1 - s$. Therefore $\nu(x \wedge y) \leq \min\{\nu(x), \nu(y)\}$. Hence $\langle \mu, \nu \rangle$ is an intuitionistic fuzzy meet semi L-ideal of A.

Case (3): Let $x, y \in A \sim F$, $\mu(x) = t = \mu(y)$ and $\nu(x) = 1 - t = \nu(y)$. As $x, y \in A \sim F$, $x \wedge y \in A \sim F$ (or) F. If $x \wedge y \in A \sim F$, then $\mu(x \wedge y) = t$ and $\max\{\mu(x), \mu(y)\} = \max\{t, t\} = t$. Therefore $\mu(x \wedge y) \geq \max\{\mu(x), \mu(y)\}$. If $x \wedge y \in A \sim F$, then $\nu(x \wedge y) = 1 - t$ and $\min\{\nu(x), \nu(y)\} = \min\{1 - t, 1 - t\} = 1 - t$. Therefore $\nu(x \wedge y) \leq \min\{\nu(x), \nu(y)\}$. Hence $\langle \mu, \nu \rangle$ is an intuitionistic fuzzy meet semi L-ideals of A in all the three cases. □

Theorem 2.3. *The intersection of two intuitionistic fuzzy meet semi L-ideal is also an intuitionistic fuzzy meet semi L-ideal of a intuitionistic fuzzy meet semilattice.*

Proof. Let A be a intuitionistic fuzzy meet semilattice. Let $\langle \mu_1, \nu_1 \rangle$ and $\langle \mu_2, \nu_2 \rangle$ be any two intuitionistic fuzzy meet semi L-ideal of a intuitionistic fuzzy meet semilattice A

To prove that $\mu_1 \cap \mu_2$ and $\nu_1 \cap \nu_2$ is a intuitionistic fuzzy meet semi L-ideal of A. Let $a, b \in \mu_1 \cap \mu_2$. Then $a, b \in \mu_1$ and $a, b \in \mu_2 \Rightarrow a \wedge b \in \mu_1$ and $a \wedge b \in \mu_2 \Rightarrow a \wedge b \in \mu_1 \cap \mu_2$. Let $c, d \in \nu_1 \cap \nu_2$. Then $c, d \in \nu_1$ and $c, d \in \nu_2 \Rightarrow c \wedge d \in \nu_1$ and $c \wedge d \in \nu_2 \Rightarrow c \wedge d \in \nu_1 \cap \nu_2$. Hence $\mu_1 \cap \mu_2$ and $\nu_1 \cap \nu_2$ is an intuitionistic fuzzy meet semi L-ideal of A. □

Theorem 2.4. *The complement of an intuitionistic fuzzy meet semi L-ideal is an intuitionistic fuzzy meet semi L-filter.*

Proof. Let $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$ be an intuitionistic fuzzy meet semi L-ideal. (ie) if

(i). $\mu_A(x \wedge y) \geq \max\{\mu_A(x), \mu_A(y)\}$

(ii). $\nu_A(x \wedge y) \leq \min\{\nu_A(x), \nu_A(y)\}$

To prove that complement of A is an intuitionistic fuzzy meet semi L- filter. (ie)

(i). $\mu_{\bar{A}}(x \wedge y) \leq \min\{\mu_{\bar{A}}(x), \mu_{\bar{A}}(y)\}$

(ii). $\nu_{\bar{A}}(x \wedge y) \geq \max\{\nu_{\bar{A}}(x), \nu_{\bar{A}}(y)\}$

Now the complement of A is defined by $\bar{A} = \{\langle x, \nu_A(x), \mu_A(x) \rangle / x \in X\}$. Here $\mu_{\bar{A}}(x) = \nu_A(x), \nu_{\bar{A}}(x) = \mu_A(x)$.

For (i):

$$\begin{aligned}
\mu_{\bar{A}}(x \wedge y) &= \nu_A(x \wedge y) \\
&\leq \min\{\nu_A(x), \nu_A(y)\} \\
&= \min\{\mu_{\bar{A}}(x), \mu_{\bar{A}}(y)\} \\
\mu_{\bar{A}}(x \wedge y) &\leq \min\{\mu_{\bar{A}}(x), \mu_{\bar{A}}(y)\}
\end{aligned}$$

For (ii):

$$\begin{aligned}
\nu_{\bar{A}}(x \wedge y) &= \mu_A(x \wedge y) \\
&\geq \max\{\mu_A(x), \mu_A(y)\} \\
&= \max\{\nu_{\bar{A}}(x), \nu_{\bar{A}}(y)\} \\
\nu_{\bar{A}}(x \wedge y) &\geq \max\{\nu_{\bar{A}}(x), \nu_{\bar{A}}(y)\}
\end{aligned}$$

Hence A is an intuitionistic fuzzy meet semi L-filter. □

Theorem 2.5. *If $A = \langle \mu_A, \nu_A \rangle$ is an intuitionistic fuzzy meet semi L-ideal. Then the $\Box A = \langle \mu_A, 1 - \mu_A \rangle$ is an intuitionistic fuzzy meet semi L-ideal of A.*

Proof. Let A be intuitionistic fuzzy meet semi L-ideal. Let $B = \Box A$. Then $\mu_B = \mu_A, \nu_B = 1 - \mu_A$. To prove that B is an intuitionistic fuzzy meet semi L-ideal

(i). $\mu_B(x \wedge y) = \mu_A(x \wedge y)$

$$\geq \max\{\mu_A(x), \mu_A(y)\}$$

$$= \max\{\mu_B(x), \mu_B(y)\}$$

$$\mu_B(x \wedge y) \geq \max\{\mu_B(x), \mu_B(y)\}$$

(ii). $\nu_B(x \wedge y) = 1 - \mu_A(x \wedge y)$

$$\leq 1 - \max\{\mu_A(x), \mu_A(y)\}$$

$$= \min\{1 - \mu_A(x), 1 - \mu_A(y)\}$$

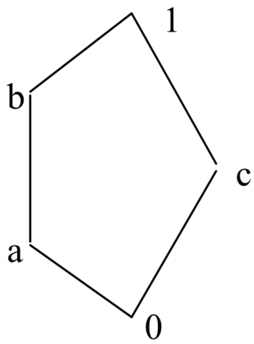
$$= \min\{\nu_A(x), \nu_A(y)\}$$

$$\nu_B(x \wedge y) \leq \min\{\nu_A(x), \nu_A(y)\}$$

Hence B is an intuitionistic fuzzy meet semi L-ideal ☐

Definition 2.6. *Let $A = \langle \mu_A, \nu_A \rangle$ be an intuitionistic fuzzy meet semi L-ideal and $t \in [0,1]$. Then $\mu_t = \{x \in A/\mu(x) \geq t\}$ and $\nu_t = \{x \in t/\nu(x) \leq t\}$ is called intuitionistic fuzzy level meet semi L-ideal of A.*

Example 2.7. *Let $A = \{0, a, b, c, 1\}$. Let $\mu : A \to [0,1]$ and $\nu : A \to [0,1]$ be a fuzzy meet subsemilattices in A defined by $\langle \mu(0), \nu(0) \rangle = \langle 0.8, 0.2 \rangle$; $\langle \mu(a), \nu(a) \rangle = \langle 0.5, 0.5 \rangle$; $\langle \mu(b), \nu(b) \rangle = \langle 0.5, 0.3 \rangle$; $\langle \mu(c), \nu(c) \rangle = \langle 0.7, 0.2 \rangle$; $\langle \mu(1), \nu(1) \rangle = \langle 0.4, 0.5 \rangle$.*



Then A is an intuitionistic fuzzy meet semi L-filter. In this case $t = 0.6$, $mu_t = \{0, c\}, \nu_t = \{0, a, b, c, 1\}$.

Definition 2.8. *Let $\langle \mu, \nu \rangle$ be a intuitionistic fuzzy meet semi L-ideal of a intuitionistic fuzzy semilattice A. The intuitionistic fuzzy level meet semi L- ideals are defined by $\mu_t = \{x \in A/\mu(x) \geq t\}$, $\nu_t = \{x \in A/\nu(x) \leq t\}$ and $\mu_s = \{x \in A/\mu(x) \geq s\}$, $\nu_s = \{x \in A/\nu(x) \leq s\}$. Clearly $\mu_t \subseteq \mu_s$, whenever $s < t$ and $\nu_t \subseteq \nu_s$, whenever $t < s$.*

Theorem 2.9. *Let A be fuzzy meet semilattice. If $\mu : A \to [0,1]$, $\nu : A \to [0,1]$ is a intuitionistic fuzzy meet semi L-ideal, then the level subsets μ_t, ν_t and $t \in [0,1]$ is a intuitionistic fuzzy level meet semi L-ideal of A.*

Proof. Let $x, y \in \mu_t$. Then $\mu(x) \geq t$, $\mu(y) \geq t$, $\mu(x \wedge y) \geq \max\{\mu(x), \mu(y)\} \geq t$. Therefore $x \wedge y \in \mu_t$. Let $x, y \in \nu_t$. Then $\nu(x) \leq t$, $\nu(y) \leq t$, $\nu(x \wedge y) \leq \min\{\nu(x), \nu(y)\} \leq t$. Therefore $x \wedge y \in \nu_t$. Hence μ_t, ν_t are intuitionistic fuzzy meet semi L-ideal of A. ☐

Theorem 2.10. *If A is a fuzzy meet semilattice. Then* $A = \langle \mu_A, \nu_A \rangle$ *is a intuitionistic fuzzy meet semi L-ideal iff the level subsets* μ_t, ν_t *and* $t \in [0,1]$ *is a intuitionistic fuzzy level meet semi L-ideal of* A.

Proof. Let A be a fuzzy meet semilattice. Assume that A is intuitionistic fuzzy meet semi L-ideal. Then μ_t, ν_t are intuitionistic fuzzy level meet semi L-ideal of A (by Theorem ?).

Conversely, assume that μ_t, ν_t are intuitionistic fuzzy level meet semi L-ideal of A. To prove that A is intuitionistic fuzzy meet semi L-ideal. Let $x, y \in \mu_t$. Then $\mu(x) \geq t$, $\mu(y) \geq t$, $\max\{\mu(x), \mu(y)\} \geq t$. Therefore $x \wedge y \in \mu_t$ (ie) $\mu(x \wedge y) \geq t$, $\mu(x \wedge y) \geq \max\{\mu(x), \mu(y)\}$. Let $x, y \in \nu_t$. Then $\nu(x) \leq t$, $\nu(y) \leq t$, $\min\{\nu(x), \nu(y)\} \leq t$. Therefore $x \wedge y \in \nu_t$ (ie) $\nu(x \wedge y) \leq t$, $\nu(x \wedge y) \leq \min\{\nu(x), \nu(y)\}$. Hence A is an intuitionistic fuzzy meet semi L-ideal. □

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