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Transformation of Signed Graphs into Trees

Research Article

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Abstract: A signed graph $G = (V, E)$ is transformed to a tree by a sequence of local switching if and only if both parties of

$$And \;\; B^m = a_1b_1, a_2b_2, \ldots, a_{m-1}a_na_1.$$

are odd.

Keywords: Transformed, Fushimitree, Degree, Vertex, Signed graph, Edges.

1. Introduction

Let $G = (V, E)$ be a graph. It is called connected if for any $u, v \in V$ there esists a sequence $u = u_0, u_1, \ldots, u_k = V$. Such that there is an edge between u_i and u_{i+1}; $0 < i < k$. If each block of a connected graph $G = (V, E)$ is a complete graph (or a perfect graph), then it is called a Fushimi tree.

If a Fushimi tree G is divided into exactly two connected components and each cut vertex in G is detected then G is called Simple Fushimi tree. A signed Simple Fushimi tree is called a Special Fushimi tree with standard sign, if one can switch all signs of edges into +1. let 'a' be a cut vertex of a Fushimi tree G. when the vertex a deleted and G is divided exactly m connected components, then we say that the Fushimi degree (or F-degree) of a cut vertex is m. if a connected sub-graph of a Fushimi tree consists of some blocks of G, then it is called a sub-Fushimi tree. If a block of Fushimi tree has only one cut vertex, then it is called pendent. It is evident that any Fushimi tree has at least two pendent blocks. A signed Fushimi tree with positive sign (or simply a positive Fushimi tree). If we can switch all signs of edges into +1. A tree is always considered as Fushimi tree with positive sign. A tree with only two leaves is called a line tree or simply a line in the present context.

$$A = k - cycle \;\; C^k = (V, E),$$

Where $V = \{a_1, a_2, a_3, \ldots, a_k\}$; $E = \{a_1a_2, a_2a_3, \ldots, a_{k-1}a_k, a_ka_1\}$ will be denoted simply by

$$C^k = \{a_1a_2, a_2a_3, \ldots, a_{k-1}a_k, a_ka_1\}$$

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For signed cycles there are two switching classes, which are distinguished by the parity or the balance, where the parity of a signed graph is the parity of the number of its edges which carry a positive sign and the balance is the product of the signs on its edges. Local switching of signed graphs is introduced by P.J. Cameron, J.J. Seidel and S.V. Tsaranov [1]. Some signed Fushimi trees are transformed to trees by a sequence of local switching [2]. Signed Cycles with odd parity are transformed to trees by a sequence of local switching, but signed cycles with even parity can't be transformed to trees by no means [2]. What kinds of graphs are transformed to trees by a sequence of local switching? It is important and interesting to give examples of signed graphs which are transformed to trees by a sequence of local switching. In this study, we give rather simple examples of such signed graphs. We have followed the terminologies of [3, 4, 5]. In [3], the following two theorems have been shown:

Theorem 1.1 ([3]). *Let G is a positive Fushimi tree whose any cut vertex has F–degree 2. We can transform G into a line tree by a sequence of local switching.*

Theorem 1.2 ([3]). *Let C^k be a k–cycle. Then, it is transformed to a tree by a sequence of local switching if and only if its parity is odd.*

2. Main Result

Theorem 2.1. *Let $G = (V, E)$ be a signed graph with $V = \{a_l, a_2, \dots, a_n, b_2, b_3, \dots, b_{m-1}\}$, and $E = \{a_1a_2, a_2a_3, \dots, a_{n-1}a_n, a_na_l, a_1b_2, b_2b_3, \dots, b_{m-1}a_n\}$. Consider two cycles*

$$A^n = a_la_2 \dots a_na_l \quad \text{and}$$
$$B^m = a_lb_2 \dots b_{m-1}a_na_1$$

Then the graph is transformed to a tree by a sequence of local switching if and only if both parities of A^n and B^m are odd.

Proof. Assume that the parity of A^n is odd. By a sequence of local switching $(a_4, J = \{a_3\}), \dots, (a_{n-2}, J = \{a_{n-3}\}), (a_{n-1}, J = \{a_1\})$, we get a signed graph with edge set

$$E = \{a_2a_3, \dots, a_{n-2}a_{n-1}, b_1b_2, b_2b_3, \dots, b_{m-1}a_na_na_{n-1}, a_{n-1}a_l\}$$

The parity of the cycles $a_1b_2b_3 \dots b_{m-i}a_na_{n-1}a_l$ is odd if and only if the parity of B^m is odd. In this case, the cycle is transformed to a tree by a sequence of local switching. If the parity of A^n is even, by a sequence of local switching $(a_2, J = \{a_3\}), (a_3, J = \{a_4\}), \dots (a_{n-2}, J = a_{n-1}\}), (a_3, J = \{a_2\}), (a_4, J = \{a_3\}), \dots, (\{a_{n-2}, J = \{a_{n-1}\})$ we get a signed graph with edge set $E = \{a_1a_2, a_2a_3, \dots, a_n - 2a_{n-1}, a_{n-1}a_n, a_{n-2}a_1, a_{n-1}a_1, a_lb_2, b_2b_3, \dots, b_{m-1}a_n\}$. As the sign of the edge $a_{n-1}a_n$ is -1, the cycle, a_1a_{n-1}, a_na_l can't be transformed to a line. Now, I am giving some examples of signed graphs which are transformed into trees.

For $j = 3, 4, \dots, 8$, set signed graph $T_j = (V, E)$ as follows:

$$V = \{a_l, a_2, \dots, a_{j+2}\},$$
$$E^+ = \{a_ia_{i+1}, a_ia_{i+2} \;\; (i = 1, 2, \dots, j), a_{j+1}a_{j+2}\}$$
$$E^- = \emptyset.$$

Then we have. □

Example 2.2. *The signed graphs T_3, T_4, T_5, T_6, T_7 are transformed to trees by a sequence of local switching, but T_8 can't be transformed to a tree by a sequence of local switching.*

Solution. By a sequence of local switching, $(a_3, J = \{a_2\})$, $(a_5, J = \{a_4\})$ from T_5, we get a tree with edge set

$$E = \{a_1a_3, a_3a_5, a_4a_5, a_2a_5\}.$$

By a sequence of local switching, $(a_3, J = \{a_2\}), (a_5, J = \{a_3, a_6\}), (a_6, J = \{a_2\})$, from T_4, we get a tree with edge set

$$E = \{a_1a_3, a_3a_5, a_4a_5, a_5a_6, a, a_6\}.$$

By a sequence of local switching, $(a_3, J\{a_2\}), (a_5, J = \{a_3, a_5, a_7\}), (a_7, J = \{a_5\}), (a_6, J = \{a_2\})$, from T_5 we get-a tree with edge set

$$E = \{a_1a_3, a_3a_5, a_5a_7, a_4a_7, a_7a_6, a_2a_6\}.$$

By a sequence of local switching, $(a_3, J = \{a_2\}), (a_5, J = \{a_3, a_6, a_7\}), (\{a_7, J = \{a_5\}), (a_6, J = \{a_7\}), (a_6, J = \{a_3\}), (a_2, J = \{a_6\})$, from T_6, we get a tree with edge set

$$E = \{a_la_3, a_3a_5, a_5a_8, a_8a_2, a_2a_6, a_2a_7, a_2a_4\}$$

By a sequence of local switching, $(a_3, J = \{a_2\}), (a_5, J = \{a_3, a6, a_7\}), (a_7, J = \{a_5\}), (a_9, J = \{a_7\}), (a_2, J = \{a_6\}, (a_4, J = \{a_8\}), (a_8, J = \{a_9\})$, from T_7, we get a tree with edge set

$$E = \{a_la_3, a_3a_5, a_3a_7, a_7a_9, a_9a_8, a_8a_4, a_8a_2, a_2a_6\}.$$

By a sequence of local switching $(a_3, J = \{a_2\}), (a_5, J = a_3, a_6, a_7\}), (a_7, J = \{a_5, a_8\}), (a_2, J = \{a_7\})$, from T_8, we get a signed graph with edge set $E^+ = \{a_1a_3, a_3a_5, a_5a_7, a_7a_9, a_9a_{10}, a_8a_{10}, a_8a_2, a_2a_7, a_2a_6\}$, $E^- = \{a_4a_7\}$. But this graph can't be transformed to a tree at all.

3. Conclusion

A signed cycle with even parity can't be transformed to a tree by a sequence of local switching. Hence, we do not make a 3-cycle with even parity by local switching. Set with even parity by local switching. Set $G_1 = (V, E)$ be a signed graph with vertex $V = \{a_l, a_2, b_1, b_2, c\}$ and edge sets $E^+ = \{a_1b_1, a_1b_2, a_2b_1, a_2b_2, a_1c\}$, $E^- = \{a_2c\}$. By local switching at b_l or b_2 we get a 3-cycle with even parity, we can't apply it. By local switching at a_l or a_2, if b_1 is in J and b_2 is in k or if the reverse holds, we get a 3-cycle with even parity. Similarly, set $G_2 = (V, E)$ be a signed graph with vertex set $V = \{a_l, a_2, b_1, \ldots, b_n, c\}$ and edge sets $E^+ = \{a_lb_1, \ldots, a_lb_n, a_2b_1, \ldots, a_2b_n, a_1c\}, E^- = \{a_2c\}$. We can't do local switching at any b_i $(1 < i < n)$. If we apply local switching at a_1 or a_2, all b_i's must be in J or in k. Let $Q_3 = (V, E)$ be a signed graph with

$$V = \{a_l, a_2, \ldots, a_7, a_8\},$$
$$E^+ = \{a_1a_2, a_1a_3, a_3a_4, a_3a_5, a_3a_6, a_5a_7, a_7a_8\} \quad \textit{and}$$
$$E^- = \{a_2a_4, a_4a_6, a_6a_8\}$$

and $Q_4 = (V, E)$ be a signed graph with

$$V = \{a_l, a_2, a_9, a_{10}\},$$
$$E^+ = \{a_1a_2, a_la_3, a_3a_4, a_3a_5, a_5a_6, a_5a_8, a_7a_8, a_7a_9, a_9a_{10}\} \quad \textit{and}$$
$$E^- = \{a_2a_4, a_4a_6, a_6a_8, a_8a_{10}\}$$

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