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A New Type of Weakly Closed Set in Ideal Topological Spaces

Research Article

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Abstract: In this paper we introduce the notation of $I_{\alpha\psi}$ closed sets in ideal topological spaces and investigate some of their properties. Further we study the weakly $I_{\alpha\psi}$ closed sets and $I_{\alpha\psi}$ normal space and derive some of their properties.

MSC: 54A05, 54A10, 54D15.

Keywords: $I_{\alpha\psi}$ closed set, weakly $I_{\alpha\psi}$ closed sets and $I_{\alpha\psi}$ normal space.

1. Introduction

O.Njastad [17] introduced the concept of α closed sets in topological spaces. The notion of ψ closed sets are introduced by M.K.R.S.Veerakumar [22]. R.Devi et.al. [4] intoduced the concept if $\alpha\psi$ closed sets in topological spaces. The first step of generalizing closed sets and semi closed sets was done by Levine in 1970 [12, 13]. The notion of weakly g closed sets was introduced and studied by P.Sundaram and N.Nagaveni [20]. In 1999, Donchev et al. studied the notion of generalized closed sets in ideal topological spaces called I_g closed sets [6]. In 2008, Navaneethakrishnan and Paulraj Joseph have studied some characterizations of normal spaces via I_g open sets [16]. J.Antony Rex Rodrigo and P.Mariappan [3], introduced the concept of g^* closed sets in ideal topological space.

In this paper, we introduce the nation of $I_{\alpha\psi}$ closed set, weakly $I_{\alpha\psi}$ closed sets in ideal topological spaces and $I_{\alpha\psi}$ normal space, we discussed about their properties and relationships.

2. Preliminaries

An ideal I on a topological space (x, τ) is a nonempty collection of subsets of X which satisfies

(1). $A \in I$ and $B \subset A \Rightarrow B \in I$ and

(2). $A \in I$ and $B \in I \Rightarrow A \cup B \in I$.

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Given a topological space (x, τ) with an ideal I on X and if $P(X)$ is the set of all subsets of X, a set operator $(.)^* : P(X) \to P(X)$, called a local function [11] of A with respect to τ and I is defined as follows: for $A \subset X$, $A^*(I,\tau)$=$\{x \in X | U \cap A \notin I$ for every $U \in \tau(X)\}$. We will make use of the basic facts about the local functions [10] with out mentioning it explicity. A kuratowski closure operator $cl^*(.)$ for a topology $\tau^*(I,\tau)$ called the $*$ topology, finer than τ is defined by $cl^*(A) = A \cup A^*(I,\tau)$ [21]. When there is no chance for confusion, we will simply write A^* for $A^*(I,\tau)$ and τ^* for $\tau^*(I,\tau)$.

Definition 2.1. *A subset A of a space (X,τ) is called*

(1). An α closed set [17] if $cl(int(cl(A))) \subseteq A$.

(2). A semi closed set [13] if $int(cl(A)) \subseteq A$.

(3). A preclosed set [15] if $cl(int(A)) \subseteq A$.

(4). A semi preclosed set [2] if $int(cl(int(A))) \subseteq A$.

(5). A regular closed set if $cl(int(A)) = A$.

The pre-closure (resp. semi-closure, α -closure, semi-preclosure) of a subset A of a space (X,τ) is the intersection of all pre-closed (resp. semi-closed, α -closed, semi-preclosed) sets that contain A and is denoted by pcl(A)(resp., scl(A), αcl(A), spcl(A)).

Definition 2.2. *A subset A of a space (X,τ) is called*

(1). A generalized semi preclosed (briefly gsp-closed) set [5] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ).

(2). A generalized α closed (briefly gα-closed) set [14] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α open in (X,τ).

(3). A $\hat{g}$ closed set [23, 24] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in (X,τ).

(4). A $\alpha\psi$ closed set [4] if $\psi cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α open in (X,τ).

(5). A I_g closed [6] if $A^* \subseteq U$ whenever $A \subseteq U$ and U is open in (X,I,τ).

(6). A I_{rg} closed if $A^* \subseteq U$ whenever $A \subseteq U$ and U is regular open in (X,I,τ).

(7). A I-R closed [1]] if $A = cl^*(int(A))$.

(8). A $*$ closed [10] if $A^* \subseteq A$,and $*$-dense in itself if $A \subseteq A^*$.

(9). A Pre^* I closed [7] if $cl^*(int(A)) \subseteq A$.

(10). A weakly I_g closed set [18] if $(int(A))^* \subseteq U$ whenever $A \subseteq U$ and U is an open set in (X,I,τ).

(11). A weakly I_{rg} closed [7] if $(int(A))^* \subseteq U$ whenever $A \subseteq U$ and U is regular open in (X,I,τ).

Lemma 2.3 ([19]). *Let (X,τ,I) be an ideal topological space $A \subseteq X$. If $A \subseteq A^*$, then $A^* = cl(A^*) = cl(A) = cl^*(A)$.*

Lemma 2.4 ([10]). *Let (X,τ,I) be an ideal topological space and A, B be subsets of X. Then the following properties hold:*

(1). $A \subset B$ implies $A^* \subset B^*$.

(2). $A^* = cl(A^*) \subset cl(A)$.

(3). $(A^*)^* \subset A^*$.

(4). $(A \cup B)^* = A^* \cup B^*$.

3. $I_{\alpha\psi}$ Closed Sets

Definition 3.1. *Let (X, τ, I) be an ideal topological space, then $A^* \subseteq U$ whenever $A \subseteq U$ and U is $\alpha\psi$ open.*

Theorem 3.2. *Let (X, τ, I) be an ideal topological space. Then the following are hold:*

(a). Every closed set is $I_{\alpha\psi}$ closed set.

(b). Every $*$ closed set is $I_{\alpha\psi}$ closed set.

(c). Every $I_{\alpha\psi}$ closed set is I_g closed set.

(d). Every $I_{\alpha\psi}$ closed set is I_{rg} closed set.

Proof.

(a). Let U be a $\alpha\psi$ - open set such that $A \subseteq U$ and A be a closed set in X then $cl(A) = A$ which implies that $cl(A) \subseteq U$. But $cl^*(A) \subseteq cl(A) \subseteq U$. Therefore $A^* \subseteq U$. Hence A is $I_{\alpha\psi}$ - closed set in X.

(b). Let $A \subseteq U$ where U is $\alpha\psi$ open. Since A is a $*$ closed set, then $A^* \subseteq A$. Hence $A^* \subseteq U$ whenever $A \subseteq U$ and U is $\alpha\psi$ open. Hence A is $I_{\alpha\psi}$ closed set.

(c). Let $A \subseteq U$ where U is open. Every open set is $\alpha\psi$ open. Hence A is $\alpha\psi$ open and $I_{\alpha\psi}$ closed set, then $A^* \subseteq U$. Therefore A is I_g closed set.

(d). Let $A \subseteq U$ where U is regular open. Every regular open set is open and every open set is $\alpha\psi$ open and A is $I_{\alpha\psi}$ closed set, then $A^* \subseteq U$. Therefore A is I_{rg} closed set. □

The converse of the above theorem need not be true as shown by the following examples.

Example 3.3.

(a). Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}\}$ and $I = \{\phi, \{a\}\}$. Then $I_{\alpha\psi}$ - closed set $=\{\phi, X, \{a\}, \{b, c\}\}$ and closed set $=\{\phi, X, \{a, b\}\}$. It is clear that $\{a\}$ is $I_{\alpha\psi}$ - closed set but it is not closed.

(b). Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ and $I = \{\phi\}$. Then $I_{\alpha\psi}$ - closed sets $=\{\phi, X, \{c\}\}$ and $*$ closed sets $= \{\phi, X, \{c\}, \{a, c\}, \{b, c\}\}$. It is clear that $\{a, b\}$ is $I_{\alpha\psi}$ - closed set but it is not $*$ closed.

(c). Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}\}$ and $I = \{\phi, \{a\}\}$. Then $I_{\alpha\psi}$ - closed set$=\{\phi, X, \{a\}, \{b, c\}\}$ and $I_g = \{P(X)\}$. It is clear that $\{b\}$ is I_g - closed set but it is not $I_{\alpha\psi}$ -closed.

(d). Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}\}$ and $I = \{\phi, \{a\}\}$. Then $I_{\alpha\psi}$ - closed set$=\{\phi, X, \{a\}, \{b, c\}\}$ and $I_{rg}$$=\{P(X)\}$. It is clear that $\{c\}$ is I_g - closed set but it is not $I_{\alpha\psi}$ -closed.

Theorem 3.4. *Let (X, τ, I) be an ideal topological space and $I = \phi$. Then the following are hold:*

(a). Every $I_{\alpha\psi}$ closed set is gsp closed set.

(b). Every $I_{\alpha\psi}$ closed set is $g\alpha$ closed set.

(c). Every $I_{\alpha\psi}$ closed set is $\hat{g}$ closed set.

(d). Every $I_{\alpha\psi}$ closed set is $\alpha\psi$ closed set.

Proof.

(a). Let $A \subseteq U$ where U is open. Every open set is $\alpha\psi$ open and A is $I_{\alpha\psi}$ closed set, then $A^* \subseteq U$, we have $cl(A) = A^* \subseteq U$. Therefore $cl(A) \subseteq U \Rightarrow spcl(A) \subseteq cl(A) \subseteq U \Rightarrow spcl(A) \subseteq U$. Hence A is gsp closed set.

(b). Let $A \subseteq U$ where U is α open. Every α open set is $\alpha\psi$ open, then $A^* \subseteq U$, we have $cl(A) = A^* \subseteq U$. Therefore $\alpha cl(A) \subseteq cl(A) \subseteq U$. Hence A is $g\alpha$ closed set.

(c). Let $A \subseteq U$ where U is semi open. Every semi open set is $\alpha\psi$ open, then $A^* \subseteq U$, we have $cl(A) = A^* \subset U$. Therefore $scl(A) \subseteq \alpha cl(A) \subseteq cl(A) \subseteq U$. Hence A is $\hat{g}$ closed set.

(d). Let $A \subseteq U$ where U is α open. Every α open set is $\alpha\psi$ open. Let A be $I_{\alpha\psi}$ closed set, then $A^* \subseteq U$, we have $cl(A) = A^* \subseteq U$. Every closed set is ψ closed set. Therefore $\psi cl(A) \subset cl(A) \subseteq U$. Hence A is $\alpha\psi$ closed set. □

The converse of the above theorem need not be true as shown by the following examples.

Example 3.5.

(a). Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}\}$ and $I = \{\phi, \{a\}\}$. Then $I_{\alpha\psi}$ - closed set=$\{\phi, X, \{a\}, \{b, c\}\}$ and gsp closed set=$\{\phi, X, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$. It is clear that $\{b\}$ is gsp closed sets but it is not $I_{\alpha\psi}$ - closed set.

(b). Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}\}$ and $I = \{\phi, \{a\}\}$. Then $I_{\alpha\psi}$ - closed set=$\{\phi, X, \{a\}, \{b, c\}\}$ and $g\alpha$ closed set= $\{\phi, X, \{b\}, \{c\}, \{b, c\}\}$. It is clear that $\{b\}$ is $g\alpha$ closed sets but it is not $I_{\alpha\psi}$ - closed set.

(c). Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ and $I = \{\phi\}$. Then $I_{\alpha\psi}$ - closed set=$\{\phi, X, \{c\}, \{a, b\}, \{b, c\}\}$ and $\hat{g}$ closed set= $\{\phi, X, \{c\}, \{b, c\}, \{a, c\}\}$. It is clear that $\{b, c\}$ is $\hat{g}$ closed sets but it is not $I_{\alpha\psi}$ - closed set.

(d). Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}\}$ and $I = \{\phi, \{a\}\}$. Then $\alpha\psi$ - closed set=$\{\phi, X, \{b\}, \{c\}, \{b, c\}\}$ and $I_{\alpha\psi}$ closed set= $\{\phi, X, \{a\}, \{b, c\}\}$. It is clear that $\{b\}$ is $\alpha\psi$ closed sets but it is not $I_{\alpha\psi}$ - closed set.

Theorem 3.6. *Union of two closed sets is $I_{\alpha\psi}$ closed set.*

Proof. Let A and B be two closed sets and hence $I_{\alpha\psi}$ closed sets in X. Let U be a $\alpha\psi$ open set such that $A \cup B \subseteq U$. Then $A \subseteq U$ and $B \subseteq U$. Since A and B are $I_{\alpha\psi}$-closed sets. We have $A^* \subseteq U$ and $B^* \subseteq U$. Hence $A^* \cup B^* = (A \cup B)^* \subseteq U$. Therefore $A \cup B$ is $I_{\alpha\psi}$-closed sets. □

Theorem 3.7. *If (X, τ, I) is an ideal topological space and $A \subseteq X$. Then the following are equivalent:*

(1). A is $I_{\alpha\psi}$ closed set.

(2). $cl^*(A) \subseteq U$, whenever $A \subseteq U$ and U is $\alpha\psi$ open in X.

(3). For all $x \in cl^*(A), \alpha\psi cl(\{x\}) \cap A \neq \phi$.

(4). $cl^*(A) - A$ contains no non empty $\alpha\psi$ closed set.

(5). $A^* - A$ contains no non empty $\alpha\psi$ closed set.

Proof. (1)⇒(2) If A is $I_{\alpha\psi}$ closed set, then $A^* \subseteq U$, whenever $A \subseteq U$ and U is $\alpha\psi$ open in X and so $cl^*(A) = A \cup A^* \subseteq U$ whenever $A \subseteq U$ and U is $\alpha\psi$ open in X.

(2)⇒(3) Suppose $x \in cl^*(A)$. If $\alpha\psi cl(\{x\}) \cap A = \phi$, then $A \subseteq X - \alpha\psi cl(\{x\})$, by (1) $cl^*(A) \subseteq X - \alpha\psi cl(\{x\})$, a contradiction. Since $x \in cl^*(A)$.

(3)⇒(4) Suppose $F \subseteq cl^*(A) - A$, F is $\alpha\psi$ closed and $x \in F$, since $F \subseteq X - A$ and F is $\alpha\psi$ closed, then $A \subseteq X - F$, $\alpha\psi cl(\{x\}) \cap A = \phi$, since $x \in cl^*(A)$ by (3) $\alpha\psi cl(\{x\} \cap A \neq \phi$. Therefore $cl^*(A) - A$ contains no non empty $\alpha\psi$ closed set.
(4)⇒(5) Since $cl^*(A) - A = (A \cup A^*) - A = (A \cup A^*) \cap A^c = (A \cap A^c) \cup (A^* \cap A^c) = (A^* \cap A^c) = (A^* - A)$. Therefore $A^* - A$ contains no non empty $\alpha\psi$ closed set.
(5)⇒(1) Let $A \subseteq U$ where U is $\alpha\psi$ open set. Therefore $X - U \subseteq X - A$ and so $A^* \cap (X - U) \subseteq A^* \cap (X - A) = A^* - A$. Therefore $A^* \cap (X - U) = A^* - A$. Since A^* is always closed set, so A^* is $\alpha\psi$ closed set and so $A^* \cap (X - U)$ is $\alpha\psi$ closed set contained in $A^* - A$. Therefore $A^* \cap (X - U) = \phi$ and hence $A^* \subseteq U$. Therefore A is $I_{\alpha\psi}$ closed set. □

Theorem 3.8. *Let (X, τ, I) be an ideal space. For every $A \in I$ is $I_{\alpha\psi}$ closed set.*

Proof. Let $A \subseteq U$ where U is $\alpha\psi$ open set. Since $A^* = \phi$ for every $A \in I$, then $cl^*(A) = A^* \cup A = A \subseteq U$. Therefore by Theorem 3.7 A is $I_{\alpha\psi}$ closed set. □

Theorem 3.9. *If (X, τ, I) be an ideal space. Then A^* is always $I_{\alpha\psi}$ closed set for every subset A of X.*

Proof. Let $A^* \subseteq U$ where U is $\alpha\psi$ open set. Since $(A^*)^* \subseteq A^*$, we have $(A^*)^* \subseteq U$ whenever $A^* \subseteq U$ and U is $\alpha\psi$ open set. Hence A^* is $I_{\alpha\psi}$ closed set. □

Theorem 3.10. *Let (X, τ, I) be an ideal space. Then every $I_{\alpha\psi}$ closed, $\alpha\psi$ open set is $*$ closed set.*

Proof. Since A is $I_{\alpha\psi}$ closed. If A is $\alpha\psi$ open set and $A \subseteq A$. Then $A^* \subseteq A$. Hence A is $*$ closed set. □

Corollary 3.11. *Let (X, τ, I) be an ideal topological space and A be an $I_{\alpha\psi}$ closed set, then the following are equivalent:*

(1). A is $*$ closed set.

(2). $cl^*(A) - A$ is an $\alpha\psi$ closed set.

(3). $A^* - A$ is an $\alpha\psi$ closed set.

Proof. (1)⇒(2) If A is $*$ closed, then $A^* \subseteq A$, and so $cl^*(A) - A = (A \cup A^*) - A = \phi$. Hence $cl^*(A) - A$ is an $\alpha\psi$ closed set.
(2)⇒(3) Since $cl^*(A) - A = A^* - A$ and so $A^* - A$ is $\alpha\psi$ closed set.
(3)⇒(1) If $A^* - A$ is $\alpha\psi$ closed set. Since A is $I_{\alpha\psi}$ closed set by Theorem 3.7, $A^* - A = \phi$ and so A is $*$ closed set. □

Theorem 3.12. *Let (X, τ, I) be an ideal space and A is a $*$ dense in itself, then A is $\alpha\psi$ closed.*

Proof. Suppose A is a $*$ dense in itself, $I_{\alpha\psi}$ closed subset X. Let $A \subseteq U$ where U is α open. Every α open set is $\alpha\psi$ open. Then by Theorem 3.7 $cl^*(A) \subset U$ whenever $A \subseteq U$ and U is $\alpha\psi$ open. Since A is $*$ dense in itself, By Lemma 2.3 $cl(A) = cl^*(A)$, every closed set is ψ closed. Therefore $\psi cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α open. Hence A is $\alpha\psi$ closed. □

Theorem 3.13. *Let (X, τ, I) be an ideal space and $A \subseteq X$ Then A is $I_{\alpha\psi}$ closed iff $A = F - N$ where F is $*$ closed and N contains no non empty $\alpha\psi$ closed set.*

Proof. If A is $I_{\alpha\psi}$ closed then by Theorem 3.7 (5), $N = A^* - A$ contains no non empty $\alpha\psi$ closed set. If $F = cl^*(A)$, then F is $*$ closed such that, $F - N = (A \cup A^*) - (A^* - A) = (A \cup A^*) \cap (A^* \cap A^c)^c = (A \cup A^*) \cap ((A^*)^c \cup A) = (A \cup A^*) \cap (A \cup (A^*)^c) = A \cup (A^* \cap (A^*)^c) = A$.
Conversely, suppose $A = F - N$ where F is $*$ closed and N contains no non empty $\alpha\psi$ closed set. Let U be an $\alpha\psi$ open set such that $A \subseteq U$. Then $F - N \subseteq U$ which implies that $F \cap (X - U) \subseteq N$. Now $A \subseteq F$ and $F^* \subseteq F$ then $A^* \subseteq F^*$ and so

$A^* \cap (X - U) \subseteq F^* \cap (X - U) \subseteq F \cap (X - U) \subseteq N$. By hypothesis, since $A^* \cap (X - U) = \phi$ and so $A^* \subseteq U$. Hence A is $I_{\alpha\psi}$ closed. □

Theorem 3.14. *Let (X, τ, I) be an ideal space. If A and B are subsets of X such that $A \subseteq B \subseteq cl^*(A)$ and A is $I_{\alpha\psi}$ closed then B is $I_{\alpha\psi}$ closed.*

Proof. Since A is $I_{\alpha\psi}$ closed the by Theorem 3.7 (4), $cl^*(A) - A$ contains no non empty $\alpha\psi$ closed set. Since $cl^*(B) - B \subseteq cl^*(A) - A$ and so $cl^*(B) - B$ contains no non empty $\alpha\psi$ closed set. Hence B is $I_{\alpha\psi}$ closed. □

Corollary 3.15. *Let (X, τ, I) be an ideal space. If A and B are subsets of X such that $A \subseteq B \subseteq A^*$ and A is $I_{\alpha\psi}$ closed then A and B are $\alpha\psi$ closed.*

Proof. Let A and B be subsets of X such that $A \subseteq B \subseteq A^*$ which implies that $A \subseteq B \subseteq A^* \subseteq cl^*(A)$ and A is $I_{\alpha\psi}$ closed by Theorem 3.14, B is $I_{\alpha\psi}$ closed. Since $A \subseteq B \subseteq A^*$ then $A^* = B^*$ and so A and B are $*$ dense in itself by Theorem 3.12, A and B are $\alpha\psi$ closed □

Theorem 3.16. *Let (X, τ, I) be an ideal space and $A \subseteq X$. Then A is $I_{\alpha\psi}$ open iff $F \subseteq int^*(A)$ whenever F is $\alpha\psi$ closed and $F \subseteq A$.*

Proof. Suppose A is $I_{\alpha\psi}$ open, If F is $\alpha\psi$ closed and $F \subseteq F$ then $X - A \subseteq X - F$ and so $cl^*(X - A) \subseteq X - F$ by Theorem 3.7, therefore $F \subseteq X - cl^*(X - A) = int^*(A)$Hence $F \subseteq int^*(A)$.
Conversely, Let U be an $\alpha\psi$open set such that $X - A \subseteq U$. Then $X - U \subseteq A$ and so $X - U \subseteq int^*(A)$. Therefore $cl^*(X - A) \subseteq U$ by Theorem 3.7, $X - A$ is $I_{\alpha\psi}$ closed. Hence A is $I_{\alpha\psi}$ open. □

Theorem 3.17. *Let (X, τ, I) be an ideal space and $A \subseteq X$. If A is $I_{\alpha\psi}$ open and $int^*(A) \subseteq B \subseteq A$, then B is $I_{\alpha\psi}$ open.*

Proof. Since A is $I_{\alpha\psi}$ open, then $X - A$ $I_{\alpha\psi}$ closed by Theorem 3.7 $cl^*(X - A) - (X - A)$ contains no non empty $\alpha\psi$ closed set. Since $int^*(A) \subseteq int^*(B)$ which implies that $cl^*(X - B) \subseteq cl^*(X - A)$ and so $cl^*(X - B) - (X - B) \subseteq cl^*(X - A) - (X - A)$. Hence B is $I_{\alpha\psi}$ open. □

Theorem 3.18. *If (X, τ, I) is an ideal topological space and $A \subseteq X$. Then the following are equivalent:*

(1). A is $I_{\alpha\psi}$ closed set.

(2). $A \cup (X - A^*)$ is $I_{\alpha\psi}$ closed set.

(3). $A^* - A$ is $I_{\alpha\psi}$ closed set.

Proof. (1)⇒(2) Suppose A is $I_{\alpha\psi}$ closed set, if U is any $\alpha\psi$ open set such that $A \cup (X - A^*) \subseteq U$ then $X - U \subseteq X - (A \cup (X - A^*)) = X \cap (A \cup (A^*)^c) = A^* \cap A^c = A^* - A$. Since A is $I_{\alpha\psi}$ closed set by Theorem 3.7(5) it follows that $X - U = \phi$ and so $X = U$. Therefore $A \cup (X - A^*) \subseteq U$ which implies that $A \cup (X - A^*) \subseteq X$ and so $(A \cup (X - A^*))^* \subseteq X^* \subseteq X = U$. Hence $A \cup (X - A^*)$ is $I_{\alpha\psi}$ closed set.
(2)⇒(1) Suppose $A \cup (X - A^*)$ is $I_{\alpha\psi}$ closed set. If F is any $\alpha\psi$ closed set such that $F \subseteq A^* - A$ then $F \subseteq A^*$ and $F \subseteq X - A$ which implies that $X - A^* \subseteq X - F$ and $A \subseteq X - F$. Therefore $A \cup (X - A^*) \subseteq A \cup (X - F) = X - F$ and $X - F$ is $\alpha\psi$ open. Since $(A \cup (X - A^*))^* \subseteq X - F$ which implies that $A^* \cup (X - A^*)^* \subseteq X - F$ and so $A^* \subseteq X - F$ which implies that, $F \subseteq X - A^*$. Since $F \subseteq A^*$, it follows that $F = \phi$. Hence A is $I_{\alpha\psi}$ closed set.
(2)⇒(3) Since $X - (A^* - A) = X \cap (A^* \cap A^c)^c = X \cap ((A^*)^c \cup A) = (X \cap (A^*)^c) \cup (X \cap A) = A \cup (X - A^*)$. □

4. Weakly $I_{\alpha\psi}$ Closed Sets

Definition 4.1. *Let (X, τ, I) be an ideal topological space, then $(int(A))^* \subseteq U$ whenever $A \subseteq U$ and U is α open.*

Theorem 4.2. *If (X, τ, I) is an ideal topological space. Then the following are hold:*

(1). Every $I_{\alpha\psi}$ closed set is weakly $I_{\alpha\psi}$ closed.

(2). Every weakly $I_{\alpha\psi}$ closed set is weakly I_{rg} closed.

(3). Every weakly $I_{\alpha\psi}$ closed set is weakly I_g closed.

Proof.

(1). Let $A \subseteq U$ and U is α open. Every α open set is $\alpha\psi$ open and A be a $I_{\alpha\psi}$ closed set then $A^* \subseteq U$ by Lemma 2.3 $(int)^* \subseteq A^* \subseteq U$.

(2). Let $A \subseteq U$ and U is regular open in X. Every regular open set is α open and A be a weakly $I_{\alpha\psi}$ closed set then $(int)^* \subseteq U$. Therefore A is weakly I_{rg} closed.

(3). Let $A \subseteq U$ and U is open. Every open set is α open and A be a weakly $I_{\alpha\psi}$ closed set then $(int)^* \subseteq U$. Therefore A is weakly I_g closed. □

The converse of the above theorem need not be true as shown by the following examples.

Example 4.3.

(1). Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ and $I = \{\phi\}$. Then $I_{\alpha\psi}$ - closed set $= \{\phi, X, \{c\}, \{a, b\}\}$ and weakly $I_{\alpha\psi}$ closed set $= \{\phi, X, \{c\}, \{a, c\}, \{b, c\}\}$. It is clear that $\{a, c\}$ is weakly $I_{\alpha\psi}$ - closed set but it is not $I_{\alpha\psi}$ closed.

(2). Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ and $I = \{\phi\}$. Then weakly I_{rg} closed set $= \{\phi, X, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ and weakly $I_{\alpha\psi}$closed set $=\{\phi, X, \{c\}, \{a, c\}, \{b, c\}\}$. It is clear that $\{a, b\}$ is weakly I_{rg} closed set but it is not weakly $I_{\alpha\psi}$ - closed set.

(3). Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}\}$ and $I = \{\phi\}$. Then weakly I_g closed set $=\{\phi, X, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ and weakly $I_{\alpha\psi}$ closed set $= \{\phi, X, \{b\}, \{c\}, \{b, c\}\}$. It is clear that $\{a, b\}$ is weakly I_g closed set but it is not weakly $I_{\alpha\psi}$ - closed set.

Theorem 4.4. *Let (X, τ, I) is an ideal topological space and $A \subseteq X$. Then the following properties are equivalent:*

(1). A is weakly $I_{\alpha\psi}$ closed.

(2). $cl^*(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is α open set in X.

Proof. (1)⇒(2) Let A be a weakly $I_{\alpha\psi}$ closed set in (X, τ, I). Suppose that $A \subseteq U$ and U is α open set in X. We have $(int(A))^* \subseteq U$. Since $int(A) \subseteq A \subseteq U$, then $(int(A))^* \cup (int(A)) \subseteq U$. This implies that $cl^*(int(A)) \subseteq U$.
(2)⇒(1) Let $cl^*(int(A)) \subseteq U$ whenever $A \subseteq U$ and U in α open in$X$. Since $(int(A))^* \cup int(A) \subseteq U$, then $(int(A))^* \subseteq U$ whenever $A \subseteq U$ and U in α open in$X$. Therefore A is A is weakly $I_{\alpha\psi}$ closed set in (X, τ, I). □

Theorem 4.5. *Let (X, τ, I) be an ideal space and $A \subseteq X$. If A is weakly $I_{\alpha\psi}$ closed set then $(int(A))^* - A$ contains no non empty α closed set.*

Proof. Since A is weakly $I_{\alpha\psi}$ closed set in (X,τ,I). Suppose that U is a α closed set such that $U \subseteq (int(A))^* - A$. Since A is a weakly $I_{\alpha\psi}$ closed set, $X-U$ is α open and $A \subseteq X-U$ then $(int(A))^* \subseteq X-U$. We have $U \subseteq X-(int(A))^*$. Hence $U \subseteq (int(A))^* \cap (X-(int(A))^*) = \phi$. Thus $(int(A))^* - A$ contains no non empty α closed set. □

Theorem 4.6. *Let (X,τ,I) be an ideal space and $A \subseteq X$. If A is weakly $I_{\alpha\psi}$ closed set then $cl^*(int(A)) - A$ contains no non empty α closed set.*

Proof. Suppose that U is a α closed set such that $U \subseteq cl^*(int(A)) - A$ by Theorem 4.5. It follows from the fact that $cl^*(int(A)) - A = ((int(A))^* \cup int(A)) - A$. □

Theorem 4.7. *Let (X,τ,I) is an ideal topological space and $A \subseteq X$. Then the following properties are equivalent:*

(1). A is pre* I closed for each weakly $I_{\alpha\psi}$ closed set A in (X,τ,I).

(2). Each $\{x\}$ of X is a α closed set (or) $\{x\}$ is pre* I open.

Proof. (1)⇒(2) Let A be pre^* I closed for each weakly $I_{\alpha\psi}$ closed set in (X,τ,I). We have $cl^*(int(A)) \subseteq A$ for each weakly $I_{\alpha\psi}$ closed set A in (X,τ,I). Assume that $\{x\}$ is not a α closed set. It follows that X is the only α open set containing $X-\{x\}$. Then $X-\{x\}$ is a weakly $I_{\alpha\psi}$ closed set A in (X,τ,I). Thus $cl^*(int(X-\{x\})) \subseteq X-\{x\}$ and hence $\{x\} \subseteq int^*(cl(\{x\}))$. Consequently, $\{x\}$ is pre^* I open. (2)⇒(1) Let A be a weakly $I_{\alpha\psi}$ closed set in (X,τ,I). Let $x \in cl^*(int(A))$. Suppose that $\{x\}$ is pre^* I open, we have $\{x\} \subseteq int^*(cl(\{x\}))$. Since $x \in cl^*(int(A))$, then $int^*(cl\,\{x\}) \cap int(A) \neq \phi$. It follows that $cl\,\{x\} \cap int(A) \neq \phi$ we have $cl(\{x\} \cap int(A)) = \phi$ and then $\{x\} \cap int(A) \neq \phi$. Hence $x \in int(A)$. Thus we have $X \in A$. Suppose that $\{x\}$ is a α closed set by Theorem 4.5 $cl^*(int(A)) - A$ does not contain $\{x\}$. Since $x \in cl^*(int(A))$ then we have $x \in G$ consequently we have $x \in A$. Thus $cl^*(int(A)) \subset A$ and hence A is pre^* I closed. □

Theorem 4.8. *Let (X,τ,I) be an ideal space and $A \subseteq X$. If A is weakly $I_{\alpha\psi}$ closed set then $int(A) = K - N$ where K is IR closed and N contains no non empty α closed set.*

Proof. Let A be a weakly $I_{\alpha\psi}$ closed set in (X,τ,I).Take $N = (int(A))^* - A$. Then by Theorem 4.5, K contains no non empty α closed set. Then $K = cl^*(int(K))$. Moreover we have $K - N = ((int(A))^* \cup int(A)) - (int(A))^* - A) = ((int(A))^* \cup int(A)) \cap (X - int(A))^* \cup A) = int(A)$. □

Theorem 4.9. *Let (X,τ,I) be an ideal space and $A \subseteq X$ be a weakly $I_{\alpha\psi}$ closed set then $A \cup (X - int(A))^*)$ is a weakly $I_{\alpha\psi}$ closed set in (X,τ,I).*

Proof. Let A be a weakly $I_{\alpha\psi}$ closed set in (X,τ,I). Suppose that U is an α open set such that $A \cup (X-(int(A))^*) \subseteq U$. We have $X - U \subseteq X - (A \cup (X-(int(A))^*) = (X-A) \cap (int(A))^* = (int(A))^* - A$. Since $X-U$ is α closed set and A is a weakly $I_{\alpha\psi}$ closed set. It follows from Theorem 4.5, that $X - U = \phi$. Hence $X = U$. Thus X is the only α open set containing, $A \cup (X-(int(A))^*)$. Consequently, $A \cup (X-(int(A))^*)$ is a weakly $I_{\alpha\psi}$ closed set in (X,τ,I). □

Corollary 4.10. *Let (X,τ,I) be an ideal space and $A \subseteq X$ be a weakly $I_{\alpha\psi}$ closed set. Then $(int(A))^* - A)$ is a weakly $I_{\alpha\psi}$ open set in (X,τ,I).*

Theorem 4.11. *Let (X,τ,I) is an ideal topological space, then the following properties are equivalent:*

(1). Each subset of (X,τ,I) is a weakly $I_{\alpha\psi}$ closed set.

(2). A is pre* I closed for each α open set A in X.

Proof. (1)⇒(2) Suppose that each subset of (X, τ, I) is weakly $I_{\alpha\psi}$ closed set. Let A be an α open set. Since A is weakly $I_{\alpha\psi}$ closed, then we have $cl^*(int(A)) \subseteq$, thus A is pre^* I closed.

(2)⇒(1) Let A be a subset of (X, τ, I) and U be an α open set such that $A \subseteq U$ by (2) we have $cl^*(int(A)) \subseteq cl^*(int(U)) \subseteq U$. Thus A is a weakly $I_{\alpha\psi}$ closed set in (X, τ, I). □

Theorem 4.12. *Let (X, τ, I) be an ideal space, if A is a weakly $I_{\alpha\psi}$ closed set and $A \subseteq B \subseteq cl^*(int(A))$, then B is a weakly $I_{\alpha\psi}$ closed set.*

Proof. Let $B \subseteq U$ and U be an α open set in X. Since $A \subseteq U$ and U is a weakly $I_{\alpha\psi}$ closed set then $cl^*(int(A)) \subseteq U$. Since $B \subseteq cl^*(int(A))$, then $cl^*(int(B)) \subseteq cl^*(int(A)) \subseteq U$. Thus $cl^*(int(B)) \subseteq U$ and hence B is a weakly $I_{\alpha\psi}$ closed set. □

Corollary 4.13. *Let (X, τ, I) be an ideal space. If A be a weakly $I_{\alpha\psi}$ closed and open set, then $cl^*(A)$ is a weakly $I_{\alpha\psi}$ closed.*

Proof. Let A be a weakly $I_{\alpha\psi}$ closed set and open set in (X, τ, I). We have $A \subseteq cl^*(A) \subseteq cl^*(int(A))$. Hence by Theorem 4.12 $cl^*(A)$ is a weakly $I_{\alpha\psi}$ closed set in (X, τ, I). □

Theorem 4.14. *Let (X, τ, I) be an ideal space and $A \subseteq X$. Then A is a weakly $I_{\alpha\psi}$ open set iff $U \subseteq int^*(cl(A))$ whenever $U \subseteq A$ and U is an α closed set.*

Proof. Let U be an α closed set in X and $U \subseteq A$. It follows that $X - U$ is a g open set and $X - A \subseteq X - U$. Since $X - A$ is a weakly $I_{\alpha\psi}$ closed set, then $cl^*(int(X - A)) \subseteq X - U$, we have $X - int^*(cl(A)) \subseteq X - U$. Thus, $U \subseteq int^*(cl(A))$. Conversely, let V be an α open set in X and $X - A \subseteq V$. Since $X - V$ is an α closed set such that $X - V \subseteq A$, then $X - V \subseteq int^*(cl(A))$. We have $X - int^*(cl(A)) = cl^*(int(X - A)) \subseteq V$. Thus $X - A$ is a weakly $I_{\alpha\psi}$ closed set. Hence A is a weakly $I_{\alpha\psi}$ open set in (X, τ, I). □

Theorem 4.15. *Let (X, τ, I) be an ideal space and $A \subseteq X$. If A is a weakly $I_{\alpha\psi}$ open set then $U = X$ whenever U is an α open set and $int^*(cl(A)) \cup (X - A) \subseteq U$.*

Proof. Let U be an α open set in X and $int^*(cl(A)) \cup (X - A) \subseteq U$. We have $X - U \subseteq (X - int^*(cl(A)) \cap A = cl^*(int(X - A)) - (X - A)$. Since $X - U$ is an α closed set and $X - A$ is a weakly $I_{\alpha\psi}$ closed set, it follows from Theorem 4.6 that $X - U = \phi$. Thus we have $U = X$. □

Theorem 4.16. *Let (X, τ, I) be an ideal space and $A \subseteq X$. If A is a weakly $I_{\alpha\psi}$ open set and $int^*(cl(A)) \subseteq B \subseteq A$, then B is a weakly $I_{\alpha\psi}$ open set.*

Proof. Let A is a weakly $I_{\alpha\psi}$ open set and $int^*(cl(A)) \subseteq B \subseteq A$. Since $int^*(cl(A)) \subseteq B \subseteq A$, then $int^*(cl(A)) = int^*(cl(B))$. Let U be an α closed set and $U \subseteq B$ we have $U \subseteq A$. Since A is a weakly $I_{\alpha\psi}$ open set, it follows from Theorem 4.14 that $U \subseteq int^*(cl(A)) = int^*(cl(B))$. Hence by Theorem 4.13 B is a weakly $I_{\alpha\psi}$ open set. □

Corollary 4.17. *Let (X, τ, I) be an ideal space and $A \subseteq X$. If A is a weakly $I_{\alpha\psi}$ open and closed set, then $int^*(A)$ is a weakly $I_{\alpha\psi}$ open set.*

Proof. Let A be a weakly $I_{\alpha\psi}$ open and closed set. Then $int^*(cl(A)) = int^*(A) \subseteq int^*(A) \subseteq A$.Thus by Theorem 4.16, $int^*(A)$ is a weakly $I_{\alpha\psi}$ open set in (X, τ, I). □

5. $I_{\alpha\psi}$-Normal Spaces

Definition 5.1. *An ideal topological space (X, τ, I), is said to be $I_{\alpha\psi}$ normal space if every pair of disjoint closed subsets A and B of X, there exist disjoint $I_{\alpha\psi}$ open sets U and V such that $A \subseteq U$ and $B \subseteq V$.*

Theorem 5.2. *Let (X, τ, I) be an ideal space. Then the following are equivalent:*

(1). X is $I_{\alpha\psi}$ normal.

(2). For every closed set A and an open set V containing A there exist an $I_{\alpha\psi}$ open set U such that $A \subset U \subset cl^*(U) \subset V$.

Proof. (1)⇒(2) Let A be a closed set and Vbe an open set containing A. Then A and $X - V$ are disjoint closed set and so there exist disjoint $I_{\alpha\psi}$ open sets U and W such that $A \subset U$ and $X - V \subset W$. Now $U \cap W = \phi$ implies that $U \cap int^*(W) = \phi$ which implies that $U \subset X - int^*(W) = \phi$ and so $cl^*(U) \subset X - int^*(W)$. Again, $X - V \subset W$ implies that $X - W \subset V$ where V is open which implies that $cl^*(X - W) \subset V$ and so $X - int^*(W) \subset V$. Thus $A \subset U \subset cl^*(U) \subset X - int^*(W) \subset V$. Therefore $A \subset U \subset cl^*(U) \subset V$, where U is $I_{\alpha\psi}$ open.
(2)⇒(1) Let A and B be two disjoint closed subsets of X, by hypothesis, there exists an $I_{\alpha\psi}$ open set Usuch that $A \subset U \subset cl^*(U) \subset X - B$. Now $cl^*(U) \subset X - B$ implies that $B \subset X - cl^*(U)$. If $X - cl^*(U) = W$ then W is an $I_{\alpha\psi}$ open, since every $*$ closed set is $I_{\alpha\psi}$ closed. Hence U and W are the required disjoint $I_{\alpha\psi}$ open sets containing A and B respectively. Therefore (X, τ, I) is $I_{\alpha\psi}$ normal. □

Theorem 5.3. *Let (X, τ, I) be an ideal space. Then the following are equivalent:*

(1). X is normal.

(2). For every disjoint closed sets A and B there exist disjoint $I_{\alpha\psi}$ open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

(3). For every closed set A and open set B containing A there exist an $I_{\alpha\psi}$ open set U such that $A \subseteq U \subseteq cl^*(U) \subseteq B$.

Proof. (1)⇒(2) The proof is obvious.
(2)⇒(3) Let A be any closed set of X and B any open set of X such that $A \subseteq B$. Then $A$and $X - B$ are disjoint closed sets of X by (2) there exist an $I_{\alpha\psi}$ open set U and V such that $A \subseteq U$ and $X - B \subseteq V$. Since V is $I_{\alpha\psi}$ open set then $X - B \subseteq int^*(V)$ and $U \cap int^*(V) = \phi$. Therefore we obtain $cl^*(U) \subseteq cl^*(X - V)$ and hence $A \subseteq U \subseteq cl^*(U) \subseteq B$.
(3)⇒(1) Let A and B be any disjoint closed sets of X. Then $A \subset X - B$ and $X - B$ is open and hence there exist an $I_{\alpha\psi}$ open set G of X such that $A \subseteq G \subseteq cl^*(G) \subseteq X - B$. Put $U = int^*(G)$ and $V = X - cl^*(G)$. Then U and V are disjoint open sets of X such that $A \subseteq U$ and $B \subseteq V$. Therefore X is normal. □

Theorem 5.4. *Let (X, τ, I) be an ideal space which is $I_{\alpha\psi}$ normal. Then the following hold:*

(1). If F is closed and A is an αclosed set such that $A \cap F = \phi$, then there exist disjoint $I_{\alpha\psi}$ open sets U and V such that $\alpha cl(A) \subset U$ and $F \subset V$.

(2). For every closed sets A and every α open set B containing A, there exist an $I_{\alpha\psi}$ open set U such that $A \subset int^*(U) \subset U \subset B$.

(3). For α closed set A and every open set B containing A, there exists an $I_{\alpha\psi}$ closed set U such that $A \subset U \subset cl^*(U) \subset B$.

Proof.

(1). Let X be an $I_{\alpha\psi}$ normal, since $A \cap F = \phi$, $A \subset X - F$ where $X - F$ is open. Therefore by hypothesis $\alpha cl(A) \cap F = \phi$. Since X is $I_{\alpha\psi}$ normal, there exist disjoint $I_{\alpha\psi}$ open sets U and Vsuch that $\alpha cl(A) \subset U$ and $F \subset V$.

(2). Let A be a closed set and B be an α open set containing A. Then $X - B$ is an α closed set such that $A \cap (x - B) = \phi$ by (1) there exist disjoint $I_{\alpha\psi}$ open sets U and V such that $A \subset U$ and $X - B \subset V$. $A \subset U$ implies that $A \subset int^*(U)$, by Theorem 3.10. Hence $A \subset int^*(U) \subset U \subset X - V \subset B$.

(3). Let$A$ be an α closed set and B be an open set containing A. Then $X - B$ is a closed set contained in the α open set $X - A$ by (2) there exists an $I_{\alpha\psi}$ open set V such that $X - B \subset int^*(V) \subset X - A$. Therefore $A \subset X - V \subset cl^*(X - V) \subset B$. If $U = X - V$ then $A \subset U \subset cl^*(X - V) \subset B$ and so U is the required $I_{\alpha\psi}$ closed set. □

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