International Journal of Mathematics And its Applications
Volume 5, Issue 4–C (2017), 353–360.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

On Intuitionistic Fuzzy γ^* Generalized Connectedness

Research Article

V.M.Riya[1]* and D.Jayanthi[1]

1 Department of Mathematics, Avinashilingam University, Coimbatore, Tamil Nadu, India.

Abstract: In this paper we have introduced the intuitionistic fuzzy γ^* generalized connected space, intuitionistic fuzzy γ^* generalized super connected space and intuitionistic fuzzy regular γ^* generalized open set. We investigated some of their properties. Also we characterized the intuitionistic fuzzy γ^* generalized super connected space.

MSC: 54A40.

Keywords: Intuitionistic fuzzy topology, intuitionistic fuzzy γ^* generalized closed set, intuitionistic fuzzy γ^* generalized continuous mapping, intuitionistic fuzzy γ^* generalized connected space.

1. Introduction

The concept of fuzzy sets was introduced by Zadeh [11] and later Atanassov [1] generalized this idea to intuitionistic fuzzy sets using the notion of fuzzy sets. On the other hand Coker [2] introduced intuitionistic fuzzy topological spaces. Recently many fuzzy topological concept such as fuzzy connectedness have been generalized for intuitionistic fuzzy topological spaces. In this paper we introduce intuitionistic fuzzy γ^* generalized connectedness in intuitionistic fuzzy topological spaces. Also we provide some characterizations of intuitionistic fuzzy γ^* generalized connectedness.

2. Preliminaries

Definition 2.1 ([1])**.** *An intuitionistic fuzzy set (IFS for short) A is an object having the form*

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X. An intuitionistic fuzzy set A in X is simply denoted by $A = \langle x, \mu_A, \nu_A\rangle$ instead of denoting $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$.

Definition 2.2 ([1])**.** *Let A and B be two IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}$. Then,*

* *E-mail: riyavm@gmail.com*

(a). $A \subseteq B$ *if and only if* $\mu_A(x) \leq \mu_B(x)$ *and* $\nu_A(x) \geq \nu_B(x)$ *for all* $x \in X$,

(b). $A = B$ *if and only if* $A \subseteq B$ *and* $A \supseteq B$,

(c). $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$,

(d). $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle : x \in X\}$,

(e). $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle : x \in X\}$.

The intuitionistic fuzzy sets $0_\sim = \langle x, 0, 1\rangle$ *and* $1_\sim = \langle x, 1, 0\rangle$ *are respectively the empty set and the whole set of* X.

Definition 2.3 ([2]). *An intuitionistic fuzzy topology (IFT in short) on X is a family* τ *of IFSs in X satisfying the following axioms:*

(1). $0_\sim, 1_\sim \in \tau$,

(2). $G_1 \cap G_2 \in \tau$ *for any* $G_1, G_2 \in \tau$,

(3). $\cup G_i \in \tau$ *for any family* $\{G_i : i \in J\} \in \tau$.

In this case the pair (X, τ) *is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in* τ *is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement Ac of an IFOS A in an IFTS* (X, τ) *is called an intuitionistic fuzzy closed set (IFCS in short) in X.*

Definition 2.4 ([9]). *Two IFSs A and B are said to be q-coincident* ($A_q B$ *in short) if and only if there exits an element* $x \in X$ *such that* $\mu_A(x) > \nu_B(x)$ *or* $\nu_A(x) < \mu_B(x)$.

Definition 2.5 ([9]). *Two IFSs A and B are said to be not q-coincident* ($A_{\bar{q}} B$ *in short) if and only if* $A \subseteq B^c$.

Definition 2.6 ([3]). *An IFS* $A = \langle x, \mu_A, \nu_A\rangle$ *in an IFTS* (X, τ) *is said to be an*

(1). intuitionistic fuzzy γ *closed set (IF*γ*CS in short) if* $cl(int(A)) \cap int(cl(A)) \subseteq A$

(2). intuitionistic fuzzy γ *open set (IF*γ*OS in short) if* $A \subseteq int(cl(A)) \cup cl(int(A))$.

Definition 2.7 ([3]). *Let A be an IFS in an IFTS* (X, τ). *Then the* γ*-interior and* γ*-closure of A are defined as*

$$\gamma\, int(A) = \cup\{G/G \text{ is an IF}\gamma OS \text{ in } X \text{ and } G \subseteq A\}$$

$$\gamma\, cl(A) = \cap\{K/K \text{ is an IF}\gamma CS \text{ in } X \text{ and } A \subseteq K\}$$

Note that for any IFS A in (X, τ), *we have* $\gamma cl(A^c) = (\gamma int(A))^c$ *and* $\gamma int(A)^c = (\gamma cl(A))^c$.

Corollary 2.8 ([2]). *Let A,* A_i $(i \in J)$ *be intuitionistic fuzzy sets in X and B,* B_j $(j \in K)$ *be intuitionistic fuzzy sets in Y and* $f : X \to Y$ *be a function. Then*

(1). $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$

(2). $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$

(3). $A \subseteq f^{-1}(f(A))$ *[If f is injective, then* $A = f^{-1}(f(A))$*]*

(4). $f(f^{-1}(B)) \subseteq B$ *[If f is surjective, then* $B = f(f^{-1}(B))$*]*

(5). $f^{-1}(\cup B_j) = \cup f^{-1}(B_j)$

(6). $f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$

(7). $f^{-1}(0_\sim) = 0_\sim$

(8). $f^{-1}(1_\sim) = 1_\sim$

(9). $f^{-1}(B^c) = (f^{-1}(B))^c$

Definition 2.9 ([5]). *An IFS A of an IFTS (X, τ) is said to be an intuitionistic fuzzy γ^* generalized closed set (briefly IFγ^*GCS) if $cl(int(A)) \cap int(cl(A)) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ).*

Definition 2.10 ([6]). *A mapping $f : (X, \tau) \to (Y, \sigma)$ is called an intuitionistic fuzzy γ^* generalized continuous (IFγ^*G continuous for short) mapping if $f^{-1}(V)$ is an IFγ^*GCS in (X, τ) for every IFCS V of (Y, σ).*

Definition 2.11 ([5]). *If every IFγ^*GCS in (X, τ) is an IFγCS in (X, τ), then the space can be called as an intuitionistic fuzzy $\gamma^* T_{1/2}$ (IF$\gamma^* T_{1/2}$ in short) space.*

Definition 2.12 ([5]). *If every IFγ^*GCS in (X, τ) is an IFCS in (X, τ), then the space can be called as an intuitionistic fuzzy $\gamma^* cT_{1/2}$ (IF$\gamma^* cT_{1/2}$ in short) space.*

Definition 2.13 ([2]). *An IFTS (X, τ) is said to be an intuitionistic fuzzy C_5-connected space if the only IFS which are both IFOS and IFCS are $0_\sim$ and $1_\sim$.*

Definition 2.14 ([10]). *An IFTS (X, τ) is said to be an intuitionistic fuzzy GO-connected space if the only IFS which are both IFGOS and IFGCS are $0_\sim$ and $1_\sim$.*

Definition 2.15 ([8]). *An IFTS (X, τ) is an intuitionistic fuzzy C_5-connected between two IFSs A and B if there is no IFOS E in (X, τ) such that $A \subseteq E$ and $E_{\bar{q}} B$.*

Remark 2.16 ([7]). *If an IFS A in an IFTS (X, τ) is an IFγ^*GCS in X, then $\gamma^* gcl(A) = A$. But the converse may not be true in general, since intersection does not exist in IFγ^*GCS [5].*

Remark 2.17 ([7]). *If an IFS A in an IFTS (X, τ) is an IFγ^*GOS in X, then $\gamma^* gint(A) = A$. But the converse may not be true in general, since union does not exist in IFγ^*GOS [5].*

3. Intuitionistic Fuzzy γ^* Generalized Connected Spaces

In this section we introduce intuitionistic fuzzy γ^* generalized connected space and investigate some of their properties.

Definition 3.1. *An IFTS (X, τ) is said to be an IFγ^* generalized (IFγ^*G for short) connected space if the only IFS which are both IFγ^*GCS and IFγ^*CS are $0_\sim$ and $1_\sim$.*

Theorem 3.2. *Every IFγ^*G connected space is an IFC_5-connected space but not conversely in general.*

Proof. Let (X, τ) be an IFγ^*G connected space. Suppose (X, τ) is not an IFC_5-connected space [3], then there exists a proper IFS A which is both an IFOS and an IFCS in (X, τ). That is A is both an IFγ^*GOS and an IFγ^*GCS in (X, τ). This implies that (X, τ) is not an IFγ^*G connected space, a contradiction. Therefore (X, τ) must be an IFC_5-connected space. □

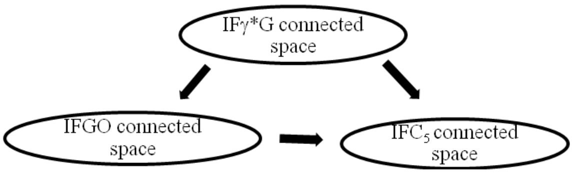
Example 3.3. *Let $X = \{a, b\}$ and $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ be an IFT on X, where $G_1 = \langle x, (0.2_a, 0.2_b), (0.7_a, 0.8_b)\rangle$ and $G_2 = \langle x, (0.6_a, 0.5_b), (0.4_a, 0.4_b)\rangle$. Then (X, τ) is an IFC_5-connected space but not an IFγ^*G connected space, since the IFS $A = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.5_b)\rangle$ is both an IFγ^*G open and an IFγ^*G closed set in (X, τ).*

Theorem 3.4. *Every IFγ^*G connected space is an IFGO-connected space but not conversely in general.*

Proof. Let (X, τ) be an IFγ^*G connected space. Suppose (X, τ) is not an IFGO-connected space, then there exists a proper IFS A which is both an IFGOS and an IFGCS in (X, τ). That is A is both an IFγ^*GOS and an IFγ^*GCS in (X, τ). This implies that (X, τ) is not an IFγ^*G connected space, a contradiction. Therefore (X, τ) must be an IFGO-connected space. □

Example 3.5. *In Example 3.3, (X, τ) is an IFGO connected space but not an IFγ^*G connected space, since the IFS $A = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.5_b)\rangle$ is both IFγ^*G open and IFγ^*G closed set in (X, τ).*

The relation between various types of intuitionistic fuzzy connectedness is given in the following diagram.



In the above diagram the reverse implications are not true in general.

Theorem 3.6. *The IFTS (X, τ) is an IFγ^*G connected space if and only if there exists no nonzero IFγ^*G open sets A and B in (X, τ) such that $A = B^c$.*

Proof. Necessity: Let A and B be two IFγ^*GOSs in (X, τ) such that $A \neq 0_\sim$, $B \neq 0_\sim$ and $A = B^c$. Therefore $A = B^c$ is an IFγ^*GCS. Since $B \neq 0_\sim$, $A = B^c \neq 1_\sim$. Hence A is a proper IFS which is both IFγ^*GOS and IFγ^*GCS in (X, τ). Hence (X, τ) is not an IFγ^*G connected space. But it is a contradiction to our hypothesis. Hence there exists no non-zero IFγ^*GOSs A and B in (X, τ) such that $A = B^c$.
Sufficiency: Suppose (X, τ) is not an IFγ^*G connected space. Then there exists an IFS A which is both an IFγ^*GOS and an IFγ^*GCS with $0_\sim \neq A \neq 1_\sim$. Now let $B = A^c$. Then B is an IFγ^*GOS and $B \neq 1_\sim$. This implies $B^c = A \neq 0_\sim$, which is a contradiction to our hypothesis. Hence (X, τ) is an IFγ^*G connected space. □

Theorem 3.7. *Let (X, τ) be an IF$\gamma^* cT_{1/2}$ space, then the following are equivalent:*

(1). (X, τ) is an IFγ^*G connected space

(2). (X, τ) is an IFGO connected space

(3). (X, τ) is an IFC5 connected space

Proof. (1)→(2) is obvious from the Theorem 3.4. (2)→(3) is obvious.
(3)→(1) Let (X, τ) be an intuitionistic fuzzy C_5 connected space. Suppose (X, τ) is not an IFγ^*G connected space, then there exists a proper IFS A in (X, τ) which is both an IFγ^*GOS and an IFγ^*GCS in (X, τ). But since (X, τ) is an IF$\gamma^* cT_{1/2}$ space, A is both an IFOS and an IFCS in (X, τ). This implies that (X, τ) is not an IFC_5 connected, which is a contradiction to our hypothesis. Therefore (X, τ) must be an IFγ^*G connected space. □

Theorem 3.8. *If f: $(X,\tau) \to (Y,\sigma)$ is an IFγ^*G continuous mapping and (X,τ) is an connected space, then (Y,σ) is an IFC_5 connected space.*

Proof. Let (X,τ) be an IFγ^*G connected space. Suppose (Y,σ) is not an IFC_5 connected space, then there exists a proper IFS A which is both an IFOS and an IFCS in (Y,σ). Since f is an IFγ^*G continuous mapping, $f^{-1}(A)$ is both an IFγ^*GOS and an IFγ^*GCS [6] in (X,τ). But it is a contradiction to our hypothesis. Hence (Y,σ) must be an IFC_5 connected space. □

Theorem 3.9. *If f: $(X,\tau) \to (Y,\sigma)$ is an IFγ^*G irresolute surjection mapping and (X,τ) is an IFγ^*G connected space, then (Y,σ) is an IFγ^*G connected space.*

Proof. Suppose (Y,σ) is not an IFγ^*G connected space, then there exists a proper IFS A such that A is both an IFγ^*GOS and an IFγ^*GCS in (Y,σ). Since f is an IFγ^*G irresolute mapping, $f^{-1}(A)$ is both an IFγ^*GOS and an IFγ^*GCS in (X,τ) [6]. But this is a contradiction to our hypothesis. Hence (Y,σ) must be an IFγ^*G connected space. □

Definition 3.10. *An IFTS (X,τ) is an IFγ^*G connected between two IFSs A and B if there is no IFγ^*GOS E in (X,τ) such that A E and $E_{\bar{q}}B$.*

Example 3.11. *Let $X = \{a,b\}$ and $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x,(0.7_a,0.8_b),(0.3_a,0.2_b)\rangle$. Then, $IF\gamma^*GO(X) = \{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]/0 \le \mu_a + \nu_a \le 1, 0 \le \mu_b + \nu_b \le 1\}$. The IFTS (X,τ) is an IFγ^*G connected between the two IFSs $A = \langle x,(0.6_a,0.7_b),(0.4_a,0.3_b)\rangle$ and $B = \langle x,(0.3_a,0.4_b),(0.7_a,0.6_b)\rangle$ as there exists no IFγ^*GO E such that A E and $E_{\bar{q}}B$.*

Theorem 3.12. *If an IFTS (X,τ) is an IFγ^*G connected between two IFSs A and B, then it is IFC_5 connected between A and B but the converse may not be true in general.*

Proof. Suppose (X,τ) is not an IFC_5 connected between A and B, then there exists an IFOS E in (X,τ) such that A E and $E_{\bar{q}}B$. Since every IFOS is an IFγ^*GOS, there exists an IFγ^*GOS E in (X,τ) such that $A \subseteq E$ and $E_{\bar{q}}B$. This implies (X,τ) is not an IFγ^*G connected between A and B. Thus we get a contradiction to our hypothesis. Therefore the IFTS (X,τ) must be an IFC_5 connected between A and B. □

Example 3.13. *Let $X = \{a,b\}$ and $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x,(0.2_a,0.3_b),(0.7_a,0.7_b)\rangle$. Then (X,τ) is IFC_5-connected between the IFSs $A = \langle x,(0.2_a,0.1_b),(0.8_a,0.9_b)\rangle$ and $B = \langle x,(0.7_a,0.8_b),(0.2_a,0.2_b)\rangle$. But (X,τ) is not IFγ^*G connected between A and B, since the IFS $E = \langle x,(0.2_a,0.1_b),(0.8_a,0.9_b)\rangle$ is an IFγ^*GOS such that $A \subseteq E$ and $E \subseteq B^c$.*

Theorem 3.14. *An IFTS (X,τ) is IFγ^*G connected between two IFSs A and B if and only if there is no IFγ^*GOS and IFγ^*GCS E in (X,τ) such that $A \subseteq E \subseteq B^c$.*

Proof. Necessity: Let (X,τ) be IFγ^*G connected between two IFSs A and B. Suppose that there exists an IFγ^*GOS and IFγ^*GCS E in (X,τ) such that $A \subseteq E \subseteq B^c$, then $E_{\bar{q}}B$ and $A \subseteq E$. This implies (X,τ) is not IFγ^*G connected between A and B, by a contradiction to our hypothesis. Therefore there is no IFγ^*GOS and an IFγ^*GCS E in (X,τ) such that $A \subseteq E \subseteq B^c$.

Sufficiency: Suppose that (X,τ) is not IFγ^*G connected between A and B. Then there exists an IFγ^*GOS E in (X,τ) such that $A \subseteq E$ and $E_{\bar{q}}B$. This implies that there is no IFγ^* GOS E in (X, τ) such that $A \subseteq E \subseteq B^c$. But this is a contradiction to our hypothesis. Hence (X,τ) is IFγ^*G connected between A and B. □

Theorem 3.15. *If an IFTS (X,τ) is IFγ^*G connected between A and B and $A \subseteq A_1$, $B \subseteq B_1$, then (X,τ) is an IFγ^*G connected between A_1 and B_1.*

Proof. Suppose that (X,τ) is not IFγ^*G connected between A_1 and B_1, then by Definition, there exists an IFγ^*GOS E in (X,τ) such that $A_1 \subseteq E$ and $E_{\bar{q}}B$. This implies $E \subseteq B_1^c$ and $A_1 \subseteq E$. That is $A \subseteq A_1 \subseteq E$. Hence $A \subseteq E$. Since $E \subseteq B_1^c$, $B_1 \subseteq E^c$. That is $B \subseteq B_1 \subseteq E^c$. Hence $E \subseteq B^c$. Therefore (X,τ) is not IFγ^*G connected between A and B, which is a contradiction to our hypothesis. Hence X must be IFγ^*G connected between A_1 and B_1. □

Theorem 3.16. *Let (X,τ) be an IFTS and A and B be IFSs in (X,τ). If $A_q B$, then (X,τ) is IFγ^*G connected between A and B.*

Proof. Suppose (X,τ) is not an IFγ^*G connected between A and B. Then there exists an IFγ^*GOS E in (X,τ) such that $A \subseteq E$ and $E \subseteq B^c$. This implies that $A \subseteq B^c$. That is $A_{\bar{q}}B$. But this is a contradiction to our hypothesis. Hence (X,τ) must be IFγ^*G connected between A and B. □

Theorem 3.17. *An IFTS (X,τ) is an IFγ^*G connected space if and only if there exists no non-zero IFγ^*G open sets A and B in (X,τ) such that $B = A^c$, $B = (cl(A))^c$, $A = (cl(B))^c$.*

Proof. Necessity: Assume that there exist IFSs A and B such that $A \neq 0_\sim \neq B$, $B = A^c$, $B = (\gamma cl(A))^c$, $A = (\gamma cl(B))^c$. Since $(\gamma cl(A))^c$ and $(\gamma cl(B))^c$ are IF γ open sets in (X,τ), A and B are IFγ^*G open sets in (X,τ). This implies (X,τ) is not an IFγ^*G connected space, which is a contradiction. Therefore there exists no non-zero IFγ^*G open sets A and B in (X,τ) such that $B = A^c$, $B = (\gamma cl(A))^c$, $A = (\gamma cl(B))^c$.
Sufficiency: Let A be both an IFγ^*GOS and an IFγ^*GCS in (X,τ) such that $1_\sim \neq A \neq 0_\sim$. Now by taking $B = A^c$, we obtain a contradiction to our hypothesis. Hence (X,τ) is an IFγ^*G connected space. □

Definition 3.18. *An IFS A is called an intuitionistic fuzzy regular γ^* generalized open set (IFRγ^*GOS for short) if $A = \gamma^* gint(\gamma^* gcl(A))$. The complement of an IFRγ^*GOS is called an intuitionistic fuzzy regular γ^* generalized closed set (IFRγ^*GCS for short).*

Definition 3.19. *An IFTS (X,τ) is called an intuitionistic fuzzy γ^* generalized super connected space (IFγ^*GS connected for short) if there exists no proper IFRγ^*GOS in (X,τ).*

Theorem 3.20. *Let (X,τ) be an IFTS, then the following are equivalent:*

(1). (X,τ) is an IFγ^*GS connected space.

(2). For every non-zero IFRγ^*GOS A, $\gamma^* gcl(A) = 1_\sim$.

(3). For every IFRγ^*GCS A with $A \neq 1_\sim$, $\gamma^* gint(A) = 0_\sim$.

(4). There exists no IFRγ^*GOSs A and B in (X,τ) such that $A \neq 0_\sim \neq B$, $A \subseteq B^c$.

(5). There exists no IFRγ^*GOSs A and B in (X,τ) such that $A \neq 0_\sim \neq B$, $B = (\gamma^* gcl(A))^c$, $A = (\gamma^* gcl(B))^c$.

(6). There exists no IFRγ^*GCSs A and B in (X,τ) such that $A \neq 1_\sim \neq B$, $B = (\gamma^* gint(A))^c$, $A = (\gamma^* gint(B))^c$.

Proof. (1)⇒(2) Let $A \neq 0_\sim$ be an IFRγ^*GOS in X and $\gamma^* gcl(A) \neq 1_\sim$. Now let $B = \gamma^* gint(\gamma^* gcl(A))^c$. Then B is a proper IFR$\gamma^*$GOS in (X,τ). But this is a contradiction to the fact that (X,τ) is an IFγ^*GS connected space. Therefore $\gamma^* gcl(A) = 1_\sim$.

(2)⇒(3) Let $A \neq 1_\sim$ be an IFRγ^*GCS in (X,τ). If $B = A^c$, then B is an IFRγ^*GOS in (X,τ) with $B \neq 0_\sim$. Hence $\gamma^*gcl(B) = 1_\sim$, by hypothesis. This implies $(\gamma^*gcl(B))^c = 0_\sim$. That is $\gamma^*gint(B^c) = 0_\sim$. Hence $\gamma^*gint(A) = 0_\sim$.

(3)⇒(4) Suppose A and B be two IFRγ^*GOSs in (X,τ) such that $A \neq 0_\sim \neq B$, $A \subseteq B^c$. Since B^c is an IFRγ^*GCS in (X,τ) and $B \neq 0_\sim$ implies $B^c \neq 1_\sim$, $B^c = \gamma^*gcl(\gamma^*gint(B^c))$ and we have $\gamma^*gint(B^c) = 0_\sim$. But $A \subseteq B^c$. Therefore $0_\sim \neq A = \gamma^*gint(\gamma^*gcl(A))\gamma^*gint(\gamma^*gcl(B^c)) = \gamma^*gint(\gamma^*gcl(\gamma^*gcl(\gamma^*gint(B^c)))) = \gamma^*gint(\gamma^*gcl(\gamma^*gint(B^c))) = \gamma^*gint(B^c) = 0_\sim$. A contradiction arises. Therefore (4) is true.

(4)⇒(1) Suppose $0_\sim \neq A \neq 1_\sim$ be an IFRγ^*GOSs in (X,τ). If we take $B = (\gamma^*gcl(A))^c$, then B is an IFRγ^*GOS, since $\gamma^*gint(\gamma^*gcl(B)) = \gamma^*gint(\gamma^*gcl(\gamma^*gcl(A))^c) = \gamma^*gint(\gamma^*gint(\gamma^*gcl(A)))^c = \gamma^*gint(A^c) = (\gamma^*gcl(A))^c = B$. Also we get $B \neq 0_\sim$, since otherwise, if $B = 0_\sim$, this implies $(\gamma^*gcl(A))^c = 0_\sim$. That is $\gamma^*gcl(A) = 1_\sim$. Hence $A = \gamma^*gint(\gamma^*gcl(A)) = \gamma^*gint(1_\sim) = 1_\sim$, which is a contradiction. Therefore $B \neq 0_\sim$ and $A \subseteq B^c$. But this is a contradiction to (4). Therefore (X,τ) is an IFγ^*GS connected space.

(1)⇒(5) Suppose A and B are any two IFRγ^*GOSs in (X,τ) such that $A \neq 0_\sim \neq B$, $B = (\gamma^*gcl(A))^c$ and $A = (\gamma^*gcl(B))^c$. Now we have $\gamma^*gint(\gamma^*gcl(A)) = \gamma^*gint(B^c) = (\gamma^*gcl(B))^c = A$, $A \neq 0_\sim$ and $A \neq 1_\sim$, since if $A = 1_\sim$, then $1_\sim = (\gamma^*gcl(B))^c \Rightarrow \gamma^*gcl(B) = 0_\sim \Rightarrow B = 0_\sim$. But $B \neq 0_\sim$. Therefore $A \neq 1_\sim \Rightarrow A$ is a proper IFRγ^*GOS in (X,τ), which is a contradiction to (1). Hence (5) is true.

(5)⇒(1) Suppose A is an IFRγ^*GOS in (X,τ) such that $0_\sim \neq A \neq 1_\sim$. Now take $B = (\gamma^*gcl(A))^c$. In this case we get $B \neq 0_\sim$ and B is an IFRγ^*GOS in (X,τ), $B = (\gamma^*gcl(A))^c$ and $(\gamma^*gcl(B))^c = (\gamma^*gcl(\gamma^*gcl(A))^c)^c = \gamma^*gint(\gamma^*gcl(A)^c)^c = \gamma^*gint(\gamma^*gcl(A)) = A$. But this is a contradiction to (5). Therefore (X,τ) is an IFγ^*GS connected space.

(5)⇒(6) Suppose A and B be two IFRγ^*GCSs in (X,τ) such that $A \neq 1_\sim \neq B$, $B = (\gamma^*gint(A))^c$ and $A = (\gamma^*gint(B))^c$. Taking $C = A^c$ and $D = B^c$, C and D become IFRγ^*GOSs in (X,τ) with $C \neq 0_\sim \neq D$, $D = (\gamma^*gcl(C))^c = (\gamma^*gcl(D))^c$, which is a contradiction to (5). Hence (6) is true.

(6)⇒(5) It can be proved easily by the similar way as in (5)⇒(6). □

References

[1] K.T.Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems, 20(1)(1986), 87-96.

[2] D.Coker, *An introduction to intuitionistic fuzzy topological spaces*, Fuzzy Sets and Systems, 88(1)(1997), 81-89.

[3] I.M.Hanafy, *Intuitionistic Fuzzy γ-Continuity*, Canad. Math. Bull., 52(4)(2009), 544554.

[4] Joung Kon Jeon, Young Bae Jun and Jin Han Park, *Intuitionistic fuzzy alpha continuity and intuitionistic fuzzy pre continuity*, International Journal of Mathematics and Mathematical Sciences, 2005(19)(2005), 3091-3101.

[5] V.M.Riya and D.Jayanthi, *Intuitionistic fuzzy γ^* generalized closed sets*, Advances in Fuzzy Mathematics, 12(3)(2017), 389-410.

[6] V.M.Riya and D.Jayanthi, *Intuitionistic fuzzy γ^* generalized continuous mappings*, Global Journal of Pure and Applied Mathematics, 13(7)(2017), 2859-2874.

[7] V.M.Riya and D.Jayanthi, *Intuitionistic fuzzy almost γ^* generalized continuous mappings*, Applied Mathematical Sciences, 11(31)(2017), 1531-1538.

[8] R.Santhi and K.Sakthivel, *Intuitionistic fuzzy alpha generalized connectedness in fuzzy topological spaces*, Int. Jour. of Appl. Math. and Physics, 3(1)(2011), 1-5.

[9] S.S.Thakur and Rekha Chaturvedi, *Regular Generalized closed sets in intuitionistic fuzzy topological spaces*, Universitatea Din Bacau, Studii Si Cercetari Stiintifice, Seria, 6(2006), 257-272.

[10] S.S.Thakur and R.Chaturvedi, *Generalized continuity in intuitionistic fuzzy topological spaces*, Notes on Intuitionistic Fuzzy Sets, 12(1)(2006), 38-44.

[11] L.A.Zadeh, *Fuzzy sets*, Information and Control, 8(3)(1965), 338-353.