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Some Labeling Techniques of Braided Star Graph

Research Article

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Abstract: This paper aims to defined a new graph, Braided Star Graph. and to focus on some labeling methods of Braided Star Graph. We investigate Braided Star Graph with six types of labeling; Cordial, H-cordial, Prime, Total prime, Vertex prime, Difference cordial.

MSC: 05C78.

Keywords: Graph Labeling, Braided Star Graph, Cordial Labeling, Prime Labeling.

1. Introduction

We begin with simple, finite, undirected graph $G = (V(G), E(G))$ where $V(G)$ and $E(G)$ denotes the vertex set and the edge set respectively. For a finite set A, $|A|$ denotes the number of elements of A. For all other terminology we follow Gross [5]. We provide some useful definitions for the present work.

Definition 1.1. *The graph labeling is an assignment of numbers to the vertices or edges or both subject to certain condition(s).*

A detailed survey of various graph labeling is explained in Gallian [4].

Definition 1.2. *For a graph $G = (V(G), E(G))$, a mapping $f : V(G) \to \{0, 1\}$ is called a binary vertex labeling of G and $f(v)$ is called the label of the vertex v of G under f. For an edge $e = uv$, the induced edge labeling $f^* : E(G) \to \{0, 1\}$ defined as $f^*(uv) = |f(u) - f(v)|$. Let $v_f(0), v_f(1)$ be the number of vertices of G having labels 0 and 1 respectively under f and let $e_f(0), e_f(1)$ be the number of edges having labels 0 and 1 respectively under f^*.*

Definition 1.3. *A binary vertex labeling f of a graph G is called a cordial labeling if $|v_f(1)-v_f(0)| \leq 1$ and $|e_f(1)-e_f(0)| \leq 1$. A graph G is said to be cordial if it admits cordial labeling.*

The concept of cordial labeling was introduced by Cahit [1]. Lee and Liu [6] proved that all complete bipartite graphs and all fans are cordial. Further, they proved that, the cycle C_n is cordial if and only if $n \not\equiv 2 \pmod 4$, the wheel W_n is cordial if and only if $n \not\equiv 3 \pmod 4$, $n \geq 3$. Prajapati and Gajjar [13] proved that complement of wheel graph and complement of cycle graph are cordial if $n \not\equiv 4 \pmod 8$ or $n \not\equiv 7 \pmod 8$. Prajapati and Gajjar [14] proved that cordial labeling in the context of duplication of cycle graph and path graph.

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Definition 1.4. *Let G be a graph with vertex set $V(G)$ and edge set $E(G)$ and let $f : E(G) \to \{0,1\}$. Define f^* on $V(G)$ by $f^* = \sum\{f(uv)/uv \in E(G)\}$ (mod 2). The function f is called an E- cordial labeling of G if $|v_f(1) - v_f(0)| \leq 1$ and $|e_f(1) - e_f(0)| \leq 1$. A graph is called E-cordial if it admits E-cordial labeling.*

In 1997 Yilmaz and Cahit [19] introduced E-cordial labeling as a weaker version of edge-graceful labeling and with the blend of cordial labeling. They proved that the trees with n vertices, K_n, C_n are E-cordial if and only if $n \not\equiv 2 \pmod 4$ while $K_{m,n}$ admits E-cordial labeling if and only if $m + n \not\equiv 2 \pmod 4$.

Definition 1.5. *A prime labeling of a graph G is an injective function $f : V(G) \to \{1, 2, ..., |V|\}$ such that for every pair of adjacent vertices u and v, $gcd(f(u), f(v)) = 1$. The graph which admits a prime labeling is called a prime graph.*

The notion of a prime labeling was originated by Roger Entringer and was discussed in a paper by Tout et al. [16] . Many researchers have studied prime graphs. For e.g. Fu and Huang [3] have proved that P_n and $K_{1,n}$ are prime graphs. Lee et al. [7] have proved that W_n is a prime graph if and only if n is even. Vaidya and Prajapati [17] has proved that if $n_1 \geq 4$ is an even integer and n_2 is a natural number, then the graph obtained by identifying any of the rim vertices of a wheel W_{n_1} with an end vertex of a path graph P_{n_2} is a prime graph. Vaidya and Prajapati [18] have proved that switching the apex vertex in W_n is a prime graph and switching a rim vertex in W_n is a prime graph if $n+1$ is prime. In the same paper it has been proved that W_n is switching invariant if n is even.

Definition 1.6. *G is called a vertex prime graph if there is a bijection $f : E(G) \to \{1, 2, ..., |E|\}$ such that for any vertex v, $\underset{uv \in E}{gcd}\{f(uv)\} = 1$. The bijection f is called a vertex prime labeling of G.*

Definition 1.7. *Let $G = (V, E)$ be a graph with p verhtices and q edges. A bijection $f : V(G) \to \{1, 2, ..., |V| + |E|\}$ is said to be a total prime labeling if for each edge $e = uv$, the labels assigned to u and v are relatively prime and for each vertex of degree at least 2, the greatest common divisor of the labels on the incident edges is 1. A graph which admits Total Prime Labeling is called total prime graph.*

Prime labeling and vertex prime labeling are introduced in [16] and [2]. Combining these two, The notion of a total prime labeling was originated by Ramasubramanian and Kala [15] have proved that paths P_n, star $K_{1,n}$, bistar, comb, cycles C_n where n is even, helm H_n, $K_{2,n}$ and fan graph are total prime graph.

Definition 1.8. *Let $G = (V, E)$ be a (p, q) graph, and f be a map from $V(G)$ to $\{1, 2, ..., p\}$. For each edge uv assign the label $|f(v) - f(u)| \leq 1$; f is called a difference cordial labeling if f is a one-to-one map and $|e_f(1) - e_f(0)| \leq 1$ where $e_f(1)$ denotes the number of edges labeled with 1 while $e_f(0)$ denotes the number of edges not labeled with 1. A graph with a difference cordial labeling is called a difference cordial graph.*

Ponraj et al. [10] first introduced the concept of difference cordial labeling in 2013. After that, they introduced many concepts and studied some types of graphs that have this kind of labeling, such as path, cycle, complete graph, complete bipartite graph, bistar, wheel, web, sun ower graph, lotus inside a circle, pyramid, permutation graph, book with n pentagonal pages, t -fold wheel, and double fan, and some more standard graphs were investigated in [8–12]. In this paper, for every natural number n the set $\{1, 2, ..., n\}$ will be denoted by $[n]$. Origami is an ancient Japanese art of folding paper. The word origami comes from two Japanese words: "ori", which means to fold, and "kami", which means paper. Usually origami models are made strictly by folding paper. There is no cutting or gluing involved. Even if origami is mainly an artistic product, it has received a great deal of attention from mathematicians, because of its interesting algebraic and geometrical properties. We present a new graph inspired from a model of origami namely Braided Star.

Definition 1.9. *Let a_0 be the apex vertex and $a_1, a_2, ..., a_{n-1}, a_n$ be consecutive n rim vertices of wheel graph W_n, $n \geq 3$; let $b_1, b_2, b_3, ..., b_{2n-1}, b_{2n}$ be consecutive $2n$ vertices of cycle C_{2n}; let $c_1, c_2, c_3, ..., c_{2n-1}, c_{2n}$ be consecutive $2n$ vertices of cycle C_{2n}. Join each a_i to b_{2i-1} by an edge and b_{2i} to c_{2i} by an edge. Take a new vertex d_i; Join each d_i to c_{2i-1} and c_{2i+1} by an edge, for each $i \in [n]$, subscript are taken modulo n. The resulting graph is called braided star graph BRS_n. which shown in Figure 1.*

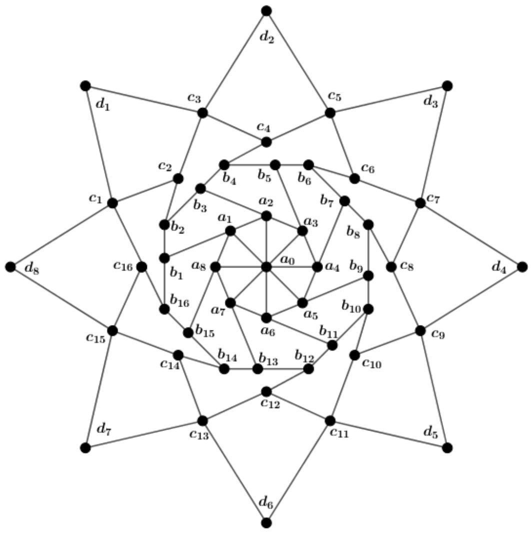

Figure 1. **The Braided Star Graph BRS_8.**

2. Main Results

Theorem 2.1. *BRS_n is cordial.*

Proof. For the graph BRS_n, $V(BRS_n) = \{a_0, a_i, d_i/1 \leq i \leq n\} \cup \{b_i, c_i/1 \leq i \leq 2n\}$ and $E(BRS_n) = \{a_0a_i, a_ib_{2i-1}, b_{2i}c_{2i}, c_{2i-1}d_i/1 \leq i \leq n\} \cup \{b_ib_{i+1}, c_ic_{i+1}/1 \leq i \leq 2n-1\} \cup \{a_na_1, b_{2n}b_1, c_{2n}c_1, d_nc_1\} \cup \{a_ia_{i+1}, d_ic_{2i+1}/1 \leq i \leq n-1\}$. Therefore $|V(BRS_n)| = 6n+1$ and $|E(BRS_n)| = 10n$. Define $f : V(BRS_n) \to \{0,1\}$ as follows:

$$f(x) = \begin{cases} 0, & \text{if } x = a_0; \\ 1, & \text{if } x \in \{a_i, b_{2i-1}, d_i\},\ i \in [n]; \\ 0, & \text{if } x \in \{b_{2i}, c_{2i}, c_{2i-1}\},\ i \in [n]. \end{cases}$$

Thus $v_f(1) = 3n$ and $v_f(0) = 3n+1$. The induced edge labeling $f^* : E(BRS_n) \to \{0,1\}$ is $f^*(uv) = |f(u) - f(v)|$, for every edge $e = uv \in E$. Therefore

$$f^*(e) = \begin{cases} 1 & \text{if } e = b_ib_{i+1},\ i \in [2n-1]; \\ 0 & \text{if } e = c_ic_{i+1},\ i \in [2n-1]; \\ 1 & \text{if } e \in \{a_0a_i, d_ic_{2i-1}\},\ i \in [n]; \\ 0 & \text{if } e \in \{a_ib_{2i-1}, b_{2i}c_{2i}\},\ i \in [n]; \\ 0 & \text{if } e = a_ia_{i+1},\ i \in [n-1]; \\ 1 & \text{if } e = d_ic_{2i+1},\ i \in [n-1]; \\ 1 & \text{if } e \in \{b_{2n}b_1, d_nc_1\}; \\ 0 & \text{if } e \in \{a_na_1, c_{2n}c_1\}. \end{cases}$$

Thus $e_f(1) = 5n$ and $e_f(0) = 5n$. Therefore f satisfies the conditions $|v_f(1) - v_f(0)| \leq 1$ and $|e_f(1) - e_f(0)| \leq 1$ for cordial labeing . So, f admits cordial labeling of BRS_n. Hence BRS_n is cordial. □

Theorem 2.2. *BRS_n is E-cordial.*

Proof. For the graph BRS_n, $V(BRS_n) = \{a_0, a_i, d_i/1 \leq i \leq n\} \cup \{b_i, c_i/1 \leq i \leq 2n\}$ and $E(BRS_n) = \{a_0a_i, a_ib_{2i-1}, b_{2i}c_{2i}, c_{2i-1}d_i/1 \leq i \leq n\} \cup \{b_ib_{i+1}, c_ic_{i+1}/1 \leq i \leq 2n-1\} \cup \{a_na_1, b_{2n}b_1, c_{2n}c_1, d_nc_1\} \cup \{a_ia_{i+1}, d_ic_{2i+1}/1 \leq i \leq n-1\}$. Therefore $|V(BRS_n)| = 6n+1$ and $|E(BRS_n)| = 10n$. Define $f : E(BRS_n) \to \{0,1\}$ as follows:

$$f(e) = \begin{cases} 0 & \text{if } e \in \{a_ia_{i+1}, b_{2i}b_{2i+1}, c_{2i}c_{2i+1}, c_{2i+1}d_i\},\ i \in [n-1]; \\ 0 & \text{if } e = a_0a_i,\ i \in [n]; \\ 1 & \text{if } e \in \{a_ib_{2i-1}, b_{2i-1}b_{2i}, b_{2i}c_{2i}, c_{2i-1}c_{2i}, c_{2i-1}d_i\},\ i \in [n]; \\ 0 & \text{if } e \in \{a_na_1, b_{2n}b_1, c_{2n}c_1, c_1d_n\}. \end{cases}$$

Thus $e_f(1) = 5n$ and $e_f(0) = 5n$. The induced vertex labeling $f^* : V(BRS_n) \to \{0,1\}$ is $f^*(v) = \sum\{f(uv)/uv \in E(BRS_n)\}(mod 2)$. Therefore

$$f^*(x) = \begin{cases} 0 & \text{if } x = a_0; \\ 1 & \text{if } x \in \{a_i, d_i, c_{2i-1}\},\ i \in [n]; \\ 0 & \text{if } x \in \{b_{2i}, b_{2i-1}, c_{2i}\},\ i \in [n]. \end{cases}$$

Thus $v_f(1) = 3n$ and $v_f(0) = 3n+1$. Therefore f satisfies the conditions $|v_f(1) - v_f(0)| \leq 1$ and $|e_f(1) - e_f(0)| \leq 1$ for E-cordial labeing. So, f admits E-cordial labeling of BRS_n. Hence BRS_n is E-cordial. □

Theorem 2.3. *BRS_n is prime graph.*

Proof. For the graph BRS_n, $V(BRS_n) = \{a_0, a_i, d_i/1 \leq i \leq n\} \cup \{b_i, c_i/1 \leq i \leq 2n\}$ and $E(BRS_n) = \{a_0a_i, a_ib_{2i-1}, b_{2i}c_{2i}, c_{2i-1}d_i/1 \leq i \leq n\} \cup \{b_ib_{i+1}, c_ic_{i+1}/1 \leq i \leq 2n-1\} \cup \{a_na_1, b_{2n}b_1, c_{2n}c_1, d_nc_1\} \cup \{a_ia_{i+1}, d_ic_{2i+1}/1 \leq i \leq n-1\}$. Therefore $|V(BRS_n)| = 6n+1$ and $|E(BRS_n)| = 10n$. Define $f : V(BRS_n) \to [6n+1]$ as follows:

$$f(x) = \begin{cases} 1 & \text{if } x = a_0; \\ 6i - \left(1 + 3(\frac{1+(-1)^i}{2})\right) & \text{if } x = a_i,\ i \in [n]\,; \\ 6i - \left(1 + 3(\frac{1+(-1)^{i+1}}{2})\right) & \text{if } x = b_{2i-1},\ i \in [n]\,; \\ 6i - 3 & \text{if } x = b_{2i},\ i \in [n]\,; \\ 6i & \text{if } x = d_i,\ i \in [n]\,; \\ 6i - 2 & \text{if } x = c_{2i},\ i \in [n]\,; \\ 6i + 1 & \text{if } x = c_{2i-1},\ i \in [n]\,. \end{cases}$$

Clearly f is an injective function. Let e be an arbitrary edge of BRS_n . To prove f is a prime labeling of BRS_n we have the following cases:

If $e = a_0a_i,\ gcd(f(a_0), f(a_i)) = gcd(1, 6i - \left(1 + 3(\frac{1+(-1)^i}{2})\right)) = 1,\ i \in [n]$.

If $e = a_ib_{2i-1},\ gcd(f(a_i), f(b_{2i-1})) = gcd(6i - \left(1 + 3(\frac{1+(-1)^i}{2})\right), 6i - \left(1 + 3(\frac{1+(-1)^{i+1}}{2})\right)) = 1,\ i \in [n]$.

If $e = a_ia_{i+1},\ gcd(f(a_i), f(a_{i+1})) = gcd(6i - \left(1 + 3(\frac{1+(-1)^i}{2})\right), 6(i+1) - \left(1 + 3(\frac{1+(-1)^{i+1}}{2})\right)) = 1,\ i \in [n-1]$.

If $e = b_{2i}c_{2i},\ gcd(f(b_{2i}), f(c_{2i})) = gcd(6i - 3, 6i - 2) = 1,\ i \in [n]$.

If $e = c_{2i-1}d_i,\ gcd(f(c_{2i-1}), f(d_i)) = gcd(6i + 1, 6i) = 1,\ i \in [n]$.

If $e = d_ic_{2i+1},\ gcd(f(d_i), f(c_{2i+1})) = gcd(6i, 6i + 7) = 1,\ i \in [n-1]$.

If $e = c_{2i-1}c_{2i},\ gcd(f(c_{2i-1}), f(c_{2i})) = gcd(6i + 1, 6i - 2) = 1,\ i \in [n]$.

If $e = c_{2i}c_{2i+1},\ gcd(f(c_{2i}), f(c_{2i+1})) = gcd(6i-2, 6i+7) = 1,\ i \in [n-1]$.

If $e = b_{2i-1}b_{2i},\ gcd(f(b_{2i-1}), f(b_{2i})) = gcd(6i - \left(1 + 3(\frac{1+(-1)^{i+1}}{2})\right), 6i-3) = 1,\ i \in [n]$.

If $e = b_{2i}b_{2i+1},\ gcd(f(b_{2i}), f(b_{2i+1})) = gcd(6i-3, 6i+2+\left(3(\frac{1+(-1)^{i+1}}{2})\right)) = 1,\ i \in [n-1]$.

If $e = c_{2n}c_1,\ gcd(f(c_{2n}), f(c_1)) = gcd(6n-2, 7) = 1$.

If $e = d_nc_1,\ gcd(f(d_n), f(c_1)) = gcd(6n, 7) = 1$.

If $e = b_{2n}b_1,\ gcd(f(b_{2n}), f(b_1)) = gcd(6n-3, 2) = 1$.

If $e = a_na_1,\ gcd(f(a_n), f(a_1)) = gcd(6n - \left(1 + 3(\frac{1+(-1)^n}{2})\right), 5) = 1$.

So, f is an injection and $gcd(f(v), f(u)) = 1$ for every pair of adjacent vertices u and v of BRS_n. Then f admits prime labeling of BRS_n. Hence BRS_n is a prime graph. □

Theorem 2.4. *BRS_n is vertex prime graph.*

Proof. For the graph BRS_n, $V(BRS_n) = \{a_0, a_i, d_i/1 \le i \le n\} \cup \{b_i, c_i/1 \le i \le 2n\}$ and $E(BRS_n) = \{a_0a_i, a_ib_{2i-1}, b_{2i}c_{2i}, c_{2i-1}d_i/1 \le i \le n\} \cup \{b_ib_{i+1}, c_ic_{i+1}/1 \le i \le 2n-1\} \cup \{a_na_1, b_{2n}b_1, c_{2n}c_1, d_nc_1\} \cup \{a_ia_{i+1}, d_ic_{2i+1}/1 \le i \le n-1\}$. Therefore $|V(BRS_n)| = 6n+1$ and $|E(BRS_n)| = 10n$. Define $f : E(BRS_n) \to [10n]$ as follows:

$$f(x) = \begin{cases} 10i-9 & \text{if } x = a_0a_i,\ i \in [n]\,; \\ 10i-7 & \text{if } x = a_ib_{2i-1},\ i \in [n]\,; \\ 10i-4 & \text{if } x = b_{2i}c_{2i},\ i \in [n]\,; \\ 10i & \text{if } x = c_{2i-1}d_i,\ i \in [n]\,; \\ 10i-8 & \text{if } x = a_ia_{i+1},\ i \in [n-1]\,; \\ 10i-1 & \text{if } x = d_ic_{2i+1},\ i \in [n-1]\,; \\ 10i-6 & \text{if } x = b_{2i-1}b_{2i},\ i \in [n]\,; \\ 10i-5 & \text{if } x = b_{2i}b_{2i+1},\ i \in [n-1]\,; \\ 10i-3 & \text{if } x = c_{2i-1}c_{2i},\ i \in [n]\,; \\ 10i-2 & \text{if } x = c_{2i}c_{2i+1},\ i \in [n-1]\,; \\ 10n-8 & \text{if } x = a_na_1; \\ 10n-5 & \text{if } x = b_{2n}b_1; \\ 10n-2 & \text{if } x = c_{2n}c_1; \\ 10n-1 & \text{if } x = d_nc_1. \end{cases}$$

Clearly f is an bijection. Let v be an arbitrary vertex of BRS_n . To prove f is a vertex prime labeling of BRS_n we have the following cases:

If $v = a_0,\ gcd(f(a_0a_1), f(a_0a_2), ..., f(a_0a_n)) = gcd(1, 11, ..., 10n-9) = 1$.

If $v = a_i,\ gcd(f(a_ia_0), f(a_ia_{i+1}), f(a_ib_{2i-1}), f(a_ia_{i-1})) = gcd(10i-9, 10i-8, 10i-7, 10i-18) = 1,\ i \in [n-1] - \{1\}$.

If $v = a_1,\ gcd(f(a_1a_0), f(a_1a_2), f(a_1b_1), f(a_1a_n)) = gcd(1, 2, 3, 10n-8) = 1$.

If $v = a_n,\ gcd(f(a_na_0), f(a_na_1), f(a_nb_{2n-1}), f(a_na_{n-1})) = gcd(10n-9, 10n-8, 10n-7, 10n-18) = 1$.

If $v = b_{2i},\ gcd(f(b_{2i}b_{2i+1}), f(b_{2i}b_{2i-1}), f(b_{2i}c_{2i})) = gcd(10i-5, 10i-6, 10i-4) = 1,\ i \in [n-1]$.

If $v = b_{2n},\ gcd(f(b_{2n}b_1), f(b_{2n}b_{2n-1}), f(b_{2n}c_{2n})) = gcd(10n-5, 10n-6, 10n-4) = 1$.

If $v = b_{2i-1},\ gcd(f(b_{2i-1}b_{2i}), f(b_{2i-1}b_{2i-2}), f(b_{2i-1}a_i)) = gcd(10i-6, 10i-15, 10i-7) = 1,\ i \in [n] - \{1\}$.

If $v = b_1,\ gcd(f(b_1b_2), f(b_1b_{2n}), f(b_1a_1)) = gcd(4, 10n-5, 3) = 1$.

If $v = c_{2i-1},\ gcd(f(c_{2i-1}c_{2i-2}), f(c_{2i-1}c_{2i}), f(c_{2i-1}d_{i-1}), f(c_{2i-1}d_i)) = gcd(10i-12, 10i-3, 10i-11, 10i) = 1,\ i \in [n] - \{1\}$.

If $v = c_1,\ gcd(f(c_1c_2), f(c_1c_{2n}), f(c_1d_1), f(c_1d_n)) = gcd(7, 10n-2, 10, 10n-1) = 1$.

If $v = c_{2i},\ gcd(f(c_{2i}c_{2i-1}), f(c_{2i}v_{2i+1}), f(c_{2i}b_{2i})) = gcd(10i-3, 10i-2, 10i-4) = 1,\ i \in [n-1]$.

If $v = c_{2n},\ gcd(f(c_{2n}c_{2n-1}), f(c_{2n}c_1), f(c_{2n}b_{2n})) = gcd(10n-3, 10n-2, 10n-4) = 1$.

If $v = d_i,\ gcd(f(d_i c_{2i-1}), f(d_i c_{2i+1})) = gcd(10i, 10i-1) = 1,\ i \in [n-1]$.

If $v = d_n,\ gcd(f(d_n c_1), f(d_n c_{2n-1})) = gcd(10n, 10n-1) = 1$.

So, f is an bijection and $\underset{uv\in E}{gcd}\{f(uv)\} = 1$. The edges are labeled such that for any vertex v_i, the g.c.d of all the edges incident with v_i is 1. Then f admits vertex prime labeling of BRS_n. Hence BRS_n is a vertex prime graph. □

Theorem 2.5. *BRS_n is Total prime graph*

Proof. For the graph BRS_n, $V(BRS_n) = \{a_0, a_i, d_i/1 \le i \le n\} \cup \{b_i, c_i/1 \le i \le 2n\}$ and $E(BRS_n) = \{a_0a_i, a_ib_{2i-1}, b_{2i}c_{2i}, c_{2i-1}d_i/1 \le i \le n\} \cup \{b_ib_{i+1}, c_ic_{i+1}/1 \le i \le 2n-1\} \cup \{a_na_1, b_{2n}b_1, c_{2n}c_1, d_nc_1\} \cup \{a_ia_{i+1}, d_ic_{2i+1}/1 \le i \le n-1\}$. Therefore $|V(CS_n)| = 6n+1$ and $|E(CS_n)| = 10n$. Define $f : V(BRS_n) \cup E(BRS_n) \to [16n+1]$ as follows:

$$f(x) = \begin{cases} 1 & \text{if } x = a_0; \\ 6i - \left(1 + 3(\frac{1+(-1)^i}{2})\right) & \text{if } x = a_i,\ i \in [n]; \\ 6i - \left(1 + 3(\frac{1+(-1)^{i+1}}{2})\right) & \text{if } x = b_{2i-1},\ i \in [n]; \\ 6i-3 & \text{if } x = b_{2i}\ i \in [n]; \\ 6i & \text{if } x = d_i\ i \in [n]; \\ 6i-2 & \text{if } x = c_{2i}\ i \in [n]; \\ 6i+1 & \text{if } x = c_{2i-1},\ i \in [n]; \\ 6n+10i-8 & \text{if } x = a_0a_i,\ i \in [n]; \\ 6n+10i-6 & \text{if } x = a_ib_{2i-1},\ i \in [n]; \\ 6n+10i-3 & \text{if } x = b_{2i}c_{2i},\ i \in [n]; \\ 6n+10i+1 & \text{if } x = c_{2i-1}d_i,\ i \in [n]; \\ 6n+10i-7 & \text{if } x = a_ia_{i+1},\ i \in [n-1]; \\ 6n+10i & \text{if } x = d_ic_{2i+1},\ i \in [n-1]; \\ 6n+10i-5 & \text{if } x = b_{2i-1}b_{2i},\ i \in [n]; \\ 6n+10i-4 & \text{if } x = b_{2i}b_{2i+1},\ i \in [n-1]; \\ 6n+10i-2 & \text{if } x = c_{2i-1}c_{2i},\ i \in [n]; \\ 6n+10i-1 & \text{if } x = c_{2i}c_{2i+1},\ i \in [n-1]; \\ 16n-7 & \text{if } x = a_na_1; \\ 16n-4 & \text{if } x = b_{2n}b_1; \\ 16n-1 & \text{if } x = c_{2n}c_1; \\ 16n & \text{if } x = d_nc_1. \end{cases}$$

Clearly f is a bijection. Let e and v be an arbitrary edge and vertex of BRS_n. To prove f is a total prime labeling of BRS_n we have the following cases: Then for any edge,

If $e = a_0a_i,\ gcd(f(a_0), f(a_i)) = gcd(1, 6i - \left(1 + 3(\frac{1+(-1)^i}{2})\right)) = 1,\ i \in [n]$.

If $e = a_ib_{2i-1},\ gcd(f(a_i), f(b_{2i-1})) = gcd(6i - \left(1 + 3(\frac{1+(-1)^i}{2})\right), 6i - \left(1 + 3(\frac{1+(-1)^{i+1}}{2})\right)) = 1,\ i \in [n]$.

If $e = a_ia_{i+1},\ gcd(f(a_i), f(a_{i+1})) = gcd(6i - \left(1 + 3(\frac{1+(-1)^i}{2})\right), 6(i+1) - \left(1 + 3(\frac{1+(-1)^{i+1}}{2})\right)) = 1,\ i \in [n-1]$.

If $e = b_{2i}c_{2i},\ gcd(f(b_{2i}), f(c_{2i})) = gcd(6i-3, 6i-2) = 1,\ i \in [n]$.

If $e = c_{2i-1}d_i,\ gcd(f(c_{2i-1}), f(d_i)) = gcd(6i+1, 6i) = 1,\ i \in [n]$.

If $e = d_ic_{2i+1},\ gcd(f(d_i), f(c_{2i+1})) = gcd(6i, 6i+7) = 1,\ i \in [n-1]$.

If $e = c_{2i-1}c_{2i},\ gcd(f(c_{2i-1}), f(c_{2i})) = gcd(6i+1, 6i-2) = 1,\ i \in [n]$.

If $e = c_{2i}c_{2i+1},\ gcd(f(c_{2i}), f(c_{2i+1})) = gcd(6i-2, 6i+7) = 1,\ i \in [n-1]$.

If $e = b_{2i-1}b_{2i},\ gcd(f(b_{2i-1}), f(b_{2i})) = gcd(6i - \left(1 + 3(\frac{1+(-1)^{i+1}}{2})\right), 6i-3) = 1,\ i \in [n]$.

If $e = b_{2i}b_{2i+1},\ gcd(f(b_{2i}), f(b_{2i+1})) = gcd(6i-3, 6i+2+\left(3(\frac{1+(-1)^{i+1}}{2})\right)) = 1,\ i \in [n-1]$.

If $e = c_{2n}c_1,\ gcd(f(c_{2n}), f(c_1)) = gcd(6n-2, 7) = 1$.

If $e = d_nc_1,\ gcd(f(d_n), f(c_1)) = gcd(6n, 7) = 1$.

If $e = b_{2n}b_1,\ gcd(f(b_{2n}), f(b_1)) = gcd(6n-3, 2) = 1$.

If $e = a_na_1,\ gcd(f(a_n), f(a_1)) = gcd(6n - \left(1 + 3(\frac{1+(-1)^n}{2})\right), 5) = 1$.

Then for any vertex,

If $v = a_0,\ gcd(f(a_0a_1), f(a_0a_2), ..., f(a_0a_n)) = gcd(6n+2, 6n+12, ..., 16n-9) = 1$.

If $v = a_i,\ gcd(f(a_ia_0), f(a_ia_{i+1}), f(a_ib_{2i-1}), f(a_iv_{i-1})) = gcd(6n+10i-8, 6n+10i-17, 6n+10i-6, 6n+10i-7) = 1,\ i \in [n-1] - \{1\}$.

If $v = a_1,\ gcd(f(a_1a_0), f(a_1a_2), f(a_1b_1), f(a_1a_n)) = gcd(6n+2, 6n+3, 6n+4, 6n+10n-7) = 1$.

If $v = a_n,\ gcd(f(a_na_0), f(a_na_1), f(a_nb_{2n-1}), f(a_na_{n-1})) = gcd(16n-8, 16n-7, 16n-6, 16n-17) = 1$.

If $v = b_{2i},\ gcd(f(b_{2i}b_{2i+1}), f(b_{2i}b_{2i-1}), f(b_{2i}c_{2i})) = gcd(6n+10i-5, 6n+10i-4, 6n+10i-3) = 1,\ i \in [n-1]$.

If $v = b_{2n},\ gcd(f(b_{2n}b_1), f(b_{2n}b_{2n-1}), f(b_{2n}c_{2n})) = gcd(16n-5, 16n-4, 16n-3) = 1$.

If $v = b_{2i-1},\ gcd(f(b_{2i-1}b_{2i}), f(b_{2i-1}b_{2i-2}), f(b_{2i-1}a_i)) = gcd(6n+10i-5, 6n+10i-14, 6n+10i-6) = 1,\ i \in [n] - \{1\}$.

If $v = b_1,\ gcd(f(b_1b_2), f(b_1b_{2n}), f(b_1a_1)) = gcd(6n+5, 16n-4, 6n+4) = 1$.

If $v = c_{2i-1},\ gcd(f(c_{2i-1}c_{2i-2}), f(c_{2i-1}c_{2i}), f(c_{2i-1}d_{i-1}), f(c_{2i-1}d_i)) = gcd(6n+10i-11, 6n+10i-2, 6n+10i-10, 6n+10i+1) = 1,\ i \in [n] - \{1\}$.

If $v = c_1,\ gcd(f(c_1c_2), f(c_1c_{2n}), f(c_1d_1), f(c_1d_n)) = gcd(6n+8, 16n-1, 6n+11, 16n) = 1$.

If $v = c_{2i},\ gcd(f(c_{2i}c_{2i-1}), f(c_{2i}v_{2i+1}), f(c_{2i}b_{2i})) = gcd(6n+10i-2, 6n+10i-1, 6n+10i-3) = 1,\ i \in [n-1]$.

If $v = c_{2n},\ gcd(f(c_{2n}c_{2n-1}), f(c_{2n}c_1), f(c_{2n}b_{2n})) = gcd(16n-2, 16n-1, 16n-3) = 1$.

If $v = d_i,\ gcd(f(d_ic_{2i-1}), f(d_ic_{2i+1})) = gcd(6n+10i+1, 6n+10i) = 1,\ i \in [n-1]$.

If $v = d_n,\ gcd(f(d_nc_1), f(d_nc_{2n-1})) = gcd(16n+1, 16n) = 1$.

So, f is a bijection. According to this pattern, the vertices are labeled such that for any edge $e = uv \in BRS_n$, $\gcd(f(u), f(v)) = 1$. Also the edges are labeled such that for any vertex v_i, the g.c.d of all the edges incident with v_i is 1. So, f admits total labeling of BRS_n. Hence BRS_n is total prime graph. □

Theorem 2.6. *BRS_n is difference cordial graph.*

Proof. For the graph BRS_n, $V(BRS_n) = \{a_0, a_i, d_i/1 \leq i \leq n\} \cup \{b_i, c_i/1 \leq i \leq 2n\}$ and $E(BRS_n) = \{a_0a_i, a_ib_{2i-1}, b_{2i}c_{2i}, c_{2i-1}d_i/1 \leq i \leq n\} \cup \{b_ib_{i+1}, c_ic_{i+1}/1 \leq i \leq 2n-1\} \cup \{a_na_1, b_{2n}b_1, c_{2n}c_1, d_nc_1\} \cup \{a_ia_{i+1}, d_ic_{2i+1}/1 \leq i \leq n-1\}$. Therefore $|V(BRS_n)| = 6n+1$ and $|E(BRS_n)| = 10n$. Define $f : V(BRS_n) \to [6n+1]$ as follows:

$$f(x) = \begin{cases} 1 & \text{if } x = a_0; \\ 6i+1 & \text{if } x = a_i,\ i \in [n]; \\ 6i & \text{if } x = b_{2i-1},\ i \in [n]; \\ 6i-1 & \text{if } x = b_{2i},\ i \in [n]; \\ 6i-2 & \text{if } x = c_{2i},\ i \in [n]; \\ 6i-3 & \text{if } x = c_{2i-1},\ i \in [n]; \\ 6i-4 & \text{if } x = d_i,\ i \in [n]. \end{cases}$$

The induced edge labeling $f^*: E(BRS_n) \to \{0,1\}$ is $f^*(uv) = |f(u) - f(v)|$, for every edge $e = uv \in E$. Therefore

$$f^*(e) = \begin{cases} 0 & \text{if } e \in \{a_i a_{i+1}, b_{2i}b_{2i+1}, c_{2i}c_{2i+1}, c_{2i+1}d_i\},\ i \in [n-1]\,; \\ 0 & \text{if } e = a_0 a_i,\ i \in [n]\,; \\ 1 & \text{if } e \in \{a_i b_{2i-1}, b_{2i-1}b_{2i}, b_{2i}c_{2i}, c_{2i-1}c_{2i}, c_{2i-1}d_i\},\ i \in [n]\,; \\ 0 & \text{if } e \in \{a_n a_1, b_{2n}b_1, c_{2n}c_1, c_1 d_n\}. \end{cases}$$

Since $e_f(0) = e_f(1) = 5n$. Therefore f satisfies the conditions $|e_f(1) - e_f(0)| \leq 1$ for difference cordial labeing. So, f admits difference cordial labeling of BRS_n. Hence BRS_n is difference cordial. $\square$

3. Conclusion

we have derived six new results by investigating some labeling techniques in Braided star graph. More exploration is possible for other graph families and in the context of different graph labeling problems.

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