International Journal of Mathematics And its Applications
Volume 5, Issue 4–D (2017), 457–460.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

A Note on L-cordial Labeling of Graphs

Research Article

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Abstract: We discuss L-cordial labeling of some families of graph.We show that Book graph $\theta(C_4, n)$, Book graph $\theta(C_5, n)$, One point union of C_3 i.e., $(C_3)^n$, a double triangular snake $2S(C_3, n)$ are L-cordial.

MSC: 05C78.

Keywords: L-cordial labeling, double snake, Book graph, union.

1. Introduction and Preliminaries

Prof.Cahit was the first person to use the word cordial [3]. In any cordial labeling of graph the values assigned to vertex are restricted to 0 and 1. Even in edge cordial labeling the similar observation is followed. After [3] a number of papers on graph cordial labeling are published. We have explained L-Cordial labeling in [1]. A graph whose L-cordial labelling is available is called as L-cordial. Not much work has been done in this sort of labeling. We show that Book graph $\theta(C_4, n)$, Book graph $\theta(C_5, n)$, One point union of C_3 i.e., $(C_3)^n$ a double triangular snake $2S(C_3, n)$ are L-cordial.

Definition 1.1 (Fusion of vertex). *Let $v \in V(G_1)$, $v' \in V(G_2)$ where G_1 and G_2 are two graphs. We fuse v and v' by replacing them with a single vertex say w and all edges incident with v in G_1 and that with v' in G_2 are incident with u in the new graph $G = G_1FG_2$.*

$$Deg\ Gu = deg\ G_1(v) + deg\ G_2(v')\ \ and\ \ |V(G)| = |V(G_1)| + |V(G_2)| - 1,\ |E(G)| = |E(G_1)| + |E(G_2)|$$

Definition 1.2. *Book graph $\theta(G, n)$ is having n copies n of graph with a common edge. The common edge is same and fixed one in all copies of G. It has $1 + 3n$ edges and $2n + 2$ vertics. Let the fixed edge be $e = (uv)$.*

Definition 1.3. *One point union of G i.e $(G)^n$ At fixed point on G_n copies of G are fused. It has $n|V(G)| - n + 1$ vertices and $n|E(G)|$ edges.*

Definition 1.4. *Double snake on C_3 i.e., $2S(C_3, n)$ a double triangular snake consists of two triangular snakes that have a common path. That is, a double triangular snake is obtained from a path $v_1, v_2, \ldots, v_n, v_{n+1}$ by joining v_i and v_{i+1} to a new vertex w_i for $i = 1, 2, \ldots, n-1$ and to a new vertex u_i for $i = 1, 2, \ldots, n$.*

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2. Main Results

Theorem 2.1. *Book graph $\theta(C_4, n)$ is L-cordial.*

Proof. There are n copies of $G = \theta(C_4, n)$ with fixed edge $e = (uv)$ common to all pages. C^i be the i^{th} page of book and is given by $(u, e_1^i, w_1^i, e_2^i, w_2^i, e_3^i, v)$ where $i = 1, 2, \ldots, n$. We define a function $f : E(G) \to \{1, 2, 3, \ldots, n\}$ a bijective function as follows:

$$f(e) = 1; \;\; f(e_1^i) = 3i; \;\; f(e_2^i) = 3(i-1) + 2; \;\; f(e_3^i) = 3i + 1; \;\; i = 1, 2, \ldots, n.$$

It follows that Every time we go on adding a page to book the label of u and v alternates. For a book with n pages we have $v_f(1) = v_f(0) = n + 1$. Clearly f is L-cordial. □

Theorem 2.2. *Book graph $\theta(C_5, n)$ is L-cordial.*

Proof. The common edge $e = (uv)$ and i^{th} page of the book is given by $(u, e_1^i, w_1^i, e_2^i, w_2^i, e_3^i, w_3^i, e_4^i, v)$.

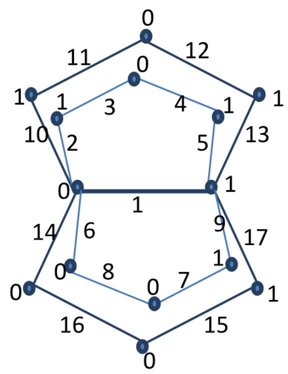


Figure 1: Book graph $\theta(C_5, 4)$. The vertex labels are very close to vertex, the other numbers are edge labels

Define a function $f : E(G) \to \{1, 2, 3, \ldots, |E|\}$ given by

$$f(e_1^i) = 4(i-1) + 2, \; f(e_2^i) = 4(i-1) + 3, \; f(e_3^i) = 4(i-1) + 4, \; f(e_4^i) = 4(i-1) + 5 \quad \text{for i = 1,3,5,7,...}$$

$$f(e_1^i) = 4(i-1) + 2, \; f(e_2^i) = 4(i-1) + 4, \; f(e_3^i) = 4(i-1) + 3, \; f(e_4^i) = 4(i-1) + 5 \quad \text{for i= 2,4,6,8,...}$$

$v_f(0) = \frac{(3n+2)}{2} = v_f(1)$ for n is even $v_f(0) = (3n + 1)$, $v_f(1) = v_f(0) + 1$ for n is odd. □

Theorem 2.3. *One point union of C_3 i.e., $(C_3)^n$ is L-cordial.*

Proof. The i^{th} copy on $(C_3)^n$ be given by $C^i = (v, e_1^i, u_1, e_2^i, u_2, e_3^i, v)$. Define a function $f : E(G) \to \{1, 2, 3, \ldots, q-1\}$ given by $f(e_j^i) = 3(i-1) + j$ for $j = 1, 2, 3, \ldots$ and for all i.

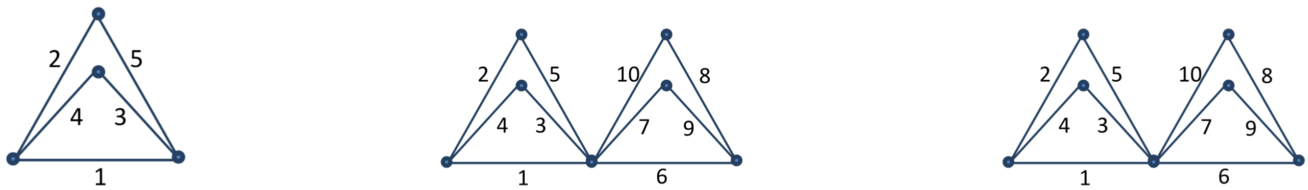
$$v_f(0) = n, \;\; v_f(1) = n + 1 \;\text{ for odd n}$$

$$v_f(0) = n + 1, \; v_f(1) = n \;\; \text{ for even n}$$

□

Theorem 2.4. *A double triangular snake $2S(C_3, n)$ is L-cordial.*

Proof. To obtain a double snake on C_3 i.e $DS(C_3, n)$ we start with a path



(a) $DS(C_3, 1)$ with edge labels

(b) $DS(C_3, 2)$ Edge label numbers are shown

(c) $DS(C_3, 3)$ edge numbers are shown

Figure 2:

$P_{n+1} = (v_1, e_1, v_2, e_2, \ldots, e_n, v_{n+1})$. Between every two vertices v_i and v_{i+1} of P_{n+1} two vertices w_i and u_i are taken. Each w_i and u_i are joined to v_i and v_{i+1} giving edge p_i, p_{i+1} and q_i, q_{i+1} respectively. Define a function $f : E(G) \rightarrow \{1, 2, 3, \ldots, |E|\}$ as follows:

For $n = 1$, $n = 2$ we have shown the labelling in Figure 2 (a), (b) and (c) above. For the snakes of length greater than 3 we use the labelling above for first three blocks.

For $i \equiv 0, 2\ (mod\ 4)$ we have,

$$f(p_j^i) = 20 + (i-4)5 \text{ for } j = 1$$
$$f(p_j^i) = 18 + (i-4)5 \text{ for } j = 2$$
$$f(q_j^i) = 17 + (i-4)5 \text{ for } j = 1$$
$$f(p_j^i) = 19 + (i-4)5 \text{ for } j = 1$$
$$f(e_i) = 16 + (i-4)5 \text{ for } j = 1$$

For $i \equiv 1, 3\ (mod\ 4)$ we get,

$$f(p_j^i) = 25 + (i-5)5 \text{ for } j = 1$$
$$f(p_j^i) = 23 + (i-5)5 \text{ for } j = 2$$
$$f(q_j^i) = 24 + (i-5)5 \text{ for } j = 1$$
$$f(p_j^i) = 22 + (i-5)5 \text{ for } j = 2$$
$$f(e_i) = 21 + (i-5)5$$

This produces vertex numbers as follows:

For $n = 4$ we have $v_f(0) = 7$ and $v_f(1) = 6$

For $n = 5$ we have $v_f(0) = 8$ and $v_f(1) = 8$.

$$v_f(0) = 7 + \left(\frac{(n-6)}{2} + 1\right)3,\ v_f(1) = v_f(0) - 1 \text{ for } n = 2x,\ x = 3, 4, 5, \ldots$$
$$v_f(0) = 7 + \left(\frac{(n-6)}{2} + 1\right)3 + 1 = v_f(1) \text{ for } n = 2x+1,\ x = 3, 4, 5, \ldots$$

It follows that f is L-cordial function. □

References

[1] Mukund V.Bapat, *Some vertex Prime Graphs and A New Type of Graph Labeling*, IJMTT, 47(2017), 23-29.

[2] Mukund V.Bapat and N.B.Limaye, *Some families of E3-cordial graphs*, Proceedings of the National conference on Graphs, Combinatorics, Algorithm & application at Anandnagar, Krishnankoli, (2004).

[3] I.Cahit, *Cordial graphs, A weaker version of graceful and harmonious graphs*, Ars combinatoria, 23(1987), 201-207.

[4] I.Cahit and R.Yilmaz, *E3-cordial graphs*, Ars Combinatoria, 54(2000), 119-127.

[5] T.Deretsky, S.M.Lee and J.Mitchem, *On vertex prime labelings of graphs, in Graph Theory*, Combinatorics and Applications, Vol. 1, J.Alavi, G.Chartrand, O.Oellerman and A.Schwenk, eds., Proceedings 6th International Conference Theory and Applications of Graphs (Wiley, New York, 1991), 359-369.

[6] J.A.Gallian, *A dynamic survey of graph labellings*, Electronic Journal of Combinatorics, 7(2015), #DS6.

[7] F.Harary, *Graph Theory*, Narosa Publishing House, New Delhi.