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Ricci Solitons on (ε, δ) Trans-Sasakian Manifolds

Research Article

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Abstract: The object of this paper is to study the characterisation of Ricci Solitons in generalised ricci recurrent and ϕ recurrent (ε, δ) trans-sasakian manifolds based on the 1-form.

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Keywords: Ricci, ϕ-recurrent, pseudo-projective, (ε, δ) Trans sasakian, Ricci recurrent, shrinking, expanding, steady.

1. Introduction

In the year 1982, Hamilton [11] introduced solution to the Ricci flow known as Ricci Soliton. In Contact Riemannian geometry RameshSharma [10] initiated the study of Ricci Solitons. Later many authors [5, 7, 9] extensively studied Ricci Solitons in contact metric manifolds. Ricci Soliton in a Riemannian manifold (M, g) is a natural generalisation of an Einstein metric and is defined as a triple (g, V, λ) with g a Riemannian metric, V a vector field, λ a real scalar such that

$$(L_V g)(X, Y) + 2S(X, Y) + 2\lambda g(X, Y) = 0, \tag{1}$$

where L_v represents the Lie derivative operator along the vector field V and S is the Ricci tensor of M. Depending on the λ value, whether it is negative, zero, and positive, the Ricci Soliton will be shrinking, steady, and expanding respectively. In 1985, Obulina [7] introduced a new class of contact manifold namely Trans-Sasakian manifold. Many authors [3, 4, 6] have studied this manifold and obtained many interesting results. In this paper, we study the conditions which characterise Ricci Solitons in Trans-sasakian manifolds. Section 2 contains a review of Trans-sasakian manifolds and Ricci solitons. In section 3-6 we proove characterizing conditions for ricci Solitons in generalised Ricci recurrent, generalised ϕ -recurrent, generalised pseudo-projective ϕ -recurrent, generalised concircular ϕ -recurrent (ε, δ) Trans-sasakian Manifolds.

2. Preliminaries

A manifold M of dimension n is an almost contact manifold if it admits an almost contact metric structure (ϕ, ξ, η, g) consisting of a vector field ξ, a 1-form η, a tensor field ϕ of type (1,1) and a Riemannian metric g compatible with (ϕ, ξ, η)

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satisfying

$$\phi^2 X = -X + \eta(X)\xi, \tag{2}$$

$$\eta(\xi) = 1, \ \phi\xi = 0, \ \eta \circ \phi = 0. \tag{3}$$

An almost contact metric manifold M is called an (ε)-almost contact metric manifold if

$$g(\xi, \xi) = \varepsilon, \tag{4}$$

$$\eta(X) = \varepsilon g(X, \xi), \tag{5}$$

$$g(\phi X, \phi Y) = g(X, Y) - \varepsilon\eta(X)\eta(Y), \tag{6}$$

for all vector fields X, Y on M, where $\varepsilon = g(\xi, \xi) = \pm 1$. An (ε)-almost contact metric manifold is said to be (ε, δ)-Trans-sasakian manifold if

$$(\nabla_X \phi)(Y) = \alpha[g(X, Y)\xi - \varepsilon\eta(Y)X] + \beta[g(\phi X, Y)\xi - \delta\eta(Y)\phi X], \tag{7}$$

holds for some smooth functions α and β on M and $\varepsilon = \pm 1$, $\delta = \pm 1$. For $\beta = 0$, $\alpha = 1$ (ε, δ)-trans sasakian manifold reduces to (ε)-sasakian and for $\alpha = 0$, $\beta = 1$ it reduces to a (δ)-kenmotsu manifold. In an (ε)-almost contact metric manifold M is an (ε, δ)-Trans-sasakian manifold if and only if [6]

$$\nabla_X \xi = -\varepsilon\alpha\phi X - \delta\beta\phi^2 X, \tag{8}$$

where ∇ denotes the Riemannian connection of g. In an (ε, δ)-Trans-sasakian manifold M, the following relation holds with (α, β) are constants

$$(\nabla_X \phi)(Y) = \varepsilon g(\phi(\nabla_X \xi), Y)\xi - \eta(Y)\phi(\nabla_X \xi), \tag{9}$$

$$(\nabla_X \eta)Y = \delta\beta[\varepsilon g(X, Y) - \eta(X)\eta(Y)] - \alpha g(\phi X, Y), \tag{10}$$

$$R(X, Y)\xi = (\beta^2 - \alpha^2)[\eta(X)Y - \eta(Y)X], \tag{11}$$

where R is the Riemannian Curvature tensor

$$S(X, \xi) = (n-1)(\varepsilon\alpha^2 - \beta^2\delta)\eta(X), \tag{12}$$

Let (g, V, λ) be a Ricci Soliton in Trans-sasakian manifold M. Put $V = \xi$ then from (8) and (1) we get

$$S(X, Y) = \beta\delta\varepsilon\eta(X)\eta(Y) - g(X, Y)(\beta\delta + \lambda). \tag{13}$$

The above equation gives

$$QX = \beta\delta\varepsilon\eta(X)\xi - (\beta\delta + \lambda)X, \tag{14}$$

$$S(X, \xi) = -\varepsilon\lambda\eta(X), \tag{15}$$

$$r = -[(n - \varepsilon)\beta\delta + n\lambda], \tag{16}$$

Also by the covariant derivative definition, we have

$$(\nabla_W S)S(Y, \xi) = \nabla_W S(Y, \xi) - S(\nabla_W Y, \xi) - S(Y, \nabla_W \xi), \tag{17}$$

The following results will be used later.

Lemma 2.1. *In a ϕ-recurrent (ε,δ)-Trans-sasakian manifold (M^n,g) the characteristic vector field ξ and the vector fields ρ_1, ρ_2 associated with two 1-forms A and B are co-directional and the 1-forms A and B are defined as follows*

$$A(W)=\eta(\rho_1)\eta(W), B(W)=\eta(\rho_2)\eta(W), \tag{18}$$

Replacing W by ξ in (18) it follows that

$$A(\xi)=\eta(\rho_1), B(\xi)=\eta(\rho_2), \tag{19}$$

3. Generalised Ricci-Recurrent (ε,δ)-Trans-Sasakian Manifold

A trans-sasakian manifold is said to be generalised Ricci-recurrent manifold if there exist two non-zero 1-forms A and B such that

$$(\nabla_W S)(Y,Z)=A(W)S(Y,Z)+(n-1)B(W)g(Y,Z), \tag{20}$$

Replacing Z by ξ in (20) and using (12), We have

$$(\nabla_W S)(Y,\xi)=(n-1)(\varepsilon\alpha^2-\beta^2\delta)A(W)\eta(Y)+\varepsilon(n-1)B(W)\eta(Y), \tag{21}$$

Using (8) and (12) we get

$$(\nabla_W S)(Y,\xi)=\beta\delta\varepsilon(n-1)(\varepsilon\alpha^2-\beta^2\delta)g(W,Y)-\alpha(n-1)(\varepsilon\alpha^2-\beta^2\delta)g(\phi W,Y)+\varepsilon\alpha S(Y,\phi W)-\beta\delta S(Y,W), \tag{22}$$

On comparing (21) with (22) we have

$$\begin{aligned}\beta\delta S(Y,W)=&\beta\delta\varepsilon(n-1)(\varepsilon\alpha^2-\beta^2\delta)g(W,Y)-\alpha(n-1)(\varepsilon\alpha^2-\beta^2\delta)g(\phi W,Y)\\&+\varepsilon\alpha S(Y,\phi W)-(n-1)(\varepsilon\alpha^2-\beta^2\delta)A(W)\eta(Y)-\varepsilon(n-1)B(W)\eta(Y),\end{aligned} \tag{23}$$

Substituting $Y=\xi$ in (23) we get,

$$S(\xi,W)=(n-1)(\varepsilon\alpha^2-\beta^2\delta)\eta(W)-\frac{(n-1)(\varepsilon\alpha^2-\beta^2\delta)}{\beta\delta}A(W)-\frac{\varepsilon}{\beta\delta}B(W)(n-1), \tag{24}$$

Using Lemma (18), (24) reduces to

$$S(\xi,W)=(n-1)\eta(W)[(\varepsilon\alpha^2-\beta^2\delta)-\frac{(\varepsilon\alpha^2-\beta^2\delta)}{\beta\delta}\eta(\rho_1)-\frac{\varepsilon}{\beta\delta}\eta(\rho_2)], \tag{25}$$

Substitute (15) and (19) in (25), we obtain

$$\lambda=-\frac{(n-1)(\varepsilon\alpha^2-\beta^2\delta)}{\varepsilon}[1-\frac{1}{\beta\delta}A(\xi)]+\frac{(n-1)}{\beta\delta}B(\xi). \tag{26}$$

Theorem 3.1. *Ricci Soliton in Generalised Ricci-Recurrent (ε,δ)-Trans-sasakian manifold (M,g) with two 1-forms A and B is*

$$\begin{cases}\textit{Expanding}, & \textit{if } A(\xi)>1, B(\xi)>1.\\ \textit{Steady}, & \textit{if } (\varepsilon\alpha^2-\beta^2\delta)=\dfrac{\varepsilon B(\xi)}{(\beta\delta-A(\xi))}.\\ \textit{Shrinking}, & \textit{if } A(\xi)<1, B(\xi)<1.\end{cases}$$

4. Generalised ϕ-Recurrent (ε, δ)-Trans-Sasakian Manifold

A (ε, δ)-Trans-sasakian Manifold is said to be generalised ϕ-Recurrent manifold [2] if its curvature tensor R satisfies the condition

$$\phi^2((\nabla_W R)(X,Y)Z) = A(W)R(X,Y)Z + B(W)[g(Y,Z)X - g(X,Z)Y], \tag{27}$$

for arbitrary vector fields X, Y, Z, W. Let us consider a generalised ϕ-Recurrent Trans-sasakian Manifold. By virtue of (2) and (27) we have,

$$-((\nabla_W R)(X,Y)Z) + \eta((\nabla_W R)(X,Y)Z)\xi = A(W)R(X,Y)Z + B(W)[g(Y,Z)X - g(X,Z)Y], \tag{28}$$

Contracting (28) with respect to U, we obtain

$$-g((\nabla_W R)(X,Y)Z,U)+\eta((\nabla_W R)(X,Y)Z)\varepsilon\eta(U) = A(W)g(R(X,Y)Z,U)+B(W)[g(Y,Z)g(X,U)-g(X,Z)g(Y,U)], \tag{29}$$

Let $e_i (i = 1, 2, 3, \dots, n)$, be an orthonormal basis of the tangent space at any point of the manifold. Put $X = U = e_i$ in (29) and take summation over i, $1 \le i \le n$, we get

$$-(\nabla_W S)(Y,Z) = A(W)S(Y,Z) + B(W)g(Y,Z)(n-1), \tag{30}$$

Replacing Z by ξ in (30) and using (12) we have

$$(\nabla_W S)(Y,\xi) = -(n-1)\eta(Y)[(\varepsilon\alpha^2 - \beta^2\delta)A(W) + \varepsilon B(W)], \tag{31}$$

Substitute (8), (12) in (17), we get

$$(\nabla_W S)(Y,\xi) = \beta\delta\varepsilon(n-1)(\varepsilon\alpha^2 - \beta^2\delta)g(W,Y) - \alpha(n-1)(\varepsilon\alpha^2 - \beta^2\delta)g(\phi W,Y) + \varepsilon\alpha S(Y,\phi W) - \beta\delta S(Y,W), \tag{32}$$

On comparison of (31) with (32), we have

$$\begin{aligned}\beta\delta S(Y,W) = {} & \beta\delta\varepsilon(n-1)(\varepsilon\alpha^2 - \beta^2\delta)g(W,Y) - \alpha(n-1)(\varepsilon\alpha^2 - \beta^2\delta)g(\phi W,Y) \\ & + \varepsilon\alpha S(Y,\phi W) + (n-1)\eta(Y)[(\varepsilon\alpha^2 - \beta^2\delta)A(W) + \varepsilon B(W)],\end{aligned} \tag{33}$$

Put $Y = \xi$ in (33), we get

$$S(\xi,W) = (n-1)(\varepsilon\alpha^2 - \beta^2\delta)\eta(W) + \frac{(n-1)}{\beta\delta}[(\varepsilon\alpha^2 - \beta^2\delta)A(W) + \varepsilon B(W)], \tag{34}$$

Applying Lemma (18), (34) reduces to

$$S(\xi,W) = (n-1)\eta(w)[(\varepsilon\alpha^2 - \beta^2\delta) + \frac{1}{\beta\delta}(\varepsilon\alpha^2 - \beta^2\delta)\eta(\rho_1) + \varepsilon\eta(\rho_2)], \tag{35}$$

Make use of (15) and (19) in (35), we obtain

$$\lambda = -\frac{(n-1)}{\varepsilon}(\varepsilon\alpha^2 - \beta^2\delta)[1 + \frac{1}{\beta\delta}A(\xi)] - \frac{(n-1)}{\beta\delta}B(\xi). \tag{36}$$

Theorem 4.1. *Ricci Soliton in Generalised ϕ -Recurrent (ε, δ)-Trans-sasakian manifold (M, g) with two 1-forms A and B is*

$$\begin{cases} \textit{Expanding} & \textit{if, } A(\xi) < 1, B(\xi) < 1\ . \\ \textit{Steady} & \textit{if, } (\varepsilon\alpha^2 - \beta^2\delta) = \dfrac{-\varepsilon B(\xi)}{(\beta\delta + A(\xi))}. \\ \textit{Shrinking} & \textit{if, } A(\xi) > 1, B(\xi) > 1. \end{cases}$$

5. Generalised Pseudo-Projective ϕ-Recurrent (ε, δ)-Trans-Sasakian Manifold

In a (ε, δ)-Trans-sasakian Manifold M, the Pseudo- Projective curvature tensor $\overline{P}$ is given by [1]

$$\overline{P}(X,Y)Z = aR(X,Y)Z + b[S(Y,Z)X - S(X,Z)Y] - \frac{r}{n}\left(\frac{a}{n-1} + b\right)[g(Y,Z)X - g(X,Z)Y], \tag{37}$$

where a and b are constants such that $a, b \neq 0$. A (ε, δ)-Trans-sasakian manifold is said to be Pseudo-Projective ϕ-Recurrent manifold if there exists two non-zero 1-forms A and B such that

$$\phi^2((\nabla_W \overline{P})(X,Y)Z) = A(W)\overline{P}(X,Y)Z + B(W)[g(Y,Z)X - g(X,Z)Y], \tag{38}$$

for arbitrary vector fields X, Y, Z, W. Let us consider a Pseudo-Projective ϕ-Recurrent (ε, δ)-Trans-sasakian Manifold. By virtue of (2) and (38), we have

$$-((\nabla_W \overline{P})(X,Y)Z) + \eta((\nabla_W \overline{P})(X,Y)Z)\xi = A(W)\overline{P}(X,Y)Z + B(W)[g(Y,Z)X - g(X,Z)Y], \tag{39}$$

Contracting (39) with respect to U, we obtain

$$-g((\nabla_W \overline{P})(X,Y)Z,U) + \eta((\nabla_W \overline{P})(X,Y)Z)\varepsilon\eta(U) = A(W)g(\overline{P}(X,Y)Z,U) + B(W)[g(Y,Z)g(X,U) - g(X,Z)g(Y,U)], \tag{40}$$

Let $e_i(i = 1, 2, 3, ...,n)$, be an orthonormal basis of the tangent space at any point of the manifold. Put $X = U = e_i$ in (40) and take summation over i, $1 \leq i \leq n$, we get

$$-(\nabla_W S)(Y,Z) = A(W)[S(Y,Z) - \frac{r}{n}g(Y,Z)] + \frac{(n-1)}{a + b(n-1)}B(W)g(Y,Z), \tag{41}$$

Replacing Z by ξ in(41) and using (2) and (12), we have

$$(\nabla_W S)(Y,\xi) = \left[A(W)\left(\frac{\varepsilon r}{n} - (n-1)(\varepsilon\alpha^2 - \beta^2\delta)\right) - B(W)\frac{(n-1)\varepsilon}{a + b(n-1)}\right]\eta(Y), \tag{42}$$

Substitute (8), (12) in (17), we get

$$(\nabla_W S)(Y,\xi) = \beta\delta\varepsilon(n-1)(\varepsilon\alpha^2 - \beta^2\delta)g(W,Y) - \alpha(n-1)(\varepsilon\alpha^2 - \beta^2\delta)g(\phi W,Y) + \varepsilon\alpha S(Y,\phi W) - \beta\delta S(Y,W), \tag{43}$$

On comparing (42) with (43) we have

$$\begin{aligned} \beta\delta S(Y,W) &= \beta\delta\varepsilon(n-1)(\varepsilon\alpha^2 - \beta^2\delta)g(W,Y) - \alpha(n-1)(\varepsilon\alpha^2 - \beta^2\delta)g(\phi W,Y) \\ &+ \varepsilon\alpha S(Y,\phi W) + A(W)\left[\left(-\frac{\varepsilon r}{n} + (n-1)\right)(\varepsilon\alpha^2 - \beta^2\delta)\right]\eta(Y) + B(W)\left[\frac{(n-1)\varepsilon}{a + b(n-1)}\right]\eta(Y), \end{aligned} \tag{44}$$

Put $Y = \xi$ in (44), we get

$$S(\xi,W) = (n-1)(\varepsilon\alpha^2 - \beta^2\delta)\eta(W) + \frac{A(W)}{\beta\delta}\left[(n-1)(\varepsilon\alpha^2 - \beta^2\delta) - \frac{\varepsilon r}{n}\right] + \frac{B(W)}{\beta\delta}\left[\frac{(n-1)\varepsilon}{a + b(n-1)}\right], \tag{45}$$

Applying Lemma (18), (45) reduces to

$$S(\xi,W) = (n-1)(\varepsilon\alpha^2 - \beta^2\delta)\eta(W) + \frac{\eta(\rho_1)\eta(w)}{\beta\delta}\left[(n-1)(\varepsilon\alpha^2 - \beta^2\delta) - \frac{\varepsilon r}{n}\right] + \frac{\eta(\rho_2)\eta(w)}{\beta\delta}\left[\frac{(n-1)\varepsilon}{a + b(n-1)}\right], \tag{46}$$

Make use of (15), (16) and (19) in (46), we obtain

$$\lambda = -\frac{(n-1)(\varepsilon\alpha^2 - \beta^2\delta)}{\varepsilon} - \left[\frac{(n-\varepsilon)}{n}\frac{\beta\delta A(\xi)}{(\beta\delta + A(\xi)} + \frac{B(\xi)}{(\beta\delta + A(\xi))}\frac{(n-1)}{(a + b(n-1))}\right]. \tag{47}$$

Theorem 5.1. *Ricci Soliton in a Generalised Pseudo-Projective ϕ-Recurrent (ε, δ)-Trans-sasakian manifold (M, g) with two 1-forms A and B is shrinking provided $A(\xi)$ and $B(\xi)$ is non-negative.*

6. Generalised Concircular ϕ-Recurrent (ε, δ)-Trans-sasakian Manifold

The concircular curvature tensor of (M, g) is given by [8]

$$\overline{C}(X, Y)Z = R(X, Y)Z - \frac{r}{n(n-1)}[g(Y, Z)X - g(X, Z)Y]. \tag{48}$$

A (ε, δ)-Trans-sasakian manifold is said to be Concircular ϕ-Recurrent manifold if there exists two non-zero 1-forms A and B such that

$$\phi^2((\nabla_W \overline{C})(X, Y)Z) = A(W)\overline{C}(X, Y)Z + B(W)[g(Y, Z)X - g(X, Z)Y], \tag{49}$$

for arbitrary vector fields X, Y, Z, W. Let us consider ϕ-Recurrent (ε, δ)-Trans-sasakian Manifold. By virtue of (2) and (49), we have

$$-(\nabla_W \overline{C})(X, Y)Z + \eta((\nabla_W \overline{C})(X, Y)Z)\xi = A(W)\overline{C}(X, Y)Z + B(W)[g(Y, Z)X - g(X, Z)Y], \tag{50}$$

Contracting (50) with respect to U, we have

$$-g((\nabla_W \overline{C})(X, Y)Z, U) + \eta((\nabla_W \overline{C})(X, Y)Z)\varepsilon\eta(U) = A(W)g(\overline{C}(X, Y)Z, U) + B(W)[g(Y, Z)g(X, U) - g(X, Z)g(Y, U)], \tag{51}$$

Let $e_i (i = 1, 2, 3, ..., n)$, be an orthonormal basis of the tangent space at any point of the manifold. Then put $X = U = e_i$ in (51) and take summation over i, $1 \leq i \leq n$, we get

$$(\nabla_W S)(Y, Z) = \frac{\nabla_W r}{n} g(Y, Z) - A(W)S(Y, Z) + g(Y, Z)\left[\frac{A(W)r}{n} - B(W)(n-1)\right], \tag{52}$$

Replacing Z by ξ in (52) and using (2) and (12), we have

$$(\nabla_W S)(Y, \xi) = \frac{\nabla_W r}{n} \varepsilon\eta(Y) - A(W)(n-1)(\varepsilon\alpha^2 - \beta^2\delta)\eta(Y) + \varepsilon\eta(Y)\left[\frac{A(W)r}{n} - B(W)(n-1)\right], \tag{53}$$

For a constant r (53) reduces to

$$(\nabla_W S)(Y, \xi) = -A(W)(n-1)(\varepsilon\alpha^2 - \beta^2\delta)\eta(Y) + \varepsilon\eta(Y)\left[\frac{A(W)r}{n} - B(W)(n-1)\right], \tag{54}$$

Substitute (8) and (12) in (17) we get

$$(\nabla_W S)(Y, \xi) = \beta\delta\varepsilon(n-1)(\varepsilon\alpha^2 - \beta^2\delta)g(W, Y) - \alpha(n-1)(\varepsilon\alpha^2 - \beta^2\delta)g(\phi W, Y) + \varepsilon\alpha S(Y, \phi W) - \beta\delta S(Y, W), \tag{55}$$

On comparison of (54) with (55), we have

$$\begin{aligned} \beta\delta S(Y, W) = {} & \beta\delta\varepsilon(n-1)(\varepsilon\alpha^2 - \beta^2\delta)g(W, Y) - \alpha(n-1)(\varepsilon\alpha^2 - \beta^2\delta)g(\phi W, Y) \\ & + \varepsilon\alpha S(Y, \phi W) + A(W)(n-1)(\varepsilon\alpha^2 - \beta^2\delta)\eta(Y) - \varepsilon\eta(Y)\left[\frac{A(W)r}{n} - B(W)(n-1)\right], \end{aligned} \tag{56}$$

Take $Y = \xi$ in (56) we get

$$\beta\delta S(\xi, W) = \beta\delta(n-1)(\varepsilon\alpha^2 - \beta^2\delta)\eta(W) + A(W)(n-1)(\varepsilon\alpha^2 - \beta^2\delta) - \varepsilon\left[\frac{A(W)r}{n} - B(W)(n-1)\right], \tag{57}$$

Applying Lemma (18), (57) reduces to

$$S(\xi, W) = (n-1)(\varepsilon\alpha^2 - \beta^2\delta)\eta(W) + \frac{(n-1)(\varepsilon\alpha^2 - \beta^2\delta)}{\beta\delta}\eta(\rho_1)\eta(W) - \frac{\varepsilon}{\beta\delta}\eta(W)\left[\frac{\eta(\rho_1)r}{n} - \eta(\rho_2)(n-1)\right], \tag{58}$$

Make use of (15), (16) and (19) in (58), we obtain

$$\lambda = -\frac{(n-1)(\varepsilon\alpha^2 - \beta^2\delta)}{\varepsilon} - \left[\frac{(n-\varepsilon)}{n}\frac{\beta\delta A(\xi)}{(\beta\delta + A(\xi)} + \frac{B(\xi)(n-1)}{(\beta\delta + A(\xi)}\right]. \tag{59}$$

Theorem 6.1. *Ricci Soliton in a Generalised Concircular ϕ-Recurrent (ε, δ)-Trans-sasakian manifold (M, g) with two 1-forms A and B and a constant scalar curvature r is shrinking for non-negative $A(\xi)$ and $B(\xi)$.*

7. Conclusion

Based on the nature of two one forms associated with the curvature conditions, Ricci solitons in generalised Ricci recurrent, ϕ-recurrent, pseudo-projective ϕ-recurrent and concircular ϕ-recurrent curvatures under (ε, δ)- Trans-sasakian manifolds is classified into expanding, shrinking and steady. This study may be extended to η -ricci solitons in real hyper surfaces of complex space forms.

References

[1] B.Prasad, *A Pseudo Projective Curvature tensor on Riemannian Manifold*, Bull. Cal. Math. Soc., 94(2002), 163-169.

[2] C.S.Bagewadi and A.Dakshayani Patil, *On Generalised ϕ Recurrent (ε, δ)-Trans-sasakian Manifolds*, Chinese Journal of Mathematics, 2014(2014), Article ID736846.

[3] C.S.Bagewadi and E Girish Kumar, *Note on Trans-sasakian manifolds*, Tensor N.S., 65(1)(2004), 80-88.

[4] D.E.Blair, *Contact Manifolds in Riemannian geometry*, Lecture notes in Mathematics, Springer-Verlag, Berlin-Newyork, 509(1970).

[5] H.G.Nagaraja and K Venu, *Ricci Solitons in Kenmotsu Manifold*, Journal of Informatics and Mathematical Sciences, 8(1)(2016), 29-36.

[6] H.G.Nagaraja, R.C.Premalatha and G.Somashekhara, *On an (ε, δ) -Trans sasakian structure*, Proceedings of the Estonian Academy of Sciences, 61(1)(2012), 20-23.

[7] J.A.Obulina, *New classes of almost contact metric structure*, Publications Mathematicae Debrecen, 32(3-4)(1985), 187-193.

[8] K.Yano, *Concircular geometry- I. Concircular transformations*, Proceedings of the Japan Academy, 16(1940), 195-200.

[9] M.M.Tripathi, *Ricci Solitons in contact metric manifolds*, arxiv:0801, 4222(2008).

[10] R.Sharma, *Certain results on K-contact and (k, μ)-contact manifolds*, J.Geom., 89(2008), 138-147.

[11] R.S.Hamilton, *The Ricci flow on surfaces, Mathematical and General Relativity*, American Math Soc. Contemp. Math., 71(1988), 237-262.