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Equilibrium Solution of the System in Case of Circular Orbit of the Centre of Mass

Research Article

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Abstract: In this paper we have studied the effect of the shadow of the earth due to solar radiation pressure and obsoleteness of earth on the motion and stability of system of two satellites connected by light flexible and extensible cable in the central gravitational field of the earth in case of circular orbit of the centre of mass. First of all derived the differential equations of motion of the system for two dimensional case in circular orbit of the centre of mass.

Keywords: Flexible, obsoleteness, Circular orbit, Gravitational field, Jacotrian.

1. Introduction

This work is direct generalization of the work Sinha and Singh [4] studied effect of solar radiation. Pressure on the motion and stability of the system of two interconnected satellites when their centre of mass moves in circular orbit. Again Sinha and Singh [4] could generalize the above problem by considering the centre of mass of the system moving in elliptical orbit. Kumar and Srivastava [6] studied about Evolution and non-evolution motion of a system of two cable-connected artificial satellite under the some perturbative forces. Again Kumar and Prasad [8] studied about non-linera planner oscillation of a cable connected satellites system in non-resonance and many author direct generalization of problem related to measures that our references.

2. Equation of Motion in Case of Circular Orbit

The equation of motion of the system given takes the form of two dimensional cases:

$$\begin{aligned} x'' - 2y' - -(3+4B)x + A\psi_1 \cos\epsilon \cos(v-\alpha) &= \bar{\alpha}\lambda\left[1 - \frac{lo}{r}\right]x \\ y'' + 2x' + By + A\psi_1 \cos\epsilon \sin(v-\alpha) &= \bar{\alpha}\lambda\left[1 - \frac{lo}{r}\right]y \end{aligned} \tag{1}$$

Where

$$A = \frac{\rho^3}{\mu}\left(\frac{B_1}{m_1} - \frac{B_2}{m_2}\right)$$

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$$B = \frac{3K_2}{\rho^2}$$
$$\lambda^{\bar{\alpha}} = \frac{\rho^3}{\mu}\alpha_\alpha$$

and

$$r = \sqrt{x^2 + y^2} \tag{2}$$

in case of circular orbit, the true anomaly V for the elliptic orbit will be replaced by τ whose values is as follows:

$$\tau = wt \tag{3}$$

where w is the angular velocity of the centre of mass of the system in case of circular orbit an 't' is the time of the system respectively the form.

$$\begin{aligned} x'' - 2y' - (3+4B)x + A\psi_1 \cos\epsilon \cos(\tau - \alpha) &= -\lambda_{\bar{\alpha}}\left[1 - \frac{lo}{r}\right]x \\ y'' - 2x' + By - A\psi_1 \cos\epsilon \cos(\tau - \alpha) &= -\lambda_{\bar{\alpha}}\left[1 - \frac{lo}{r}\right]y \end{aligned} \tag{4}$$

The condition of constraint takes the form

$$x^2 + y^2 \leq l_0^2 \tag{5}$$

Now there will be two types of motion

(i). Free motion ($\lambda_{\bar{\alpha}} = 0$)

(ii). Constrained motion ($\lambda_{\bar{\alpha}} \neq 0$)

In case of free motion the motion takes with tight string. Thus for free motion we must have from (4).

$$\begin{aligned} x'' - 2y' - (3+4B)x + A\psi_1 \cos\epsilon \cos(\tau - \alpha) &= 0 \\ y'' - 2x' + By - A\psi_1 \cos\epsilon \cos(\tau - \alpha) &= 0 \end{aligned} \tag{6}$$

The system of differential equation (6) can be easily integrated in terms of elementary functions. These integrated clearly indicate that free motion of the satellites of mass m_1 is bound to be converted into constrained motion after some time. Hence here after it is assumed that the system is moving with tight cable like a dumbbell satellites.

The equation of motion can be averaged with respect to t, which varies from 0 to 2π the averaged equations will describe the smaller secular perturbations and long periodic effects due to the periodic forces on the motion of the satellites of mass m_1. And the small secular and long periodic effects of the solar pressure with consideration of the effect of the earth's shadow on the system may be analysed by averaging the periodic term in (4) with respect to τ from θ to $2\pi - \theta$ for the period when the system being under the influence of su rays directly i.e. for $\psi_1 = 1$ and from $-\theta$ to θ for the period when the system passing through the shadow beam i.e. $\psi_1 = 0$. Thus after averaging the periodic term (4) we have

$$\begin{aligned} \frac{1}{2\pi}\left[\int_{-\theta}^{\theta} A\cos\epsilon\cos(\tau - \alpha)d\tau + \int_{\theta}^{2\pi-\theta} A\cos\epsilon\cos(\tau - \alpha)d\tau\right]\psi_1 = 1 \\ = \frac{-A\cos\epsilon\cos\alpha\sin\theta}{\pi} \\ \frac{1}{2\pi}\left[\int_{-\theta}^{\theta} A\cos\epsilon\sin(\tau - \alpha)d\tau + \int_{\theta}^{2\pi-\theta} A\cos\epsilon\cos(\tau - \alpha)d\tau\right]\psi_1 = 1 \end{aligned}$$

$$= \frac{-A\cos\epsilon\cos\alpha\sin\theta}{\pi} \tag{7}$$

Thus, the equations of motion (4) of the system are being described by using averaged values (7) in the form.

$$\begin{aligned} x'' - 2y' - (3+4B)x - \frac{-A\cos\epsilon\cos\alpha\sin\theta}{\pi} &= -\lambda_{\bar{\alpha}}\left[1 - \frac{lo}{r}\right]x \\ y'' + 2x' + By - \frac{-A\cos\epsilon\cos\alpha\sin\theta}{\pi} &= -\lambda_{\bar{\alpha}}\left[1 - \frac{lo}{r}\right]y \end{aligned} \tag{8}$$

We see that the equation (8) do not contain time explicitly therefore there must exists a Jocotrian's integrals of the problem. Multiply (8) by $2x'$, and $2y'$ and adding both sides. We get after integrating

$$x'^2 + y'^2 - (3+4B)x^2 + By^2 - \frac{-A\cos\epsilon\cos\alpha\sin\theta}{\pi} - \frac{2Ay}{\pi}\cos\epsilon\sin\alpha\sin\theta + \lambda_{\bar{\alpha}}\left[\left(x^2+y^2\right) - 2l_0\sqrt{x^2+y^2}\right] = h \tag{9}$$

Where h is the constraint of integration equation (9) is below as Jcasian's integrals of the problem. From equation (9) follows the chance of zero velocity can be obtained in this form

$$(3+4B)x^2 - By^2 + \frac{2Ax\cos\epsilon\cos\alpha\sin\theta}{\pi} + \frac{2Ay\cos\epsilon\sin\alpha\sin\theta}{\pi} - \lambda_{\bar{\alpha}}\left[\left(x^2+y^2\right) - 2l_0\sqrt{x^2+y^2}\right] = h \tag{10}$$

3. Equilibrium Solution of the Problem

The Equilibrium positions of the system are given by the constant values of the Co-ordinates in the rating from of reference. Now Let $x = x_0$ and $y = y_0$, where x_0 and y_0 are constants give the equilibrium positions then

$$\left. x' = x_0' = 0 = x_0'' y' = y_0' = 0 = y_0'' \right\} \tag{11}$$

using (11) in (8) we get

$$\begin{aligned} -(3+4B)x_0 - \frac{A}{\pi}\cos\epsilon\cos\alpha\sin\theta &= -\lambda_{\bar{\alpha}}\left[1 - \frac{lo}{r_0}\right]x_0 \\ By_0 - \frac{A}{\pi}\cos\epsilon\sin\alpha\sin\theta &= -\lambda_{\bar{\alpha}}\left[1 - \frac{lo}{r_0}\right]y_0 \end{aligned} \tag{12}$$

Where $r_0 = \sqrt{x_0^2 + y_0^2}$. Now we shall discuss the particular solutions of the equation (12) as follows. Thus putting $\alpha = 0$ the equations of motion given (12) takes the form.

$$\begin{aligned} -(3+4B)x_0 - \frac{A}{\pi}\cos\epsilon\sin\theta &= -\lambda_{\bar{\alpha}}\left[1 - \frac{lo}{r_0}\right]x_0 \\ By_0 &= -\lambda_{\bar{\alpha}}\left[1 - \frac{lo}{r_0}\right]y_0 \end{aligned} \tag{13}$$

Hence the equilibrium position in this case are given below:

The system may be wholly extended along x-axis and in this case $y = 0$. Thus equilibrium pair is taken as $(a, 0)$. So, putting $x_0 = a$, $y_0 = 0$ in (13), we get,

$$\begin{aligned} -(3+4B)a - \frac{A}{\pi}\cos\epsilon\sin\theta &= -\lambda_{\bar{\alpha}}\left[1 - \frac{lo}{r_0}\right]a \\ 0 &= -\lambda_{\bar{\alpha}}\left[1 - \frac{lo}{r_0}\right]0 \end{aligned} \tag{14}$$

Where $r_0 = \sqrt{x_0^2 + 0} = a$. From the first equation of (14), we get,

$$-(3+4B)a = \frac{A}{\pi}\cos\epsilon\sin\theta = -\lambda_{\bar{\alpha}}a + \lambda_{\bar{\alpha}}l_0$$
$$(\lambda_{\bar{\alpha}} - 3 - 4b)a = \frac{\lambda_{\bar{\alpha}}l_0\pi + A\cos\epsilon\sin\theta}{\pi}$$
$$a = \frac{\lambda_{\bar{\alpha}}l_0\pi + A\cos\epsilon\sin\theta}{\pi(\alpha_a - 3 - 4B)}$$

Here the first equilibrium point is given by

$$[a, o] = \left[\frac{\lambda_{\alpha}^{-}l_0\pi + A\cos\epsilon\sin\theta}{\pi(\alpha_a - 3 - 4\ B)}, 0\right] \tag{15}$$

4. Treatment of Problem Equilibrium Position (a, 0)

Now taken the moments of different masses about the satellites m_1 in equilibrium position, we get

$$m_2 l_E = (m_1 + m_2)\frac{r_{0V}}{l_0} = \frac{(m_1+m_2)}{l_0}.r_0\frac{l_0 m_2}{m_1+m_2}$$
$$= m_2 r_0$$
$$l_E = r_0 \tag{16}$$

Now we proceed to analyze the value of the Hook's Modulus of elasticity λ at the two equilibrium position separately. In this case we have:

$$r_0 = a\frac{\lambda_{\bar{\alpha}}\pi l_0 + A\cos\epsilon\sin\theta}{\pi(\lambda_{\bar{\alpha}} - 3 - 4B)} \tag{17}$$

Comparing (16) and (17) we get,

$$l_E = \frac{\lambda_{\bar{\alpha}}\pi l_0 + A\cos\epsilon\sin\theta}{\pi(\lambda_{\bar{\alpha}} - 3 - 4B)} \tag{18}$$

But

$$\lambda_{\bar{\alpha}} = \frac{\rho^3\,(m_1+m_2)}{\mu m_1 m_2 l_0}\lambda$$
$$\lambda = \frac{\mu l_0}{\rho^3}\frac{m_1 m_2}{(m_1+m_2)}\lambda_{\bar{\alpha}} \tag{19}$$

Using (18) in (19) we get

$$\lambda = \frac{\mu}{\rho^3}\frac{m_1 m_2}{(m_1+m_2)}\left[\frac{A\cos\epsilon\sin\theta + \pi(3+4B)l_E}{\pi(l_E - l_0)}\right] l_0 > 0 \tag{20}$$

Now we proceed to test the stability of the first equilibrium position $(a, 0)$

$$a = \frac{\lambda_{\alpha}^{-}\pi l_0 + A\cos\epsilon sin\theta}{\pi(\lambda_{\bar{\alpha}} - 3 - 4B)} \tag{21}$$

Let n_1 and n_2 denote the small variations in the co-ordinates for the equilibrium position then we have.

$$x = a + n_1, \quad and \quad y = n_2$$
$$x' = n'' \quad and \quad y' = n_2'$$
$$x'' = n_1'' \quad and \quad y'' = n_2'' \tag{22}$$

Using (22) in (8), we get the variational equational of motion for the system in the form after putting $\alpha = 0$ in (8),

$$n_1^{''} = 2n_2' - (3+4B)(a+n_1) - \frac{A\cos\epsilon\sin\theta}{\pi} = -\lambda_{\bar{\alpha}}\left[1 - \frac{lo}{r_0}\right](a+n_1)$$
$$n_2^{''} = 2n_1' + (Bn_2) = -\lambda_{\bar{\alpha}}\left[1 - \frac{lo}{r_0}\right]n_2 \tag{23}$$

where

$$r^2 = (a+n_1)\,2 + n_2 2 \tag{24}$$

If two equations (23) by $2(a+n_1)^1$, $2n_2^1$ respectively and then adding them together and integrating in form

$$n_1^{12} + n_1^{22} - (3+4B)(a+n_1)2 + Bn_2^2 - \frac{2An_1\cos\epsilon\sin\theta}{\pi} + \lambda_{\bar{\alpha}}\left[(a+n_1)^2 + n_2^2\right] - 2\alpha_{\bar{\alpha}}l_0[(a+n_1)2 + n_2^2]^{1/2} = h \tag{25}$$

where h is the constant of integration. Equation (25) is known as Jacobils integral for variational equations of motion of the system at the equilibrium position $[a, 0]$ to test the stability at $(a, 0)$ in the sense of liapunov, we take Jacobian integral as the Liapunov function $V\left(n_1^1, n_2^1, n_1, n_2\right)$ and is obtained by expanding (25).

$$V\left(n_1^1, n_2^1, n_1, n_2\right) = n_1^{1^2} + n_2^{1^2} + n_1^2[\lambda_{\bar{\alpha}} - 3 - 4B - \frac{\lambda_{\bar{\alpha}}l_0}{a}] + n_2^2[\lambda_{\bar{\alpha}} + B - \frac{\lambda_{\bar{\alpha}}l_0}{a}]$$
$$+ n_1[-2(3+4B)a - \frac{2A\cos\epsilon\sin\theta}{\pi} + 2\lambda_{\bar{\alpha}}a - 2\lambda_{\bar{\alpha}}l_0] \tag{26}$$
$$+ a[-(3+4B)a + \lambda_{\bar{\alpha}}a - 2\lambda_{\bar{\alpha}}l_0] + 0(3) \tag{27}$$

where $0(3)$ stands for 3rd and higher order term in the small gravity $n_1$and n_2.

5. Conclusions

Therefore we come to the conclusion that the satellite of mass m_1 will move inside the boundary of different curve of zero velocity. Represented by (10) for different value of Tacobian Constant h and (26) that the sufficient conditions for the stability of the system at the said equilibrium position $(a, 0)$ in the sense of Liapunov are:

(i). $-2(3+4B)a - \frac{2A\cos\epsilon\sin\theta}{\pi} + 2\lambda_{\bar{\alpha}}a - 2\lambda_{\bar{\alpha}}l_0 = 0$

(ii). $\lambda_{\bar{\alpha}} - 3 - 4B - \frac{\lambda_{\bar{\alpha}}l_0}{a} > 0$ and

(iii). $\lambda_{\bar{\alpha}} + B - \frac{\lambda_{\bar{\alpha}}l_0}{a} > 0$

Now in order to have the clear the satisfactory of the system at the equilibrium position $(a, 0)$ we have to examine the above condition one by one separately.

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