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Two-person Zero-sum Game Problem Solution: Integer Simplex Method

Research Article

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Abstract: In this paper, we have calculated the optimal strategies of two-person zero-sum game problems which does not have a saddle point using Integer simplex method (Gomory's cutting plane method).

Keywords: Linear programming problem (LPP), Game problem, optimal strategies, Integer simplex method (Gomory's cutting plane method).

1. Introduction

Game theory is the study of the strategic interaction (conflict, cooperation, etc.) of two or more decision makers, or "players", who are attentive that their actions affect each other. Game theory describes the situations involving conflict in which the payoff is affected by the actions and counter-actions of clever opponents. Two-person zero-sum games play a crucial role in the progress of the theory of games [1]. In order to know the theory of game, consider the game in which player X has four choices from which to select, and player Y has five alternatives for each choice of player Y. The payoff matrix A is given below:

	Player Y					
		$j=1$	$j=2$	$j=3$	$j=4$	$j=5$
Player X	$i=1$	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}
	$i=2$	S_{21}	S_{22}	S_{23}	S_{24}	S_{25}
	$i=3$	S_{31}	S_{32}	S_{33}	S_{34}	S_{35}
	$i=4$	S_{41}	S_{42}	S_{43}	S_{44}	S_{45}

In the payoff matrix, the two rows $(i = 1, 2, 3, 4, 5)$ represents the two possible strategies that player X can employ, and the three columns $(j = 1, 2, 3, 4, 5)$ represent the three possible strategies that player Y can utilize. The payoff matrix is oriented to player X, meaning that a positive A_{ij} is a gain for player X and a loss for player Y, and a negative A_{ij} is a gain for player Y and a loss for player X. For example, if player X uses strategy 3 and player Y uses strategy 4, player X receives $A_{34} = S_{34}$ units and player Y thus losses S_{34} units. Clearly, in our example player Y always loses; however, the objective is to minimize the payoff to player X [1, 2].

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2. Solution of Game Problem

Consider a two-person zero-sum game without saddle point, having the payoff matrix for player X as,

	Player Y			
Player X	1	3	-3	7
	2	5	4	-6

Since, Maximin value$= -3$, Minimaxi value$= 2$, the payoff matrix does not possess saddle point. Therefore, value of the game lie between -3 and 2. Let the optimal strategies of two players be:

$$S_X = (p_1, p_2),\ S_Y = (q_1, q_2, q_3, q_4),$$

where, $p_1 + p_2 = 1$ and $q_1 + q_2 + q_3 + q_4 = 1$. Then, the linear programming problem (LPP) can be written as:

For Player X:

$$\begin{aligned} \textit{Minimize} \quad & X = x_1 + x_2 = \frac{1}{v} \\ \textit{Subject to} \quad & x_1 + 2x_2 \geq 1 \\ & 3x_1 + 5x_2 \geq 1 \\ & 3x_1 + 4x_2 \geq 1 \\ & 7x_1 - 6x_2 \geq 1 \\ & \textit{and } x_1, x_2 \geq 0. \end{aligned}$$

For Player Y:

$$\begin{aligned} \textit{Maximize} \quad & Y = y_1 + y_2 + y_3 + y_4 = \frac{1}{v} \\ \textit{Subject to} \quad & y_1 + 3y_2 - 3y_3 + 7y_4 \leq 1 \\ & 2y_1 + 5y_2 + 4y_3 - 6y_4 \leq 1 \\ & \textit{and } y_1, y_2, y_3, y_4 \geq 0. \end{aligned}$$

Standard form of LPP:

For Player X:

$$\begin{aligned} \textit{Maximize} \quad & X^* = -Minimize\ X = -(x_1 + x_2) + 0.s_1 + 0.s_2 + 0.s_3 + 0.s_4 \\ \textit{Subject to} \quad & x_1 + 2x_2 - s_1 = 1 \\ & 3x_1 + 5x_2 - s_2 = 1 \\ & -3x_1 + 4x_2 - s_3 = 1 \\ & 7x_1 - 6x_2 - s_4 = 1 \\ & \textit{and } x_1, x_2, s_1, s_2, s_3, s_4 \geq 0, \end{aligned}$$

where, s_1, s_2, s_3, s_4 are surplus variables.

For Player Y:

$$\textit{Maximize} \quad Y = y_1 + y_2 + y_3 + y_4 + 0.s_1 + 0.s_2 = \frac{1}{v}$$

$$\text{Subject to} \qquad y_1 + 3y_2 - 3y_3 + 7y_4 + s_1 = 1$$
$$2y_1 + 5y_2 + 4y_3 - 6y_4 + s_2 = 1$$
$$\text{and } y_1, y_2, y_3, y_4, s_1, s_2 \geq 0,$$

where, s_1, s_2 are slack variables.

Integer simplex method (Gomory's cutting plane method), solution of LPP, for Player X is $x_1 = 1$, $x_2 = 1$ and for player Y is $y_1 = 0$, $y_2 = 0$, $y_3 = \frac{13}{10}$, $y_4 = \frac{7}{10}$. Value of the game, $v = \frac{1}{2}$.

Optimal strategies for player X,

$$p_1 = vx_1 = \frac{1}{2}, \;\; p_2 = vx_2 = \frac{1}{2}$$

Optimal strategies for player Y,

$$q_1 = vy_1 = 0, \; q_2 = vy_2 = 0, \; q_3 = vy_3 = \frac{13}{20}, \; q_4 = vy_4 = \frac{7}{20}$$

3. Conclusion

Game problem was successfully solved using Integer simplex method (Gomory's cutting plane method). It has been observed that the optimal strategies for players are same when compared to the solution using graphical method.

References

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