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Metro Domination of Square Cycle

Research Article

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Abstract: Let $G = (V, E)$ be a graph.A set $S \subseteq V$ is called resolving set if for every $u, v \in V$ there exist $w \in V$, such that $d(u, w) \neq d(v, w)$. The resolving set with minimum cardinality is called metric basis and its cardinality is called metric dimention and it is denoted by $\beta(G)$. A set $D \subseteq V$ is called dominating set if every vertex not in D is adjacent to at least one vertex in D. The dominating set with minimum cardinality is called domination number of G and it is denoted by $\gamma(G)$. A set which is both resolving set as well as dominating set is called metro dominating set. The minimum cardinality of a metro dominating set is called metro domination number of G and it is denoted by $\gamma_\beta(G)$. In this paper we determine metro domination number of square cycle.

Keywords: Power graph, metric dimension, landmark, distance matrix.

1. Introduction

All the graphs considered are simple,finite and connected.A set of vertices S resolves a graph G if every vertex of G is uniquely determined by its vector of distances to the vertices in S.A set of vertices S resolves a graph G if every vertex of G is uniquely determined by its vector of distances to the vertices in S. This work undertakes a general study of resolving sets in square cycle of graphs. In 1976, F.Harary and R.A.Melter [4] introduce the notion of metric dimension.The vertex set and edge set of a graph$G$ are denoted by $V(G)$ and $E(G)$. The distance between vertices $u, w \in V(G)$ is denoted by $d_G(v, w)$, or $d(v, w)$ if the graph G is clear from the context.A vertex $x \in V(G)$ resolves a pair of vertices $v, w \in V(G)$ if $d(v.x) \neq d(w, x)$. A set of vertices$S \subseteq V(G)$ resolves G, and S is a resolving set of G, if every pair of distinct vertices of G are resolved by some vertex in S. A resolving set S of G with minimum cardinality is a metric basis of G and its cardinality is the metric dimension of G, denoted by $\beta(G)$. In 1976, F.Harary and R.A.Melter [4] introduce the notion of metric dimension A work place can be denoted as node in the graph, and edges denote the connections between the places. The problem of minimum number of machines ($or Robots$) to be placed at certain nodes to trace each and every node exactly once is a classical one. this problem can be solved by using networks where places are interconnected in which, the navigating agent moves from one node to another in the network. The places or nodes of a network where we place the machines ($robot$) are called landmarks. The minimum number of machines required to locate each and every node of the network is termed the metric dimension and the set of all minimum possible number of landmarks constitute metric basis. The machines, where they placed at nodes of the network know, their distances to sufficiently large set of landmarks and the position of these machines on the network are uniquely determined.However that a Robot navigating on a Graph can sense the distance to

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a set of landmarks. For safeguard analysis of a facility, such as a fire protection, study of nuclear power plants, the facility can be modeled by a graph or network. Suppose we have a detection device located at vertex and the device can supply three outputs,

(1). There is an object at that vertex

(2). There is an object at one of the vertices adjacent to that vertex,

(3). There is no object at that vertex or any adjacent vertex.

It is necessary to determine a collection of vertices at which to place detection device so that if there is an object at any vertex in the graph, it can be detected, and its position uniquely identified. In order to detect an object, which might be at any vertex in V(G), it is necessary to have a dominating set. The additional problem of uniquely identifying the location of the object requires a metric dimension (locating) feature. These two concept motivated for the investigation of the new graph invariants called locating domination number by P.J.Slater and which is perfectly refined as a metro domination number by us in this paper. We recall the following result which we use in next sections.

Theorem 1.1 ([4]). *The metric dimension of a non-trivial complete graph of order n is $n-1$.*

Theorem 1.2 ([9]). *For any non trivial graph G on $n \geq 2$ vertices, $\beta_k(G)$ = n-1 if and only if diam$(G) \leq K$, where $K \geq 1$ is any integer.*

Theorem 1.3 ([7]). *A graph with metric dimension two cannot have a sub graph isomorphic to K_5 or $K_{3,3}$.*

Theorem 1.4 ([7]). *If C_n is cycle of order n,then $\beta(C_n) = 2$.*

Theorem 1.5 ([4]). *The metric dimension of a complete bipartite graph Km, n is $m + n - 2$.*

Theorem 1.6 ([11]). *For any two positive integers n,k with $k < n$, $\beta_b(C_n^k) = \beta(C_n^k)$.*

2. Locating Number

A subset D of $V(G)$ is called a dominating set, if every vertex is $V - D$ is adjacent to at least one vertex in D. The minimum cardinality of a dominating set is called the domination number of the graph G 1976. The metric dimension of a graph $G(V, E)$ is the cardinality of a minimal subset S of V such that for each pair of vertices $u, vofV$ there is a vertex w in S such that the length of the shortest path from $wtou$ is different from the length of a shortest path from $wtov$. The metric dimension of a graph G is also called as a locating number of G and studied its dominating property independently by P.J.Slater.A dominating set D is called a locating dominating set or simply $LD-set$ if for each pair of vertices $u, v \in V - D$, $ND(u) \neq ND(v)$, where $ND(u) = N(u) \cap D$. The minimum cardinality of an $LD-set$ of the graph G is called the locating domination number of G denoted by $\gamma_L(G)$. For example, the set of darkened vertices of the graph G of Figure 1, serves as a minimal locating dominating set, so $\gamma_L(G) = 3$.

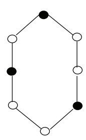

Figure 1. **The graph G and its $LD - set$**

We recall the following result which we use in next sections.

Theorem 2.1 ([8]). *Domination number of square of cartesian products of cycles.*

Theorem 2.2 ([8]). *Let S be a dominating set of G^2.Then S is a minimal dominating set of G^2 if and only if each vertex $u \in S$ satisfies at least one of the following conditions:*

(a). There exists a vertex $v \in V(G) - S$ for which $N_2(v) \cap S = \{u\}$.

(b). $d(u, w) \geq 2$ for every vertex $w \in S - \{u\}$.

Theorem 2.3 ([8]). *If G is a graph with no isolated vertices and S is a minimal dominating set of G, then $V(G) - S$ is a dominating set of G.*

Corollary 2.4. $\gamma(c_n) = \lceil n/3 \rceil$.

3. Metro Domination Number

We now define a metro dominating set, which can be served as a better alternating for the locating dominating set as, A dominating set $D of V(G)$ having the property that for each pair of vertices u, v there exists a vertex $w in D$ such that $d(u, w) \neq d(v, w)$ is called the metro dominating set of G or simply an $MD - set$. The minimum cardinality of a metro dominating set of G is called metro domination number of G and it is denoted by $\gamma_\beta(G)$. For example, the set of darkened vertices of the graph G, of $figure 2$, is a minimal metro dominating set and hence $\gamma_\beta(G) = 3$.
We recall the following result which we use in next sections.

Theorem 3.1 ([6]). *The metro dominating number of a graph G is $\lceil n/3 \rceil$ if and only if G is a cycle.*

Theorem 3.2 ([6]). *Let G be a graph on n vertices. Then $\gamma_\beta(G) = n - 1$ if and only if G is K_n or $K_{1,n-1}$, for $n \geq 1$.*

Theorem 3.3 ([6]). *If $\gamma_\beta(G) = 2$, then G cannot have K_4 as a sub graph of G.*

Remark 3.4. *For any connected graph G, $\gamma_\beta(G) \geq max\{\gamma(G), \beta(G)\}$.*

4. Power Graph

Let $G = (V, E)$ connected a graph. K^{th} power of is denoted by G^k. Whose vertex is same as that of G and two vertices U, V in G^k are adjacent K. If and only if $d(u, v) \leq K$ in G.

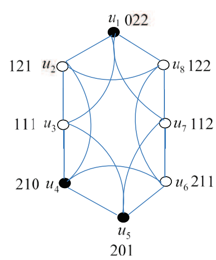

Figure 2. $\gamma_\beta(c_8^2) = 3$

Theorem 4.1. *For any integer $n \geq 3$,* $\gamma_\beta(c_n^2) = \begin{cases} 3 & if \quad n = 4, 6, 7, 8, 11, 15 \\ 4 & if \quad n = 5, 9, 10, 12, 13, 14 \\ \lceil n/5 \rceil & if \quad n \geq 16 \end{cases}.$

Proof. By Theorem 1.4 and the Remark 3.4, $\gamma_\beta(c_n^2) \geq 3$ for all n. For $n = 4$ choose $S = \{u_1, u_2u_3\}$ then S is called dominating set as well as metric basis,hence $\gamma_\beta(c_n^2) = 3$. For $n = \{6, 7, 8\}$ choose $S = \{u_1, u_2, u_6\}$ is both metric basis as well as dominating set, hence $\gamma_\beta(c_n^2) = 3$. For $n = 11$ choose $S = \{u_1, u_2, u_7\}$ is both metric basis as well as dominating set, hence $\gamma_\beta(c_n^2) = 3$. For $n = 15$ choose $S = \{u_1, u_6, u_{11}\}$ is both dominating set as well as metric basis,hence $\gamma_\beta(c_n^2) = 3$. For $n = 5$ choose $S = \{u_1, u_2, u_3, u_4\}$ is both dominating set as well as metric basis, hence $\gamma_\beta(c_n^2) = 4$.For $n = 9, 10$ choose $S = \{u_1, u_2, u_5, u_6\}$ is both dominating set as well as metric basis,hence $\gamma_\beta(c_n^2) = 4$. For $n = 12, 13, 14$ choose $S = \{u_1, u_2, u_6, u_{10}\}$ is both dominating set as well as metric basis, hence $\gamma_\beta(c_n^2) = 4$. □

Lemma 4.2 ([8]). *For any integer n, $\gamma(c_n^2) = \lceil n/5 \rceil$.*

Lemma 4.3. *For any integer n, $\gamma_\beta(c_n^2) = \lceil n/5 \rceil$. For $n \geq 16$.*

Proof. Let D be a dominating set C_n^2. Let $u_1, u_2 .... u_n$ be vertices of C_n, such that u_i is adjacent to u_{i+1} for $i = 1, 2, 3, ... n-1$. By using [11] and $\gamma(C_n^2) = \lceil n/5 \rceil$ by the lemma 4.2. since a metro dominating set D is also a dominating set then we show that

$$\gamma_\beta(C_n^2) \geq \lceil n/5 \rceil \tag{1}$$

To prove the reverse inequality we find a metro dominating set of cardinality $\lceil n/5 \rceil$. Choose $D = \{u_{i+(5k-2)} : k \geq 1\}$ if $6k-2$, $6k-1$, $6k$, $6+1$ and $6k+2$ then $|D| = \lceil n/5 \rceil$ proved and D is metro dominating set, infact for every $u_j \in V - D$, by the choice of D, at least one of $u_{j-2}, u_{j-1}, u_{j+1} or u_{j+2}$ must be in D and which dominates u_j, by the Lemma 4.3, D is resolving set. Hence

$$\gamma_\beta(C_n^2) \leq \lceil n/5 \rceil \tag{2}$$

Therefore from (1) and (2), $\gamma_\beta(C_n^2) = \lceil n/5 \rceil$. □

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