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k-cordial Labeling of Triangular Belt, Alternate Triangular Belt, Braid Graph and Z- P_n

Research Article

N.B. Rathod^{1*} and K.K. Kanani²

- 1 School of Science, R.K. University, Rajkot, Gujarat, India.
- 2 Department of Mathematics, Government Engineering College, Rajkot, Gujarat, India.

Abstract: In this research paper, we find k-cordial labeling of some special graphs. We prove that Triangular Belt $TB(n)(\downarrow^n)$,

Alternate Triangular Belt $ATB(n)(\downarrow\uparrow\downarrow\uparrow\dots)$ and Braid Graph B(n) are k-cordial. Moreover, the graph Z- P_n is k-cordial

for all odd k.

MSC: 05C78

Keywords: Abelian Group, k-cordial Labeling, Triangular Belt, Alternate Triangular Belt, Braid graph, Z- P_n .

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1. Introduction

In this research work we consider only finite, connected, undirected and simple graph G = (V(G), E(G)). We denote |V(G)| = total number of vertices of graph G and |E(G)| = total number of edges of graph G respectively. A graph labeling is an assignment of numbers to the vertices or edges or both subject to certain condition(s). To understand more about graph labeling as well as bibliographic references we refer Gallian [1].

1.1. Preliminaries

Definition 1.1. Let $\langle A, * \rangle$ be any Abelian group. A graph G = (V(G), E(G)) is said to be A-cordial if there is a mapping $f : V(G) \to A$ which satisfies the following two conditions when the edge e = uv is labeled as f(u) * f(v)

(i).
$$|v_f(a) - v_f(b)| \le 1$$
; for all $a, b \in A$,

(ii).
$$|e_f(a) - e_f(b)| \le 1$$
; for all $a, b \in A$.

Where

 $v_f(a)$ =the number of vertices with label a;

 $v_f(b)$ =the number of vertices with label b;

 $e_f(a)$ =the number of edges with label a;

 $e_f(b)$ =the number of edges with label b.

We note that if $A = \langle Z_k, +_k \rangle$, that is additive group of modulo k then the labeling is known as k-cordial labeling.

 $^{^*}$ E-mail: rathodneha005@gmail.com

The concept of A-cordial labeling was introduced by Hovey [2] and proved the following results:

- All the Connected graphs are 3-cordial.
- All the Trees are 3-cordial and 4-cordial.
- Cycles are k-cordial for all odd k.

Youssef [3] proved the following results:

- The Complete Graph K_n is 4-cordial $\iff n \leq 6$.
- The Complete Bipartite Graph $K_{m,n}$ is 4-cordial $\iff m \text{ or } n \not\equiv 2 \pmod{4}$.

Modha and Kanani [4] proved the following result:

• The Fan f_n is k-cordial for all k.

Modha and Kanani [5] proved the following results:

- The Bistar $B_{m,n}$ is k-cordial for all k.
- The Restricted Square Graph $B_{n,n}^2$ of bistar is k-cordial for odd k.
- The One Point Union of cycle C_3 with star graph $K_{1,n}$ is k-cordial for all k.
- The Comb Graph $P_n \odot K_1$ is k-cordial for all k.

Modha and Kanani [6] proved the following results:

- The Wheel W_n is k-cordial for all odd k and for all $n=mk+j, m\geq 0, 1\leq j\leq k-1$ except for $j=\frac{k-1}{2}$.
- The Total Graph $T(P_n)$ of path P_n is k-cordial for all k.
- The Square Graph C_n^2 of cycle C_n is k-cordial for all odd k and $n \ge k$.
- The Path Union of n Copies of cycle C_k is k-cordial graph for odd k.

Rathod and Kanani [7] proved the following results:

- The Square Graph P_n^2 of path P_n is k-cordial.
- The Pan Graph C_n^{+1} is k-cordial for all even k and $n=k+j,\, 0\leq j\leq k-1.$
- The Pan Graph C_n^{+1} is k-cordial for all even k and n=2tk+j, where $t\in N\cup\{0\}$ and $0\leq j\leq k-1$.
- The Pan Graph C_n^{+1} is k-cordial for all even k and n=2tk+k+j, where $t\in N$ and $0\leq j\leq k-1$.

Rathod and Kanani [8] proved the following results:

- \bullet The Triangular Book B(3,n) with n-pages is k-cordial.
- The Triangular Book with Book Mark $TB_n(u, v)(v, w)$ is k-cordial.
- The Jewel Graph J_n is k-cordial.

We consider the following useful definitions to understand the results of this research paper.

Definition 1.2. Let $L_n = P_n \times P_2$ $(n \ge 2)$ be the ladder graph with vertex set u_i and v_i , i = 1, 2, ..., n. The Triangular Belt is obtained from the ladder by adding the edges $u_i v_{i+1}$ for all $1 \le i \le n-1$. This graph is denoted by $TB(n)(\downarrow^n)$.

Definition 1.3. Let $L_n = P_n \times P_2$ $(n \ge 2)$ be the ladder graph with vertex set u_i and v_i , i = 1, 2, ..., n. The Alternate Triangular Belt is obtained from the ladder by adding the edges $u_{2i+1}v_{2i+2}$ for all i = 0, 1, 2, ..., n - 1 and $v_{2i}u_{2i+1}$ for all i = 1, 2, ..., n - 1. This graph is denoted by $ATB(n)(\downarrow\uparrow\downarrow\uparrow...)$.

Definition 1.4. The Braid Graph B(n) is obtained from a pair of paths P_n and P_n . Let $v_1, v_2, ..., v_n$ be the vertices of path P_n and $u_1, u_2, ..., u_n$ are the vertices of path P_n . To obtain braid graph B(n) join i^{th} vertex of path P_n with $(i+1)^{th}$ vertex of path P_n and i^{th} vertex of path P_n with $(i+2)^{th}$ vertex of path P_n with the new edges for all $1 \le i \le n-2$.

Definition 1.5. The graph Z-P_n is obtained from a pair of paths P_n and P_n . Let $v_1, v_2, ..., v_n$ be the vertices of path P_n and $u_1, u_2, ..., u_n$ are the vertices of path P_n . To obtain Z-P_n join i^{th} vertex of path P_n with $(i+1)^{th}$ vertex of path P_n for all $1 \le i \le n-1$.

Here, all terminologies are considered from Gross and Yellen [9].

2. Main Results

Theorem 2.1. The Triangular Belt $TB(n)(\downarrow^n)$ is k-cordial for all n.

Proof. Let G = TB(n) be the triangular belt. Let $L_n = P_n \times P_2$ $(n \ge 2)$ be the ladder graph with vertex set u_i and v_i , i = 1, 2, ..., n. The triangular belt is obtained from the ladder by adding the edges $u_i v_{i+1}$ for all $1 \le i \le n-1$. Let n = mk + j, where $m \ge 0$ and $1 \le j \le k-1$. We divide n vertices into two blocks of mk and j, which are denoted by $u_1, u_2, ..., u_{mk}$ and $u'_1, u'_2, ..., u'_j$ also $v_1, v_2, ..., v_{mk}$ and $v'_1, v'_2, ..., v'_j$. We note that |V(G)| = 2n and |E(G)| = 4n - 3.

To define k-cordial labeling $f: V(G) \to Z_k$ we consider the following cases.

<u>Case 1</u>: m > 0 and j = 0.

The labeling pattern of mk vertices is defined as follows:

Subcase 1: $k \equiv 0 \pmod{2}$, $1 \le i \le mk$.

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f(u_i) = 2p_i - 1; i \equiv p_i \pmod{k}; 1 \le i \le k,
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$$f(v_i) = 2p_i - 2$$
; $i \equiv p_i \pmod{k}$; $1 \le i \le k$,

Subcase 2: $k \equiv 1 \pmod{2}$, $1 \le i \le mk$.

$$f(u_i) = 2p_i - 1; i \equiv p_i \pmod{k}; 1 \le i \le \frac{k-1}{2},$$

$$f(u_i) = 2p_i$$
; $i \equiv p_i \pmod{k+1}$; $\frac{k-1}{2} + 1 \le i \le k$,

$$f(v_i) = 2p_i - 2$$
; $i \equiv p_i \pmod{k}$; $1 \le i \le \frac{k-1}{2} + 1$,

$$f(v_i) = 2p_i - 1$$
; $i \equiv p_i \pmod{k+1}$; $\frac{k-1}{2} + 2 \le i \le k$,

Case 2: $m \ge 0$ and $1 \le j \le k - 1$.

Subcase 1: $k \equiv 0 \pmod{2}$.

The labeling pattern of first block of mk vertices is defined as follows:

$$f(u_i) = 2p_i - 1$$
; $i \equiv p_i \pmod{k}$; $1 \le i \le k$,

$$f(v_i) = 2p_i - 2$$
; $i \equiv p_i \pmod{k}$; $1 \le i \le k$,

The labeling pattern of second block of j vertices is defined as follows:

$$f(u'_j) = 2j - 1$$
; $1 \le j \le k - 1$,

$$f(v_i') = 2j - 2$$
; $1 \le j \le k - 1$.

Subcase 2: $k \equiv 1 \pmod{2}$.

The labeling pattern of first block of mk vertices is defined as follows:

$$f(u_i) = 2p_i - 1;$$
 $i \equiv p_i \pmod{k};$ $1 \le i \le \frac{k-1}{2},$ $f(u_i) = 2p_i;$ $i \equiv p_i \pmod{k+1};$ $\frac{k-1}{2} + 1 \le i \le k,$

$$f(u_i) = 2p_i$$
; $i \equiv p_i \pmod{k+1}$; $\frac{k-1}{2} + 1 \le i \le k$,

$$f(v_i) = 2p_i - 2$$
; $i \equiv p_i \pmod{k}$; $1 \le i \le \frac{k-1}{2} + 1$, $f(v_i) = 2p_i - 1$; $i \equiv p_i \pmod{k+1}$; $\frac{k-1}{2} + 2 \le i \le k$,

The labeling pattern of second block of j vertices is defined as follows:

$$\begin{split} f(u_j') &= 2j-1 \; ; & 1 \leq j \leq \frac{k-1}{2}, \\ f(u_j') &= 2j \; ; & \frac{k-1}{2} + 1 \leq j \leq k-1, \\ f(v_j') &= 2j-2 \; ; & 1 \leq j \leq \frac{k-1}{2} + 1, \\ f(v_j') &= 2j-1 \; ; & \frac{k-1}{2} + 2 \leq j \leq k-1. \end{split}$$

In each possibility the graph under consideration satisfies the vertex conditions and edge conditions for k-cordial labeling. Hence, triangular belt $TB(n)(\downarrow^n)$ is k-cordial for all n.

Example 2.2.

1. The Triangular Belt $TB(9)(\downarrow^9)$ and its 8-cordial labeling is shown in Figure 1.

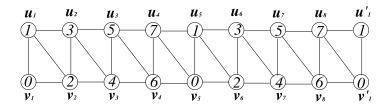


Figure 1. 8-cordial labeling of Triangular Belt $TB(9)(\downarrow^n)$

2. The Triangular Belt $TB(10)(\downarrow^{10})$ and its 7-cordial labeling is shown in Figure 2.

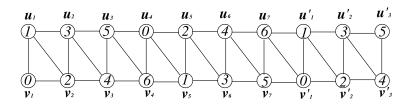


Figure 2. 7-cordial labeling of Triangular Belt $TB(10)(\downarrow^{10})$

Theorem 2.3. The Alternate Triangular Belt $ATB(n)(\downarrow\uparrow\downarrow\uparrow...)$ is k-cordial for all n.

Proof. Let G = ATB(n) be the alternate triangular belt. Let $L_n = P_n \times P_2$ $(n \ge 2)$ be the ladder graph with vertex set u_i and v_i , i=1,2,...,n. The alternate triangular belt is obtained from the ladder by adding the edges $u_{2i+1}v_{2i+2}$ for all i = 0, 1, 2, ..., n - 1 and $v_{2i}u_{2i+1}$ for all i = 1, 2, ..., n - 1. Let n = mk + j, where $m \ge 0$ and $1 \le j \le k - 1$. We divide n vertices into two blocks of mk and j, which are denoted by $u_1, u_2, ..., u_{mk}$ and $u'_1, u'_2, ..., u'_j$ also $v_1, v_2, ..., v_{mk}$ and $v'_1, v'_2, ..., v'_j$. We note that |V(G)| = 2n and |E(G)| = 4n - 3.

To define k-cordial labeling $f:V(G)\to Z_k$ we consider the following cases.

Case 1: m > 0 and j = 0.

The labeling pattern of mk vertices is defined as follows:

Subcase 1: $k \equiv 0 \pmod{2}$, $1 \le i \le mk$.

$$f(u_i) = 2p_i - 1$$
; $i \equiv p_i \pmod{k}$; $1 \le i \le k$,

$$f(v_i) = 2p_i - 2$$
; $i \equiv p_i \pmod{k}$; $1 \le i \le k$,

Subcase 2: $k \equiv 1 \pmod{2}, 1 \leq i \leq mk$.

$$f(u_i) = 2p_i - 1$$
; $i \equiv p_i \pmod{k}$; $1 \le i \le \frac{k-1}{2}$,

$$f(u_i) = 2p_i$$
; $i \equiv p_i \pmod{k+1}$; $\frac{k-1}{2} + 1 \le i \le k$,

$$f(v_i) = 2p_i - 2$$
; $i \equiv p_i \pmod{k}$; $1 \le i \le \frac{k-1}{2} + 1$,

$$f(v_i) = 2p_i - 1$$
; $i \equiv p_i \pmod{k+1}$; $\frac{k-1}{2} + 2 \le i \le k$,

Case 2: $m \ge 0$ and $1 \le j \le k - 1$.

Subcase 1: $k \equiv 0 \pmod{2}$.

The labeling pattern of first block of mk vertices is defined as follows:

$$f(u_i) = 2p_i - 1$$
; $i \equiv p_i \pmod{k}$; $1 \le i \le k$,

$$f(v_i) = 2p_i - 2$$
; $i \equiv p_i \pmod{k}$; $1 \le i \le k$,

The labeling pattern of second block of j vertices is defined as follows:

$$f(u'_j) = 2j - 1$$
; $1 \le j \le k - 1$,

$$f(v_j') = 2j - 2$$
; $1 \le j \le k - 1$.

Subcase 2: $k \equiv 1 \pmod{2}$.

The labeling pattern of first block of mk vertices is defined as follows:

$$f(u_i) = 2p_i - 1$$
; $i \equiv p_i \pmod{k}$; $1 \le i \le \frac{k-1}{2}$,

$$f(u_i) = 2p_i$$
; $i \equiv p_i \pmod{k+1}$; $\frac{k-1}{2} + 1 \le i \le k$,

$$f(v_i) = 2p_i - 2$$
; $i \equiv p_i \pmod{k}$; $1 \le i \le \frac{k-1}{2} + 1$,

$$f(v_i) = 2p_i - 1$$
; $i \equiv p_i \pmod{k+1}$; $\frac{k-1}{2} + 2 \le i \le k$,

The labeling pattern of second block of j vertices is defined as follows:

$$f(u'_j) = 2j - 1$$
; $1 \le j \le \frac{k-1}{2}$,

$$f(u'_j) = 2j$$
; $\frac{k-1}{2} + 1 \le j \le k-1$,

$$f(v_i') = 2j - 2$$
; $1 \le j \le \frac{k-1}{2} + 1$,

$$f(v'_j) = 2j - 1$$
; $\frac{k-1}{2} + 2 \le j \le k - 1$.

In each possibility the graph under consideration satisfies the vertex conditions and edge conditions for k-cordial labeling.

Hence, alternate triangular belt $ATB(n)(\downarrow\uparrow\downarrow\uparrow...)$ is k-cordial for all n.

Example 2.4.

1. The Alternate Triangular Belt ATB(6) and its 8-cordial labeling is shown in Figure 3.

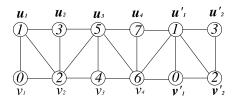


Figure 3. 8-cordial labeling of Alternate Triangular Belt ATB(6)

2. The Alternate Triangular Belt ATB(8) and its 7-cordial labeling is shown in Figure 4.

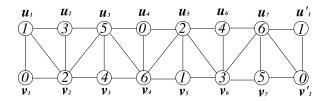


Figure 4. 7-cordial labeling of Alternate Triangular Belt ATB(8)

Theorem 2.5. The Braid graph B(n) is k-cordial for all n.

Proof. Let G = B(n) be the braid graph. The braid graph is obtained from a pair of paths P_n and P_n . Let $u_1, u_2, ..., u_n$ be the vertices of path P_n and $v_1, v_2, ..., v_n$ are the vertices of path P_n . To find braid graph join i^{th} vertex of path P_n with $(i+1)^{th}$ vertex of path P_n and i^{th} vertex of path P_n with $(i+2)^{th}$ vertex of path P_n with the new edges for all $1 \le i \le n-2$. Let n=mk+j, where $m \ge 0$ and $1 \le j \le k-1$. We divide n vertices into two blocks of mk and j, which are denoted by $u_1, u_2, ..., u_{mk}$ and $u'_1, u'_2, ..., u'_j$ also $v_1, v_2, ..., v_{mk}$ and $v'_1, v'_2, ..., v'_j$. We note that |V(G)| = 2n and |E(G)| = 4n-5.

To define k-cordial labeling $f:V(G)\to Z_k$ we consider the following cases.

Case 1: m > 0 and j = 0.

The labeling pattern of mk vertices is defined as follows:

Subcase 1: $k \equiv 0 \pmod{2}$, $1 \le i \le mk$.

$$f(u_i) = 2p_i - 1$$
; $i \equiv p_i \pmod{k}$; $1 \le i \le k$,

$$f(v_i) = 2p_i - 2$$
; $i \equiv p_i \pmod{k}$; $1 \le i \le k$,

Subcase 2: $k \equiv 1 \pmod{2}$, $1 \le i \le mk$.

$$f(u_i) = 2p_i - 1; i \equiv p_i \pmod{k}; 1 \le i \le \frac{k-1}{2},$$

$$f(u_i) = 2p_i$$
; $i \equiv p_i \pmod{k+1}$; $\frac{k-1}{2} + 1 \le i \le k$,

$$f(v_i) = 2p_i - 2$$
; $i \equiv p_i \pmod{k}$; $1 \le i \le \frac{k-1}{2} + 1$,

$$f(v_i) = 2p_i - 1$$
; $i \equiv p_i \pmod{k+1}$; $\frac{k-1}{2} + 2 \le i \le k$,

Case 2: $m \ge 0$ and $1 \le j \le k - 1$.

Subcase 1: $k \equiv 0 \pmod{2}$.

The labeling pattern of first block of mk vertices is defined as follows:

$$f(u_i) = 2p_i - 1$$
; $i \equiv p_i \pmod{k}$; $1 \le i \le k$,

$$f(v_i) = 2p_i - 2$$
; $i \equiv p_i \pmod{k}$; $1 \le i \le k$,

The labeling pattern of second block of j vertices is defined as follows:

$$f(u_j') = 2j - 1$$
; $1 \le j \le k - 1$,

$$f(v_i') = 2j - 2$$
; $1 \le j \le k - 1$.

Subcase 2: $k \equiv 1 \pmod{2}$.

The labeling pattern of first block of mk vertices is defined as follows:

$$f(u_i) = 2p_i - 1$$
; $i \equiv p_i \pmod{k}$; $1 \le i \le \frac{k-1}{2}$,

$$f(u_i) = 2p_i$$
; $i \equiv p_i \pmod{k+1}$; $\frac{k-1}{2} + 1 \le i \le k$,

$$f(v_i) = 2p_i - 2$$
; $i \equiv p_i \pmod{k}$; $1 \le i \le \frac{k-1}{2} + 1$,

$$f(v_i) = 2p_i - 1$$
; $i \equiv p_i \pmod{k+1}$; $\frac{k-1}{2} + 2 \le i \le k$,

The labeling pattern of second block of j vertices is defined as follows:

$$\begin{split} f(u_j') &= 2j-1 \; ; & 1 \leq j \leq \frac{k-1}{2}, \\ f(u_j') &= 2j \; ; & \frac{k-1}{2} + 1 \leq j \leq k-1, \\ f(v_j') &= 2j-2 \; ; & 1 \leq j \leq \frac{k-1}{2} + 1, \\ f(v_j') &= 2j-1 \; ; & \frac{k-1}{2} + 2 \leq j \leq k-1. \end{split}$$

In each possibility the graph under consideration satisfies the vertex conditions and edge conditions for k-cordial labeling. Hence, the braid graph B(n) is k-cordial for all n.

Example 2.6.

1. The Braid graph B(9) and its 10-cordial labeling is shown in Figure 5.

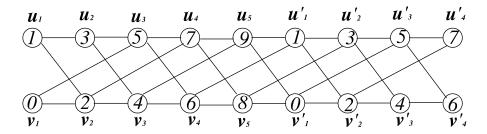


Figure 5. 10-cordial labeling of Braid graph B(9)

2. The Braid graph B(10) and its 9-cordial labeling is shown in Figure 6.

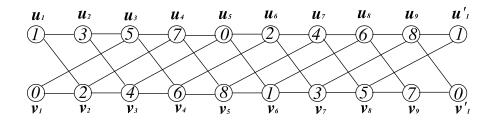


Figure 6. 9-cordial labeling of Braid graph B(10)

Theorem 2.7. The graph Z- P_n is k-cordial for all odd k and for all n.

Proof. Let $G=Z-P_n$ be the graph obtained from a pair of paths P_n and P_n . Let $v_1, v_2, ..., v_n$ be the vertices of path P_n and $u_1, u_2, ..., u_n$ are the vertices of path P_n . To find $Z-P_n$ join i^{th} vertex of path P_n with $(i+1)^{th}$ vertex of path P_n for all $1 \le i \le n-1$. Let n=mk+j, where $m \ge 0$ and $1 \le j \le k-1$. We divide n vertices into two blocks of mk and j, which are denoted by $u_1, u_2, ..., u_{mk}$ and $u'_1, u'_2, ..., u'_j$ also $v_1, v_2, ..., v_{mk}$ and $v'_1, v'_2, ..., v'_j$. We note that |V(G)| = 2n and |E(G)| = 3n-3.

The labeling pattern of first block of mk vertices is defined as follows.

$$f(u_i) = 2p_i - 1; i \equiv p_i \pmod{k}; 1 \le i \le \frac{k-1}{2},$$

$$f(u_i) = 2p_i; i \equiv p_i \pmod{k+1}; \frac{k-1}{2} + 1 \le i \le k,$$

$$f(v_i) = 2p_i - 2; i \equiv p_i \pmod{k}; 1 \le i \le \frac{k-1}{2} + 1,$$

$$f(v_i) = 2p_i - 1; i \equiv p_i \pmod{k+1}; \frac{k-1}{2} + 2 \le i \le k,$$

The labeling pattern of second block of j vertices is defined as follows:

$$\begin{split} f(u_j') &= 2j-1 \; ; & 1 \leq j \leq \frac{k-1}{2}, \\ f(u_j') &= 2j \; ; & \frac{k-1}{2} + 1 \leq j \leq k-1, \\ f(v_j') &= 2j-2 \; ; & 1 \leq j \leq \frac{k-1}{2} + 1, \\ f(v_j') &= 2j-1 \; ; & \frac{k-1}{2} + 2 \leq j \leq k-1. \end{split}$$

In each possibility the graph under consideration satisfies the vertex conditions and edge conditions for k-cordial labeling. Hence, the graph Z- P_n is k-cordial for all odd k and for all n.

Example 2.8. The graph Z- P_{10} and its 5-cordial labeling is shown in Figure 7.

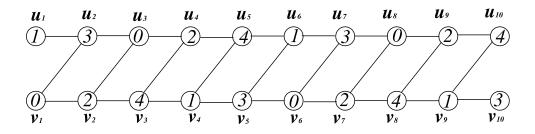


Figure 7. 5-cordial labeling of Z- P_{10}

3. Concluding Remarks

Graph labeling technique is a wide area of research. To investigate more graph families which admit k-cordial labeling for even and odd k is an open area of research.

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