International Journal of Mathematics And its Applications
Volume 5, Issue 4–E (2017), 663–667.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Revan Indices of Oxide and Honeycomb Networks

Research Article

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Abstract: There are many types of topological indices. Among degree based topological indices, Zagreb indices, Banhatti indices, Gourava indices are studied well. In this paper, we introduce the Revan indices and compute exact formulas for oxide and honeycomb networks.

MSC: 05C05, 05C07, 05C12, 05C35.

Keywords: Revan indices, molecular graph, oxide network, honeycomb network.

1. Introduction

Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to V. Let $\Delta(G)(\delta(G))$ denote the maximum(minimum) degree among the vertices of G. We refer to [1] for undefined term and notation. A molecular graph is a graph such that its vertices correspond to the atoms and the edges to the bonds. Chemical graph theory is a branch of Mathematical Chemistry which has an important effect on the development of the Chemical Sciences. A topological index is a numerical parameter mathematically derived from the graph structure. Numerous such topological indices have been considered in theoretical chemistry and have some applications, especially in $QSPR/QSAR$ research [2, 3]. The Revan vertex degree of a vertex v in G is defined as $r_G(v) = \Delta(G) + \delta(G) - d_G(v)$. The revan edge connecting the revan vertices u and v will be denoted by uv. We introduce the first and second Revan indices of a molecular graph G as follows:
The first and second Revan indices of a graph G are defined as

$$R_1(G) = \sum_{uv\in E(G)} [r_G(u) + r_G(v)], \qquad R_2(G) = \sum_{uv\in E(G)} r_G(u)\, r_G(v).$$

The first Revan vertex index of a graph G is defined as

$$R_{01}(G) = \sum_{u\in V(G)} r_G(u)^2.$$

Also we define the third Revan index of a graph G and defined it as

$$R_3(G) = \sum_{uv\in E(G)} |r_G(u) - r_G(v)|. \tag{1}$$

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The Zagreb indices were studied, for example, in [4–7]; the Banhatti indices were studied, for example, in [8–12]; the Gourava indices were studied, for example, in [13–16]. In this paper, we initiate a study of the Revan indices. For networks see [17] and references cited therein.

2. Results for Oxide Networks

Oxide networks are of vital importance in the study of silicate networks. An oxide network of dimension n is denoted by OX_n. A 5-dimensional oxide network is shown in Figure 1.

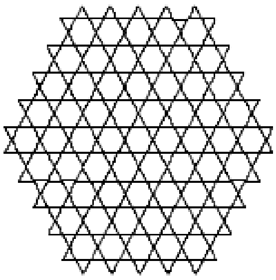

Figure 1. **A 5-dimensional oxide network**

Let G be the graph of oxide network OX_n. From Figure 1, it is easy to see that the vertices of OX_n are either of degree 2 or 4. By calculation, we obtain that G has $9n^2 + 3n$ vertices and $18n^2$ edges. We partition $V(G)$ into two sets, vertices of degree 2 and 4 respectively.

$$V_2 = \{u \in V(G) | d_G(u) = 2\}, \quad |V_2| = 6n.$$

$$V_4 = \{u \in V(G) | d_G(u) = 4\}, \quad |V_4| = 9n^2 - 3n.$$

Clearly we have $\Delta(G) + \delta(G) = 6$. For vertex $u \in V(G)$, $d_G(u) = 2$. Then $r_G(u) = 4$ and $|V_{r4}| = 6n$. For vertex $u \in V(G)$, $d_G(u) = 4$. Then $r_G(u) = 2$ and $|V_{r2}| = 9n^2 - 3n$. We compute the first Revan vertex index of OX_n.

Theorem 2.1. *The first Revan vertex index of an oxide network OX_n is given by $R_{01}(OX_n) = 36n^2 + 84n$.*

Proof. By definition, we have $R_{01}(G) = \sum\limits_{u \in V(G)} r_G(u)^2$. Thus

$$R_{01}(OX_n) = \sum_{V_{r_4}} r_G(u)^2 + \sum_{V_{r_2}} r_G(u)^2 = 6n \times 4^2 + \left(9n^2 - 3n\right) 2^2 = 36n^2 + 84n.$$

We compute the value of $R_1(OX_n)$, $R_2(OX_n)$, $R_3(OX_n)$ for oxide networks. □

Theorem 2.2. *Let OX_n be the oxide network. Then*

(1). $R_1(OX_n) = (3n + 1)\, 24n.$

(2). $R_2(OX_n) = (3n + 2)\, 24n.$

(3). $R_3(OX_n) = 24n.$

Proof. Let G be the graph of oxide network. In OX_n, by algebraic method, there are two types of edges based on the degree of the end vertices of each edge as follows:

$$E_{24} = \{uv \in E(G) | d_G(u) = 2, d_G(v) = 4\}, \quad |E_{24}| = 12n.$$

$$E_{44} = \{uv \in E(G) | d_G(u) = d_G(v) = 4\}, \qquad |E_{44}| = 18n^2 - 12n.$$

Thus there are two types of revan edges based on the degree of the end revan vertices of each revan edge as follows: we have $\Delta(G) + \delta(G) = 6$.

$$RE_{42} = \{uv \in E(G) | r_G(u) = 4, r_G(v) = 2\}, \quad |RE_{42}| = 12n.$$
$$RE_{22} = \{uv \in E(G) | r_G(u) = r_G(v) = 2\}, \qquad |RE_{22}| = 18n^2 - 12n.$$

(1) To compute $R_1(OX_n)$, we see that

$$\begin{aligned} R_1(G) &= \sum_{uv \in E(G)} [r_G(u) + r_G(v)] = \sum_{RE_{42}} [r_G(u) + r_G(v)] + \sum_{RE_{22}} [r_G(u) + r_G(v)] \\ &= 12n \times 6 + (18n^2 - 12n)4 = (3n + 1)24n. \end{aligned}$$

(2) To determine $R_2(OX_n)$, we see that

$$\begin{aligned} R_2(G) &= \sum_{uv \in E(G)} r_G(u) r_G(v) = \sum_{RE_{42}} r_G(u) r_G(v) + \sum_{RE_{22}} r_G(u) r_G(v) \\ &= 12n \times 8 + (18n^2 - 12n)4 = (3n + 2)24n. \end{aligned}$$

(3) To determine $R_3(OX_n)$, we see that

$$\begin{aligned} R_3(G) &= \sum_{uv \in E(G)} |r_G(u) - r_G(v)| = \sum_{RE_{42}} |r_G(u) - r_G(v)| + \sum_{RE_{22}} |r_G(u) - r_G(v)| \\ &= 12n \times 2 + (18n^2 - 6n) \times 0 = 24n. \end{aligned}$$

□

3. Results for Honeycomb Networks

If we recursively use hexagonal tiling in a particular pattern, honeycomb networks are formed. These networks are very useful in computer graphics and also in chemistry. A honeycomb network of dimension n is denoted by HC_n where n is the number of hexagons between central and boundary hexagon. A 4-dimensional honeycomb network is shown in Figure 2.

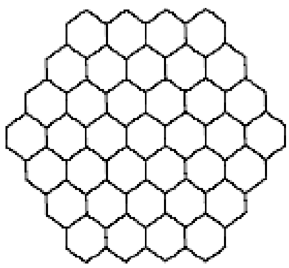

Figure 2. **A 4-dimensional honeycomb network**

Let H be the graph of honeycomb network HC_n. From Figure 2, it is easy to see that the vertices of HC_n are either of degree 2 or 3. By calculation, we obtain that H has $6n^2$ vertices and $9n^2 - 3n$ edges. We partition $V(H)$ into two sets, vertices of degree 2 and 3 respectively.

$$V_2 = \{u \in V(H) | d_H(u) = 2\}, \quad |V_2| = 6n.$$

$$V_3 = \{u \in V(H) | d_H(u) = 3\}, \quad |V_3| = 6n^2 - 6n.$$

Clearly we have $\Delta(G) + \delta(G) = 5$.

$$V_{r_3} = \{u \in V(H), d_H(u) = 2 \Rightarrow r_H(u) = 3\}, \quad |V_{r_3}| = 6n.$$
$$V_{r_2} = \{u \in V(H), d_H(u) = 3 \Rightarrow r_H(u) = 2\}, \quad |V_{r_2}| = 6n^2 - 6n.$$

We compute the first Revan vertex index of HC_n.

Theorem 3.1. *The first Revan vertex index of honeycomb network HC_n is given by $R_{01}(HC_n) = 24n^2 + 30n$.*

Proof. By Definition, we have $R_{01}(H) = \sum\limits_{u \in V(H)} r_H(u)^2$. Thus

$$\begin{aligned} R_{01}(HC_n) &= \sum_{V_{r_3}} r_H(u)^2 + \sum_{V_{r_2}} r_H(u)^2 \\ &= 6n \times 3^2 + (6n^2 - 6n)2^2 = 24n^2 + 30n. \end{aligned}$$

□

We compute the values of $R_1(HC_n)$, $R_2(HC_n)$, $R_3(HC_n)$ for honeycomb networks.

Theorem 3.2. *Let HC_n be the honeycomb network. Then*

(1). $R_1(HC_n) = 36n^2$,

(2). $R_2(HC_n) = 36n^2 + 12n + 6$,

(3). $R_3(HC_n) = 12n - 12$.

Proof. Let H be the graph of honeycomb network. In HC_n, by algebraic method, there are three types of edges based on the degree of the end vertices of each edge as follows:

$$E_{22} = \{uv \in E(H) | d_H(u) = d_H(v) = 2\}, \qquad |E_{22}| = 6.$$
$$E_{23} = \{uv \in E(H) | d_H(u) = 2, d_H(v) = 3\}, \quad |E_{23}| = 12n - 12.$$
$$E_{33} = \{uv \in E(H) | d_H(u) = d_H(v) = 3\}, \qquad |E_{33}| = 9n^2 - 15n + 6.$$

Thus there are three types of revan edges based on the degree of the end revan vertices of each revan edge as follows: we have $\Delta(H) + \delta(H) = 5$.

$$RE_{33} = \{uv \in E(H) | r_H(u) = r_H(v) = 3\}, \qquad |RE_{33}| = 6.$$
$$RE_{32} = \{uv \in E(H) | r_H(u) = 3, r_H(v) = 2\}, \quad |RE_{32}| = 12n - 12.$$
$$RE_{22} = \{uv \in E(H) | r_H(u) = r_H(v) = 2\}, \qquad |RE_{22}| = 9n^2 - 15n + 6.$$

(1) To compute $R_1(HC_n)$, we see that

$$\begin{aligned} R_1(H) &= \sum_{uv \in E(H)} [r_H(u) + r_H(v)] = \sum_{RE_{33}} [r_H(u) + r_H(v)] + \sum_{RE_{32}} [r_H(u) + r_H(v)] + \sum_{RE_{22}} [r_H(u) + r_H(v)] \\ &= 6 \times 6 + (12n - 12)5 + (9n^2 - 15n + 6)4 = 36n^2. \end{aligned}$$

(2) To determine $R_2(HC_n)$, we see that

$$R_2(H) = \sum_{uv \in E(H)} r_H(u)\, r_H(v) = \sum_{RE_{33}} r_H(u)\, r_H(v) + \sum_{RE_{32}} r_H(u)\, r_H(v) + \sum_{RE_{22}} r_H(u)\, r_H(v)$$

$$= 6 \times 9 + (12n - 12)6 + (9n^2 - 15n + 6)4 = 36n^2 + 12n + 6.$$

(3) To determine $R_3(HC_n)$, we see that

$$R_3(H) = \sum_{uv \in E(H)} |r_H(u) - r_H(v)| = \sum_{RE_{33}} |r_H(u) - r_H(v)| + \sum_{RE_{32}} |r_H(u) - r_H(v)| + \sum_{RE_{22}} |r_H(u) - r_H(v)|$$

$$= 6 \times 0 + (12n - 12) + (9n^2 - 15n + 6)0 = 12n - 12.$$

□

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