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On Nano $\wedge_{rg}$-closed Sets

Research Article

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Abstract: In this paper, we introduce nano $\wedge_{rg}$-closed sets in nano topological spaces. Some properties of nano $\wedge_{rg}$-closed sets and nano $\wedge_{rg}$-open sets are weaker forms of nano closed sets and nano open sets.

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1. Introduction and Preliminaries

In 2017, Rajasekaran et.al [4] introduced the notion of nano $\wedge_r$-sets in nano topological spaces and nano $\wedge_r$-set is a set H which is equal to its nano kernel and we introduced the notion of nano λ_r-closed set and nano λ_r-open sets. In this paper to introduce new classes of sets called nano $\wedge_{rg}$-closed sets and nano $\wedge_{rg}$-open sets in nano topological spaces. We also some properties of such sets and nano $\wedge_{rg}$-closed sets and nano $\wedge_{rg}$-open sets are weaker forms of nano closed sets and nano open sets.

Throughout this paper $(U, \tau_R(X))$ (or X) represent nano topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset H of a space $(U, \tau_R(X))$, $Ncl(H)$ and $Nint(H)$ denote the nano closure of H and the nano interior of H respectively. We recall the following definitions which are useful in the sequel.

Definition 1.1 ([3]). *Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.*

(1). The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.

(2). The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \cap X \neq \phi\}$.

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(3). The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Property 1.2 ([2]). *If (U, R) is an approximation space and $X, Y \subseteq U$; then*

(1). $L_R(X) \subseteq X \subseteq U_R(X)$;

(2). $L_R(\phi) = U_R(\phi) = \phi$ and $L_R(U) = U_R(U) = U$;

(3). $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$;

(4). $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$;

(5). $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$;

(6). $L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$;

(7). $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$;

(8). $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$;

(9). $U_R U_R(X) = L_R U_R(X) = U_R(X)$;

(10). $L_R L_R(X) = U_R L_R(X) = L_R(X)$.

Definition 1.3 ([2]). *Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by the Property 1.2, $R(X)$ satisfies the following axioms:*

(1). U and $\phi \in \tau_R(X)$,

(2). The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,

(3). The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called as nano open sets and $[\tau_R(X)]^c$ is called as the dual nano topology of $[\tau_R(X)]$.

Remark 1.4 ([2]). *If $[\tau_R(X)]$ is the nano topology on U with respect to X, then the set $B = \{U, \phi, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.*

Definition 1.5 ([2]). *If $(U, \tau_R(X))$ is a nano topological space with respect to X and if $H \subseteq U$, then the nano interior of H is defined as the union of all nano open subsets of H and it is denoted by $Nint(H)$. That is, $Nint(H)$ is the largest nano open subset of H. The nano closure of H is defined as the intersection of all nano closed sets containing H and it is denoted by $Ncl(H)$. That is, $Ncl(H)$ is the smallest nano closed set containing H.*

Definition 1.6 ([2]). *A subset H of a nano topological space $(U, \tau_R(X))$ is called nano regular-open $H = Nint(Ncl(H))$. The complement of the above mentioned set is called their respective closed set.*

Definition 1.7 ([4]). *Let H be a subset of a space $(U, \tau_R(X))$ is called nano r-Kernel of the set H, denoted by $\mathcal{N}r$-$Ker(H)$, is the intersection of all nano regular-open supersets of H.*

Definition 1.8 ([4]). *A subset H of a space $(U, \tau_R(X))$ is called*

(1). nano $\wedge_r$-set if $H = \mathcal{N}r$-$Ker(H)$.

(2). nano λ_r-closed if $H = L \cap F$ where L is a nano $\wedge_r$-set and F is nano closed.

Definition 1.9. *A subset H of a nano topological space $(U, \tau_R(X))$ is called;*

(1). nano g-closed [1] if $Ncl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano open.

(2). nano rg-closed set [5] if $Ncl(H) \subseteq G$ whenever $H \subseteq G$ and G is nano regular-open.

Lemma 1.10 ([4]). *In a space $(U, \tau_R(X))$,*

(1). Every nano closed set is nano λ_r-closed.

(2). Every nano $\wedge_r$-set is nano λ_r-closed.

Remark 1.11 ([4]). *In a nano topological space, the concepts of nano rg-closed sets and nano λ_r-closed sets are independent.*

2. Nano $\wedge_{rg}$-closed Sets

Definition 2.1. *A subset H of a space $(U, \tau_R(X))$ is called nano λ_r-open if $H^c = U - H$ is nano λ_r-closed.*

Example 2.2. *Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{b, d\}$. Then the nano topology $\tau_R(X) = \{\phi, \{b\}, \{c, d\}, \{b, c, d\}, U\}$. Then $\{b\}$ is nano λ_r-open.*

Definition 2.3. *A subset H of a space $(U, \tau_R(X))$ is called a nano $\wedge_{rg}$-closed set if $Ncl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano λ_r-open. The complement of nano $\wedge_{rg}$-open if $H^c = U - H$ is nano $\wedge_{rg}$-closed.*

Example 2.4. *In Example 2.2,*

(1). then $\{a, b\}$ is nano $\wedge_{rg}$-closed set.

(2). then $\{c, d\}$ is nano $\wedge_{rg}$-open set.

Lemma 2.5. *In a space $(U, \tau_R(X))$, every nano open set is nano $\wedge_{rg}$-open but not conversely*

Remark 2.6. *The converse of statements in Lemma 2.5 are not necessarily true as seen from the following Example.*

Example 2.7. *In Example 2.2, then $\{b, c\}$ is nano $\wedge_{rg}$-open but not nano open.*

Remark 2.8. *The following example shows that the concepts of nano $\wedge_{rg}$-closed sets and nano λ_r-closed are independent for each other.*

Example 2.9. *In Example 2.2,*

(1). then $\{a, d\}$ is nano $\wedge_{rg}$-closed but not nano λ_r-closed.

(2). then $\{b\}$ is nano λ_r-closed but not nano $\wedge_{rg}$-closed.

Theorem 2.10. *In a space $(U, \tau_R(X))$, the union of two nano $\wedge_{rg}$-closed sets is nano $\wedge_{rg}$-closed.*

Proof. Let $H \cup Q \subseteq G$, then $H \subseteq G$ and $Q \subseteq G$ where G is nano λ_r-open. As H and Q are nano $\wedge_{rg}$-closed, $Ncl(H) \subseteq G$ and $Ncl(Q) \subseteq G$. Hence $Ncl(H \cup Q) = Ncl(H) \cup Ncl(Q) \subseteq G$. □

Example 2.11. *In Example 2.2, then $H = \{a, b\}$ and $Q = \{a, d\}$ is nano $\wedge_{rg}$-closed. Clearly $H \cup Q = \{a, b, d\}$ is nano $\wedge_{rg}$-closed.*

Theorem 2.12. *In a space $(U, \tau_R(X))$, the intersection of two nano $\wedge_{rg}$-open sets are nano $\wedge_{rg}$-open.*

Example 2.13. *In Example 2.2, then $H = \{d\}$ and $Q = \{c, d\}$ is nano $\wedge_{rg}$-open. Clearly $H \cap Q = \{d\}$ is nano $\wedge_{rg}$-open.*

Remark 2.14. *In a space $(U, \tau_R(X))$, the intersection of two nano $\wedge_{rg}$-closed sets but not nano $\wedge_{rg}$-closed.*

Example 2.15. *Let $U = \{p, q, r\}$ with $U/R = \{\{p\}, \{q, r\}\}$ and $X = \{p\}$. Then the nano topology $\tau_R(X) = \{\phi, \{p\}, U\}$, Then $H = \{p, q\}$ and $Q = \{p, r\}$ is nano $\wedge_{rg}$-closed. Clearly $H \cap Q = \{p\}$ is but not nano $\wedge_{rg}$-closed.*

Theorem 2.16. *In a space $(U, \tau_R(X))$ is nano $\wedge_{rg}$-closed, then $Ncl(H) - H$ contains no nonempty nano closed.*

Proof. Let P be a nano closed subset contains in $Ncl(H) - H$. Clearly $H \subseteq P^c$ where H is nano $\wedge_{rg}$-closed and P^c is an nano open set of U. Thus $Ncl(H) \subseteq P^c$ (or) $P \subseteq (Ncl(H))^c$. Then $P \subseteq (Ncl(H))^c \cap (Ncl(H) - H) \subseteq (Ncl(H))^c \cap Ncl(H) = \phi$. This is show that $P = \phi$. □

Theorem 2.17. *A subset H of a space $(U, \tau_R(X))$ is nano $\wedge_{rg}$-closed $\iff$ $Ncl(H) - H$ contains no nonempty nano λ_r-closed.*

Proof. Necessity. Assume that H is nano $\wedge_{rg}$-closed. Let K be a nano λ_r-closed subset of $Ncl(H) - H$. Then $H \subseteq K^c$. Since H is nano $\wedge_{rg}$-closed, we have $Ncl(H) \subseteq K^c$. Consequently $K \subseteq (Ncl(H))^c$. Hence $K \subseteq Ncl(H) \cap (Ncl(H))^c = \phi$. Therefore K is empty.

Sufficiency. Assume that $Ncl(H) - H$ contains no nonempty nano λ_r-closed sets. Let $H \subseteq C$ and C be a nano λ_r-open. If $Ncl(H) \nsubseteq C$, then $Ncl(H) \cap C^c$ is a nonempty nano λ_r-closed subset of $Ncl(H) - H$. Therefore H is nano $\wedge_{rg}$-closed. □

Theorem 2.18. *In a space $(U, \tau_R(X))$, if H is a nano $\wedge_{rg}$-closed and $H \subseteq Q \subseteq Ncl(H)$, then Q is a nano $\wedge_{rg}$-closed.*

Proof. Let $H \subseteq Q$ and $Ncl(Q) \subseteq Ncl(H)$. Hence $(Ncl(Q) - Q) \subseteq (Ncl(H) - H)$. But by Theorem 2.17, $Ncl(H) - H$ contains no nonempty nano λ_r-closed subset of U and hence neither does $Ncl(B) - B$. By Theorem 2.17, Q is nano $\wedge_{rg}$-closed. □

Theorem 2.19. *In a space $(U, \tau_R(X))$, if H is nano λ_r-open and nano $\wedge_{rg}$-closed, then hence H is nano closed.*

Proof. Since H is nano λ_r-open and nano λ_r-closed, $Ncl(H) \subseteq H$ and hence H is nano closed. □

Theorem 2.20. *For each $x \in U$, either $\{x\}$ is nano λ_r-closed (or) $\{x\}^c$ is nano $\wedge_{rg}$-closed.*

Proof. Assume $\{x\}$ is not nano λ_r-closed. Then $\{x\}^c$ is not nano λ_r-open and the only nano λ_r-open set containing $\{x\}^c$ is the space of U itself. Therefore $Ncl(\{x\}^c) \subseteq U$ and so $\{x\}^c$ is nano $\wedge_{rg}$-closed. □

Theorem 2.21. *In a space $U, \tau_R(X)$, H is nano $\wedge_{rg}$-open $\iff$ $P \subseteq Nint(H)$ whenever P is nano λ_r-closed and $P \subseteq H$.*

Proof. Assume that $P \subseteq Nint(H)$ whenever P is nano λ_r-closed and $P \subseteq H$. Let $H^c \subseteq C$, where C is nano λ_r-open. Hence $C^c \subseteq H$. By assumption $C^c \subseteq Nint(H)$ which implies that $(Nint(H))^c \subseteq C$, so $Ncl(H^c) \subseteq C$. Hence H^c is nano $\wedge_{rg}$-closed that is, H is nano $\wedge_{rg}$-open.

Conversely, let H be nano $\wedge_{rg}$-open. Then H^c is nano $\wedge_{rg}$-closed. Also let P be a nano λ_r-closed set contained in H. Then P^c is nano λ_r-open. Therefore whenever $H^c \subseteq P^c$, $Ncl(H^c) \subseteq P^c$. This implies that $P \subseteq (Ncl(H^c))^c = Nint(H)$. Thus $H \subseteq Nint(H)$. □

Theorem 2.22. *In a space $(U, \tau_R(X))$, H is $\wedge_{rg}$-open $\iff$ $C = U$ whenever C is nano λ_r-open and $Nint(H) \cup H^c \subseteq C$.*

Proof. Let H be a nano $\wedge_{rg}$-open, C be a nano λ_r-open and $Nint(H) \cup H^c \subsetneq C$. Then $C^c \subsetneq (Nint(H))^c \cap (H^c)^c = (Nint(H))^c - H^c) = Ncl(H^c) - H^c$. Since H^c is nano $\wedge_{rg}$-closed and C^c is nano λ_r-closed, by Theorem 2.17 it follows that $C^c = \phi$. Therefore $U = C$. Conversely, suppose that P is nano λ_r-closed and $P \subsetneq H$. Then $Nint(H) \cup H^c \subsetneq Nint(H) \cup P^c$. It follows that $Nint(H) \cup P^c = U$ and hence $P \subsetneq Nint(H)$. Therefore H is nano $\wedge_{rg}$-open. □

Theorem 2.23. *In a space $(U, \tau_R(X))$, if $Nint(H) \subseteq Q \subseteq H$ and H is nano $\wedge_{rg}$-open, then Q is nano $\wedge_{rg}$-open.*

Proof. Assume $Nint(H) \subseteq Q \subseteq H$ and H is nano $\wedge_{rg}$-open. Then $H^c \subseteq Q^c \subseteq Ncl(H^c)$ and H^c is nano $\wedge_{rg}$-closed. By Theorem 2.18, Q is nano $\wedge_{rg}$-open. □

Theorem 2.24. *In a space $(U, \tau_R(X))$, H is nano $\wedge_{rg}$-closed $\iff$ $Ncl(H) - H$ is nano $\wedge_{rg}$-open.*

Proof. Necessity. Assume that H is nano $\wedge_{rg}$-closed. Let $P \subseteq Ncl(H) - H$, where P is nano λ_r-closed. By Theorem 2.17, $P = \phi$, Therfore $P \subseteq Nint(Ncl(H) - H)$ and by Theorem 2.21, $Ncl(H) - H$ is nano $\wedge_{rg}$-open.
Sufficiency. Let $H \subseteq C$ where C is a nano λ_r-open set. Then $Ncl(H) \cap C^c \subseteq Ncl(H) \cap H^c = Ncl(H) - H$. Since $Ncl(H) \cap C^c$ is nano λ_r-closed and $Ncl(H) - H$ is nano $\wedge_{rg}$-open, by Theorem 2.21, we have $Ncl(H) \cap C^c \subseteq Nint(Ncl(H) - H) = \phi$. Hence H is nano $\wedge_{rg}$-closed. □

Theorem 2.25. *In a nano topological space $(U, \tau_R(X))$, the following properties are equivalent:*

(1). H is nano $\wedge_{rg}$-closed.

(2). $Ncl(H) - H$ contains no nonempty nano λ_r-closed set.

(3). $Ncl(H) - H$ is nano $\wedge_{rg}$-open.

Proof. This follows from by Theorems 2.17 and 2.24. □

Definition 2.26. *A subset H of a space $(U, \tau_R(X))$ is called*

(1). a nano ${}_{rg}\wedge$-closed set if $N\lambda_r cl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano open.

(2). a nano $\wedge_r$-g-closed set if $N\lambda_r cl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano λ_r-open.

The complement of the above mentioned sets are called their respective open sets.

Example 2.27. *In Example 2.15, then $\{q\}$ is nano ${}_{rg}\wedge$-closed and nano $\wedge_r$-g-closed.*

Remark 2.28. *For a subset of a space $(U, \tau_R(X))$, we have the following implications:*

$$\begin{array}{ccc}
\textit{nano closed} & \rightarrow & \textit{nano } \lambda_r\textit{-closed} \\
\downarrow & & \downarrow \\
\textit{nano } \wedge_{rg}\textit{-closed} & \rightarrow & \textit{nano } \wedge_r\textit{-g-closed} \\
\downarrow & & \downarrow \\
\textit{nano g-closed} & \rightarrow & \textit{nano } {}_{rg}\wedge\textit{-closed}
\end{array}$$

None of the above implications are reversible.

Theorem 2.29. *In a space $(U, \tau_R(X))$, H is nano $\wedge_{rg}$-closed $\iff$ $N\lambda_r cl(\{x\}) \cap H \neq \phi$ for every $x \in Ncl(H)$.*

Proof. Necessity. Suppose that $N\lambda_r cl(\{x\}) \cap H = \phi$ for some $x \in Ncl(H)$. Then $U - N\lambda_r cl(\{x\})$ is a nano λ_r-open set containing H. Furthermore, $x \in Ncl(H) - (U - N\lambda_r cl(\{x\}))$ and hence $Ncl(H) \nsubseteq U - N\lambda_r cl(\{x\})$. This shows that H is not nano $\wedge_{rg}$-closed.

Sufficiency. Suppose that H is not nano $\wedge_{rg}$-closed. There exist a nano λ_r-open set G containing H such that $Ncl(H) - G \neq \phi$. There exist $x \in Ncl(H)$ such that $x \notin G$, hence $N\lambda_r cl(\{x\}) \cap G = \phi$. Therefore, $N\lambda_r cl(\{x\}) \cap H = \phi$ for some $x \in Ncl(H)$. □

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