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Normal-Rayleigh Stochastic Production Frontier Model

Research Article

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Abstract: In the production field, Stochastic Production Frontier Analysis (SPFA) plays a principal role in the measurement of technical efficiency. This paper presents the derivation of Normal-Rayleigh Stochastic Production Frontier Model (NR-SPFM)for the estimation of technical efficiency. By using Maximum likelihood estimation method, the parameters of the model were evaluated.

Keywords: Rayleigh distribution, Normal-Rayleigh stochastic production frontier model, Technical efficiency, Maximum likelihood estimation.

1. Introduction

Stochastic Production Frontier Analysis (SPFA) is a mathematical modelling, used to estimate the technical efficiency scores of individual firm. Aigner et al., [2], Meeusen van den Broeck [7] and Battese and Corra [3] simultaneously introduced SPFA in the form $y = f(x, \beta)e^{v-u}$, where y is scalar output, x is a vector of inputs and β is vector of technology parameter. This model consist of two error terms, inefficiency error u and noise error v.

The noise component v is a random shock which is assumed to be independent and identically distributed normal random variable with mean 0 and variance σ_v^2 and u is a non-negative variable which represents the unmeasured variables such as weather, walkout, economic difficulties etc. Distributional assumptions play a vital role in the estimation of technical efficiency of each firm. Battese and Corra [3] modelled technical inefficiency by half-normal distribution, Meeusen van den Broeck [3] considered exponential distribution, Aigner et al., [2] followed half-normal and exponential distributions, when Greene [4] considered Gamma, Stevenson [8] had a comparative study with Gamma and Truncated normal distributions and Tsionas [10] used Weibull distribution. So far most of the studies are carried out with the usual half normal, gamma and truncated distribution.

For technical inefficiency term u, Rayleigh distribution is assigned in this paper. Rayleigh distribution is a one parameter continuous probability distribution with non-zero mode. Koopmans [6] conveyed the technical efficiency as achieving maximum output from given level of inputs. The output oriented technical efficiency is the ratio of observed output to the maximum feasible output, whichcan be expressed by the formula

$$TE_i = \frac{y_i}{f(x_i, \beta)\exp\{v_i\}}$$

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(Kumbhakar and Lovell, [9]). Usually,

$$TE_i = \exp\left(-E\left(\frac{u_i}{\varepsilon_i}\right)\right)$$

which can be calculated by exponential conditional expectation of u given the composed error term ε (Jondrow et al., [5]).

2. Normal-Rayleigh Stochastic Production Frontier Model (NR-SPFM)

For the estimation of NRSPFM, the following assumptions in the distribution were made.

(1). $v \sim iid\ N(0, s_v^2)$

(2). $u \sim iid$ Rayleigh distribution in the interval $(0, \infty)$

(3). u and v are distributed independently of each other and of the regressors.

The probability density function of v is given by

$$f(v) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{v^2}{2\sigma^2}},\ -\infty < v < \infty \tag{1}$$

The probability density function of $u \geq 0$ is given by

$$f(u) = \frac{u}{\sigma_u^2} e^{-\frac{u^2}{2\sigma_u^2}},\ u > 0 \tag{2}$$

Since u and v are independent. The joint density function of u and v is

$$\begin{aligned} f(u,v) &= f(u).f(v) \\ &= \frac{1}{\sqrt{2\pi\sigma_u^2\sigma_v}} u e^{-\frac{1}{2}\left(\frac{u}{\sigma_u^2}^2 + \frac{v}{\sigma_v^2}^2\right)} \end{aligned} \tag{3}$$

Making the transformation, $\varepsilon = v - u \Rightarrow v = u + \varepsilon$. Therefore, the joint density function of u and ε is

$$\begin{aligned} f(u,\varepsilon) &= \frac{1}{\sqrt{2\pi}\sigma_u^2\sigma_v} u e^{-\frac{1}{2}\left(\frac{u}{\sigma_u^2}^2 + \frac{(u+\varepsilon)}{\sigma_v^2}^2\right)} \\ &= \frac{1}{\sqrt{2\pi}\sigma_u^2\sigma_v} u e^{-\frac{1}{2}\left(\frac{u^2\sigma_v^2 + \sigma_u^2(u+\varepsilon)^2}{\sigma_v^2\sigma_u^2}\right)} \\ &= \frac{1}{\sqrt{2\pi}\sigma_u^2\sigma_v} u e^{-\frac{1}{2}\left(\frac{u^2\sigma_v^2 + u^2\sigma_u^2 + \varepsilon^2\sigma_u^2 + 2u\varepsilon\sigma_u^2}{\sigma_v^2\sigma_u^2}\right)} \\ &= \frac{1}{\sqrt{2\pi}\sigma_u^2\sigma_v} u e^{-\frac{1}{2}\left(\frac{u^2\left(\sigma_v^2+\sigma_u^2\right) + 2u\varepsilon\sigma_u^2}{\sigma_v^2\sigma_u^2}\right)} \quad e^{\frac{-\varepsilon^2\sigma_u^2}{\sigma_v^2\sigma_u^2}} \\ &= \frac{1}{\sqrt{2\pi}\sigma_u^2\sigma_v} e^{\frac{-\varepsilon^2}{2\sigma_v^2}} u e^{-\frac{1}{2}\left(\frac{u^2\left(\sigma_v^2+\sigma_u^2\right) + 2u\varepsilon\sigma_u^2}{\sigma_v^2\sigma_u^2}\right)} \\ &= \frac{1}{\sqrt{2\pi}\sigma_u^2\sigma_v} e^{\frac{-\varepsilon^2}{2\sigma_v^2}} u e^{-\frac{\left(\sigma_v^2+\sigma_u^2\right)}{2\sigma_v^2\sigma_u^2}\left(u^2 + \frac{2u\varepsilon\sigma_u^2}{\left(\sigma_v^2+\sigma_u^2\right)}\right)} \\ &= \frac{1}{\sqrt{2\pi}\sigma_u^2\sigma_v} e^{\frac{-\varepsilon^2}{2\sigma_v^2}} u e^{-\frac{\left(\sigma_v^2+\sigma_u^2\right)}{2\sigma_v^2\sigma_u^2}\left(\left(u^2 + \frac{2u\varepsilon\sigma_u^2}{\left(\sigma_v^2+\sigma_u^2\right)} + \left(\frac{\varepsilon\sigma_u^2}{\left(\sigma_v^2+\sigma_u^2\right)}\right)^2\right) - \left(\frac{\varepsilon\sigma_u^2}{\left(\sigma_v^2+\sigma_u^2\right)}\right)^2\right)} \\ &= \frac{1}{\sqrt{2\pi}\sigma_u^2\sigma_v} e^{\frac{-\varepsilon^2}{2\sigma_v^2}} u e^{\frac{\left(\sigma_v^2+\sigma_u^2\right)}{2\sigma_v^2\sigma_u^2}\left(\frac{\varepsilon\sigma_u^2}{\left(\sigma_v^2+\sigma_u^2\right)}\right)^2} \quad e^{\left(\left(u + \left(\frac{\varepsilon\sigma_u^2}{\left(\sigma_v^2+\sigma_u^2\right)}\right)\right)^2\right)} \end{aligned} \tag{4}$$

$$= \frac{1}{\sqrt{2\pi}\sigma_u^2\sigma_v} e^{\frac{-\varepsilon^2}{2\sigma_v^2}} u e^{\frac{\left(\frac{\varepsilon\sigma_u^2}{\left(\sigma_v^2+\sigma_u^2\right)}\right)^2}{\frac{2\sigma_v^2\sigma_u^2}{\left(\sigma_v^2+\sigma_u^2\right)}}} e^{\left(\left(u-\left(-\frac{\varepsilon\sigma_u^2}{\left(\sigma_v^2+\sigma_u^2\right)}\right)\right)^2\right)} \tag{5}$$

Let $\mu = \frac{\varepsilon\sigma_u^2}{(\sigma_v^2+\sigma_u^2)}$, $\sigma^2 = \frac{\sigma_v^2\sigma_u^2}{(\sigma_v^2+\sigma_u^2)}$. Therefore

$$f(u,\varepsilon) = \frac{1}{\sqrt{2\pi}\sigma_u^2\sigma_v} u e^{\frac{-\varepsilon^2}{2\sigma_v^2}} e^{\frac{\mu^2}{2\sigma^2}} e^{\frac{(u-\mu)^2}{2\sigma^2}} \tag{6}$$

$$f(\varepsilon) = \frac{1}{\sqrt{2\pi}\sigma_u^2\sigma_v} e^{\frac{-\varepsilon^2}{2\sigma_v^2}} e^{\frac{\mu^2}{2\sigma^2}} \int_0^\infty u \; e^{\frac{(u-\mu)^2}{2\sigma^2}} du$$

Let $t = \frac{u-\mu}{\sigma} \Rightarrow t\sigma + \mu = u$, $dt = \frac{du}{\sigma} \Rightarrow \sigma\; dt = du$. Therefore

$$f(\varepsilon) = \frac{1}{\sqrt{2\pi}\sigma_u^2\sigma_v} e^{\frac{-\varepsilon^2}{2\sigma_v^2}} e^{\frac{\mu^2}{2\sigma^2}} \int_{\left(-\frac{\mu}{\sigma}\right)}^\infty (t\sigma+\mu) e^{-\frac{t^2}{2}} \sigma dt \tag{7}$$

$$= \frac{1}{\sqrt{2\pi}\sigma_u^2\sigma_v} e^{\frac{-\varepsilon^2}{2\sigma_v^2}} e^{\frac{\mu^2}{2\sigma^2}} \sigma\left(\sigma \int_{\left(-\frac{\mu}{\sigma}\right)}^\infty t e^{-\frac{t^2}{2}} dt + \mu \int_{\left(-\frac{\mu}{\sigma}\right)}^\infty e^{-\frac{t^2}{2}} dt\right)$$

$$= \frac{1}{\sigma_u^2\sigma_v} e^{\frac{-\varepsilon^2}{2\sigma_v^2}} e^{\frac{\mu^2}{2\sigma^2}} \sigma\left(\sigma \frac{1}{\sqrt{2\pi}} \int_{\left(-\frac{\mu}{\sigma}\right)}^\infty t e^{-\frac{t^2}{2}} dt + \mu \frac{1}{\sqrt{2\pi}} \int_{\left(-\frac{\mu}{\sigma}\right)}^\infty e^{-\frac{t^2}{2}} dt\right)$$

$$= \frac{1}{\sigma_u^2\sigma_v} e^{\frac{-\varepsilon^2}{2\sigma_v^2}} e^{\frac{\mu^2}{2\sigma^2}} \sigma\left(\sigma \frac{1}{\sqrt{2\pi}} \int_{\left(-\frac{\mu}{\sigma}\right)}^\infty t e^{-\frac{t^2}{2}} dt + \mu\left(1 - \Phi\left(-\frac{\mu}{\sigma}\right)\right)\right) \tag{8}$$

$$= \frac{1}{\sigma_u^2\sigma_v} e^{\frac{-\varepsilon^2}{2\sigma_v^2}} e^{\frac{\mu^2}{2\sigma^2}} \sigma\left(\sigma \frac{1}{\sqrt{2\pi}} \int_{\left(-\frac{\mu}{\sigma}\right)}^\infty t e^{-\frac{t^2}{2}} dt + \mu\left(\Phi\left(\frac{\mu}{\sigma}\right)\right)\right)$$

Let $u = \frac{t^2}{2}$, $du = tdt$ when $t = -\frac{\mu}{\sigma} \Rightarrow u = \frac{\left(\frac{\mu}{\sigma}\right)^2}{2}$, when $t = \infty \Rightarrow u = \infty$. Therefore

$$f(\varepsilon) = \frac{1}{\sigma_u{}^2\sigma_v} e^{\frac{-\varepsilon^2}{2\sigma_v{}^2}} e^{\frac{\mu^2}{2\sigma^2}} \sigma\left(\sigma \frac{1}{\sqrt{2\pi}} \int_{\frac{\left(\frac{\mu}{\sigma}\right)^2}{2}}^{\infty} e^{-u} du + \mu\left(\Phi\left(\frac{\mu}{\sigma}\right)\right)\right) \tag{9}$$

$$= \frac{1}{\sigma_u{}^2\sigma_v} e^{\frac{-\varepsilon^2}{2\sigma_v{}^2}} e^{\frac{\mu^2}{2\sigma^2}} \sigma\left(\sigma \frac{1}{\sqrt{2\pi}} \left(-e^{-u}\right)_{\frac{\left(\frac{\mu}{\sigma}\right)^2}{2}}^{\infty} + \mu\left(\Phi\left(\frac{\mu}{\sigma}\right)\right)\right)$$

$$= \frac{1}{\sigma_u{}^2\sigma_v} e^{\frac{-\varepsilon^2}{2\sigma_v{}^2}} e^{\frac{\mu^2}{2\sigma^2}} \sigma\left(\sigma \frac{1}{\sqrt{2\pi}} e^{-\frac{\left(\frac{\mu}{\sigma}\right)^2}{2}} + \mu\left(\Phi\left(\frac{\mu}{\sigma}\right)\right)\right)$$

$$f(\varepsilon) = \frac{1}{\sigma_u^2\sigma_v} e^{\frac{-\varepsilon^2}{2\sigma_v^2}} e^{\frac{\mu^2}{2\sigma^2}} \sigma\left(\sigma\phi\left(\frac{\mu}{\sigma}\right) + \mu\left(\Phi\left(\frac{\mu}{\sigma}\right)\right)\right) \tag{10}$$

3. Technical Efficiency Measurement of NRSPFM

Technical Efficiency,

$$TE_i = \exp(-\hat{u}) = \exp\left(-E\left(\frac{u}{\varepsilon}\right)\right) \tag{11}$$

Consider,

$$f\left(\frac{u}{\varepsilon}\right) = \frac{f(u,\varepsilon)}{f(\varepsilon)}$$

$$= \frac{\frac{1}{\sqrt{2\pi}\sigma_u^2\sigma_v} u e^{\frac{-\varepsilon^2}{2\sigma_v^2}} e^{\frac{\mu^2}{2\sigma^2}} e^{\frac{(u-\mu)^2}{2\sigma^2}}}{\frac{1}{\sigma_u^2\sigma_v} e^{\frac{-\varepsilon^2}{2\sigma_v^2}} e^{\frac{\mu^2}{2\sigma^2}} \sigma\left(\sigma\phi\left(\frac{\mu}{\sigma}\right)+\mu\left(\Phi\left(\frac{\mu}{\sigma}\right)\right)\right)} \tag{12}$$

$$= \frac{u e^{\frac{(u-\mu)^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma\left(\sigma\phi\left(\frac{\mu}{\sigma}\right)+\mu\left(\Phi\left(\frac{\mu}{\sigma}\right)\right)\right)}$$

$$E\left(\frac{u}{\varepsilon}\right) = \int_0^\infty u\, f\left(\frac{u}{\varepsilon}\right)\ du = \int_0^\infty u\ \frac{f(u,\varepsilon)}{f(\varepsilon)}\ du = \int_0^\infty u \frac{u\ e^{\frac{(u-\mu)^2}{2\sigma^2}}}{\sqrt{2\pi}\ \sigma\left(\sigma\ \varphi\ \left(\frac{\mu}{\sigma}\right)\ +\mu\left(\Phi\left(\frac{\mu}{\sigma}\right)\right)\right)}\ du \tag{13}$$

$$= \frac{1}{\sqrt{2\pi}\ \sigma\left(\sigma\ \varphi\ \left(\frac{\mu}{\sigma}\right)\ +\mu\left(\Phi\left(\frac{\mu}{\sigma}\right)\right)\right)} \int_0^\infty u^2\ e^{\frac{(u-\mu)^2}{2\sigma^2}}\ du$$

Let $t = \frac{u-\mu}{\sigma} \Rightarrow t\sigma + \mu = u$, $dt = \frac{du}{\sigma} \Rightarrow \sigma dt = du$ when $u = 0 \Rightarrow t = -\frac{\mu}{\sigma}$, when $u = \infty \Rightarrow t = \infty$. Therefore

$$E\left(\frac{u}{\varepsilon}\right) = \frac{1}{\sqrt{2\pi}\sigma\left(\sigma\phi\left(\frac{\mu}{\sigma}\right)+\mu\left(\Phi\left(\frac{\mu}{\sigma}\right)\right)\right)} \int_{\left(-\frac{\mu}{\sigma}\right)}^\infty (t\sigma+\mu)^2 e^{-\frac{t^2}{2}}\sigma dt$$

$$= \frac{1}{\left(\sigma\phi\left(\frac{\mu}{\sigma}\right)+\mu\left(\Phi\left(\frac{\mu}{\sigma}\right)\right)\right)} \left(\sigma^2\frac{1}{\sqrt{2\pi}}\int_{\left(-\frac{\mu}{\sigma}\right)}^\infty t^2 e^{-\frac{t^2}{2}}dt + \mu^2\frac{1}{\sqrt{2\pi}}\int_{\left(-\frac{\mu}{\sigma}\right)}^\infty e^{-\frac{t^2}{2}}dt + 2\sigma\mu\frac{1}{\sqrt{2\pi}}\int_{\left(-\frac{\mu}{\sigma}\right)}^\infty te^{-\frac{t^2}{2}}dt\right)$$

$$= \frac{1}{\left(\sigma\phi\left(\frac{\mu}{\sigma}\right)+\mu\left(\Phi\left(\frac{\mu}{\sigma}\right)\right)\right)} \left(\sigma^2\frac{1}{\sqrt{2\pi}}\int_{\left(-\frac{\mu}{\sigma}\right)}^\infty t^2 e^{-\frac{t^2}{2}}dt + \mu^2\Phi\left(\frac{\mu}{\sigma}\right) + 2\sigma\mu\phi\left(\frac{\mu}{\sigma}\right)\right) \tag{14}$$

Consider

$$\sigma^2\frac{1}{\sqrt{2\pi}}\int_{\left(-\frac{\mu}{\sigma}\right)}^\infty t^2 e^{-\frac{t^2}{2}}dt = \sigma^2\frac{1}{\sqrt{2\pi}}\left[t\int te^{-\frac{t^2}{2}}dt\right]_{\left(-\frac{\mu}{\sigma}\right)}^\infty - \int_{\left(-\frac{\mu}{\sigma}\right)}^\infty 1\left(\int te^{-\frac{t^2}{2}}dt\right)dt \tag{15}$$

We know that

$$\int xe^{-bx^2}dx = \frac{1}{(-2b)}e^{-bx^2}$$

$$= \sigma^2\frac{1}{\sqrt{2\pi}}\left(\left[t(-2)\frac{1}{2}e^{-\frac{t^2}{2}}\right]_{\left(-\frac{\mu}{\sigma}\right)}^\infty - \int_{\left(-\frac{\mu}{\sigma}\right)}^\infty (-2)\frac{1}{2}e^{-\frac{t^2}{2}}dt\right)$$

$$= \sigma^2\frac{1}{\sqrt{2\pi}}\left(-\left(0-\left(-\frac{\mu}{\sigma}\right)e\frac{-\left(-\frac{\mu}{\sigma}\right)^2}{2}\right)+\int_{\left(-\frac{\mu}{\sigma}\right)}^\infty e^{-\frac{t^2}{2}}dt\right)$$

$$= \left(\sigma^2\frac{1}{\sqrt{2\pi}}\left(\left(-\frac{\mu}{\sigma}\right)e\frac{-\left(-\frac{\mu}{\sigma}\right)^2}{2}\right)+\sigma^2\frac{1}{\sqrt{2\pi}}\int_{\left(-\frac{\mu}{\sigma}\right)}^\infty e^{-\frac{t^2}{2}}dt\right)$$

$$= \sigma^2\left(\left(\left(-\frac{\mu}{\sigma}\right)\phi\left(\frac{\mu}{\sigma}\right)\right)+\sigma^2\Phi\left(\frac{\mu}{\sigma}\right)\right) \tag{16}$$

$$E\left(\frac{u}{\varepsilon}\right) = \frac{1}{\left(\sigma\phi\left(\frac{\mu}{\sigma}\right)+\mu\left(\Phi\left(\frac{\mu}{\sigma}\right)\right)\right)}\left(\sigma^2\left(\left(\left(-\frac{\mu}{\sigma}\right)\phi\left(\frac{\mu}{\sigma}\right)\right)+\sigma^2\Phi\left(\frac{\mu}{\sigma}\right)\right)+\mu^2\Phi\left(\frac{\mu}{\sigma}\right)+2\sigma\mu\phi\left(\frac{\mu}{\sigma}\right)\right)$$

$$= \frac{1}{\left(\sigma\phi\left(\frac{\mu}{\sigma}\right)+\mu\left(\Phi\left(\frac{\mu}{\sigma}\right)\right)\right)}\left(\left(\left(-\mu\sigma\phi\left(\frac{\mu}{\sigma}\right)\right)+\sigma^2\Phi\left(\frac{\mu}{\sigma}\right)\right)+\mu^2\Phi\left(\frac{\mu}{\sigma}\right)+2\sigma\mu\phi\left(\frac{\mu}{\sigma}\right)\right) \tag{17}$$

The expected value of inefficiency term u given ε in the NRSPFM is

$$E\left(\frac{u}{\varepsilon}\right) = \frac{\left(\mu\sigma\phi\left(\frac{\mu}{\sigma}\right)+\left(\sigma^2+\mu^2\right)\Phi\left(\frac{\mu}{\sigma}\right)\right)}{\left(\sigma\phi\left(\frac{\mu}{\sigma}\right)+\mu\Phi\left(\frac{\mu}{\sigma}\right)\right)} \tag{18}$$

where $\phi(.)$, $\Phi(.)$ are and density function and the standard normal cumulative distribution respectively.

4. Parameter Estimation of NRSPFM Using Maximum Likelihood Estimation

The maximum likelihood estimate (MLE) method to estimate the parameters was introduced by Aigner and Chu (1968). The MLE method is to maximize the log likelihood function corresponding to the marginal density function f (ε). The likelihood function of the sample is the product of the density function of the individual observations, which is given as, $L\left(sample\right)=\prod_{i=1}^{i=N} f(e_i)$. The Log likelihood function for a sample of N producers is

$$InL=-\frac{N}{2}In\sigma_v^2-NIn\sigma_u^2+\frac{N}{2}In\sigma^2+\sum_{i=1}^{N}In\sigma\left(\sigma\phi\left(\frac{\mu_i}{\sigma}\right)+\mu\left(\Phi\left(\frac{\mu_i}{\sigma}\right)\right)\right)+\sum_{i=1}^{N}\frac{\varepsilon_i^2}{2\sigma_v^2}+\sum_{i=1}^{N}\frac{\mu_i^2}{2\sigma^2} \tag{19}$$

Parameters σ^2, β, μ can be estimated using the first order conditions of the maximization of log-likelihood function. Differentiate partially (19) w.r.to σ^2.

$$\begin{aligned}R^*_{\sigma^2}=\frac{\partial InL}{\partial\sigma^2}&=\frac{N}{2}\frac{1}{\sigma^2}+\sum_{i=1}^{N}\frac{\sigma\phi'\left(\frac{\mu_i}{\sigma}\right)\mu_i\left(\frac{-1}{2\sigma^3}\right)+\mu_i\phi\left(\frac{\mu_i}{\sigma}\right)\frac{1}{2\sigma}+\mu_i\phi\left(\frac{\mu_i}{\sigma}\right)\left(\frac{-\mu_i}{2\sigma^3}\right)}{\sigma\left(\sigma\phi\left(\frac{\mu_i}{\sigma}\right)+\mu\left(\Phi\left(\frac{\mu_i}{\sigma}\right)\right)\right)}-\sum_{i=1}^{N}\frac{\mu_i^2}{2\sigma}\\&=\frac{N}{2}\frac{1}{\sigma^2}+\sum_{i=1}^{N}\frac{\sigma\phi'\left(\frac{\mu_i}{\sigma}\right)\mu_i\left(\frac{-1}{2\sigma^3}\right)+\mu_i\phi\left(\frac{\mu_i}{\sigma}\right)\frac{1}{2\sigma}+\phi\left(\frac{\mu_i}{\sigma}\right)\left(\frac{-1}{2\sigma^3}\right)}{\sigma\left(\sigma\phi\left(\frac{\mu_i}{\sigma}\right)+\mu\left(\Phi\left(\frac{\mu_i}{\sigma}\right)\right)\right)}-\sum_{i=1}^{N}\frac{\mu_i^2}{2\sigma}\end{aligned} \tag{20}$$

Differentiate partially (19) w.r.to μ_i

$$\begin{aligned}R^*_{\mu}=\frac{\partial InL}{\partial\mu_i}&=\sum_{i=1}^{N}\frac{\sigma\phi'\left(\frac{\mu_i}{\sigma}\right)\mu_i\left(\frac{1}{\sigma}\right)+\mu_i\phi\left(\frac{\mu_i}{\sigma}\right)\frac{1}{\sigma}}{\sigma\left(\sigma\phi\left(\frac{\mu_i}{\sigma}\right)+\mu\left(\Phi\left(\frac{\mu_i}{\sigma}\right)\right)\right)}+\sum_{i=1}^{N}\frac{2\mu_i}{2\sigma^2}\\&=\sum_{i=1}^{N}\frac{\phi'\left(\frac{\mu_i}{\sigma}\right)\mu_i+\frac{\mu_i}{\sigma}\phi\left(\frac{\mu_i}{\sigma}\right)}{\sigma\left(\sigma\phi\left(\frac{\mu_i}{\sigma}\right)+\mu\left(\Phi\left(\frac{\mu_i}{\sigma}\right)\right)\right)}+\sum_{i=1}^{N}\frac{\mu_i}{\sigma^2}\end{aligned} \tag{21}$$

put $\varepsilon_i=y_i-x_i\beta$ in (19)

$$InL=-\frac{N}{2}In\sigma_v^2-NIn\sigma_u^2+\frac{N}{2}In\sigma^2+\sum_{i=1}^{N}In\sigma\left(\sigma\phi\left(\frac{\mu_i}{\sigma}\right)+\mu\left(\Phi\left(\frac{\mu_i}{\sigma}\right)\right)\right)+\sum_{i=1}^{N}\frac{(y_i-x_i\beta)^2}{2\sigma_v^2}+\sum_{i=1}^{N}\frac{\mu_i^2}{2\sigma^2} \tag{22}$$

Partially differentiate (22) w.r.to β

$$\begin{aligned}R^*_{\beta}=\frac{\partial InL}{\partial\beta}&=\sum_{i=1}^{N}\frac{2\left(y_i-x_i\beta\right)\left(-x_i\right)}{2\sigma_v^2}\\&=\sum_{i=1}^{N}\frac{-x_i\left(y_i-x_i\beta\right)}{\sigma_v^2}\end{aligned} \tag{23}$$

Equating the equations (20), (21), (23) to zero and then solving, we get the maximum likelihood estimation of all parameters.

5. Conclusion

In this paper, the expected value of inefficiency term u given ε in the NRSPFM. Therefore

$$E\left(\frac{u}{\varepsilon}\right)=\frac{\left(\mu\sigma\phi\left(\frac{\mu}{\sigma}\right)+\left(\sigma^2+\mu^2\right)\Phi\left(\frac{\mu}{\sigma}\right)\right)}{\left(\sigma\phi\left(\frac{\mu}{\sigma}\right)+\mu\Phi\left(\frac{\mu}{\sigma}\right)\right)}$$

was derived, to measure the efficiency of individual firm. The model derived also can be applied to identify the factors which affect the production and hence best firm can be recognised. Recommendations to increase the average production can be given based on the results. Like this, we can derive the technical efficiency and parameter estimation for other continuous distributions also.

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