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Delta Atom Bond Connectivity Indices of Certain Networks

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Abstract

In this study, we introduce the delta atom bond connectivity index and the delta atom bond sum connectivity index of a graph. Furthermore, we compute these delta atom bond connectivity indices for some networks such as silicate networks, hexagonal networks, honeycomb networks and oxide networks.

Keywords: Delta atom bond connectivity index; delta atom bond sum connectivity index; network.
2020 Mathematics Subject Classification: 05C07, 05C09, 05C92.

1. Introduction

Let $G=(V,E)$ be a finite, simple connected graph. Let $d_G(u)$ denote the degree of a vertex u [1]. In the modeling of Mathematics, a molecular or a chemical graph is a simple graph related to the structure of a chemical compound. Each vertex of this graph represents an atom of the molecule and its edges to the bonds between atoms. Topological indices [2] are useful for finding correlations between the structure of a chemical compound and its physicochemical properties [3,4]. The δ vertex degree was introduced in [5] and it is defined as

$$\delta_u = d_G(u) - \delta(G) + 1$$

The first and second δ-Banhatti indices [6] of a graph are defined as

$$\delta B_1(G) = \sum_{uv \in E(G)} (\delta_u + \delta_v) \quad \text{and} \quad \delta B_2(G) = \sum_{uv \in E(G)} \delta_u \delta_v$$

Recently, some delta Banhatti indices were studied in [7, 8, 9, 10, 11]. We introduce the delta atom bond connectivity index of a graph G, defined as

$$\delta ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{\delta_u + \delta_v - 2}{\delta_u \delta_v}}$$

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We also introduce the delta atom bond sum connectivity index of a graph G, defined as

$$\delta ABS\,(G) = \sum_{uv \in E(G)} \sqrt{\frac{\delta_u + \delta_v - 2}{\delta_u + \delta_v}}$$

The atom bond connectivity index has been found to be a useful predictive indicator in the research on heat generation in octanes, and heptanes [12]. The atom bond connectivity indices have been researched in the past [13–21]. In this paper, we compute the delta atom bond connectivity index and delta atom bond sum connectivity index of certain networks.

2. Results for Silicate Networks

Silicate networks are obtained by fusing metal oxide or metal carbonates with sand. A silicate network is denoted by SL_n. A 2-D silicate network is shown in Figure 1.

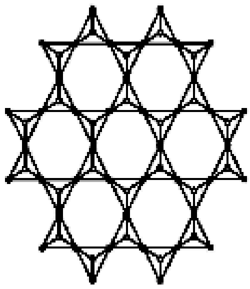

Figure 1: A 2-D silicate network

Let G be the graph of a silicate network SL_n. By calculation, we obtain that G has $15n^2 + 3n$ vertices and $36n^2$ edges. In G, there are two types of vertices as follows:

$$V_1 = \{u \in V(G) | d_G(u) = 3\}, \qquad |V_1| = 6n^2 + 6n$$
$$V_2 = \{u \in V(G) | d_G(u) = 6\}, \qquad |V_2| = 9n^2 - 3n$$

Therefore, we have $\delta(G) = 3$ and hence $\delta_u = d_G(u) - \delta(G) + 1 = d_G(u) - 2$. Thus there are two types of δ-vertices as given in Table 1.

$\delta_u \backslash u \in V(G)$	1	4
Number of vertices	$6n^2 + 6n$	$9n^2 - 3n$

Table 1: δ-vertex partition of SL_n

By calculation, in G, there are three types of edges based on degrees of end vertices of each edge as follows:

$$E_1 = \{uv \in E(G) | d_G(u) = d_G(v) = 3\}, \qquad |E_1| = 6n$$
$$E_2 = \{uv \in E(G) | d_G(u) = 3, d_G(v) = 6\}, \qquad |E_2| = 18n^2 + 6n$$

$$E_3 = \{uv \in E(G) | d_G(u) = d_G(v) = 6\}, \qquad |E_3| = 18n^2 - 12n$$

Hence there are 3 types of δ-edges as given in Table 2.

$\delta_u, \delta_v \backslash uv \in E(G)$	(1, 1)	(1, 4)	(4, 4)
Number of edges	$6n$	$18n^2 + 6n$	$18n^2 - 12$

Table 2: δ-edge partition of SL_n

Theorem 2.1. *The delta atom bond connectivity index of SL_n is given by*

$$\delta ABC\,(G) = \left(1 + \frac{1}{\sqrt{2}}\right) 9\sqrt{3}n^2 + \left(1 - \sqrt{2}\right) 3\sqrt{3}n$$

Proof. Applying definition and δ-edge partition of SL_n, we conclude

$$\begin{aligned}
\delta ABC\,(G) &= \sum_{uv \in E(G)} \sqrt{\frac{\delta_u + \delta_v - 2}{\delta_u \delta_v}} \\
&= 6n \left(\sqrt{\frac{1+1-2}{1 \times 1}}\right) + \left(18n^2 + 6n\right) \left(\sqrt{\frac{1+4-2}{1 \times 4}}\right) + \left(18n^2 - 12n\right) \left(\sqrt{\frac{4+4-2}{4 \times 4}}\right) \\
&= \left(1 + \frac{1}{\sqrt{2}}\right) 9\sqrt{3}n^2 + \left(1 - \sqrt{2}\right) 3\sqrt{3}n
\end{aligned}$$

□

Theorem 2.2. *The delta atom bond sum connectivity index of SL_n is given by*

$$\delta ABS\,(G) = \left(\frac{2}{\sqrt{5}} + 1\right) 9\sqrt{3}n^2 + \left(\frac{1}{\sqrt{5}} - 1\right) 6\sqrt{3}n$$

Proof. Applying definition and δ-edge partition of SL_n, we conclude

$$\begin{aligned}
\delta ABS\,(G) &= \sum_{uv \in E(G)} \sqrt{\frac{\delta_u + \delta_v - 2}{\delta_u + \delta_v}} \\
&= 6n \left(\sqrt{\frac{1+1-2}{1+1}}\right) + \left(18n^2 + 6n\right) \left(\sqrt{\frac{1+4-2}{1+4}}\right) + \left(18n^2 - 12n\right) \left(\sqrt{\frac{4+4-2}{4+4}}\right)
\end{aligned}$$

By solving the above equation, we get the desired result. □

3. Results for Hexagonal Networks

It is known that there exist three regular plane tilings with composition of some kind of regular polygons such as triangular, hexagonal and square. Triangular tiling is used in the construction of hexagonal networks. This network is denoted by HX_n. A hexagonal network of dimension six is shown in Figure 2.

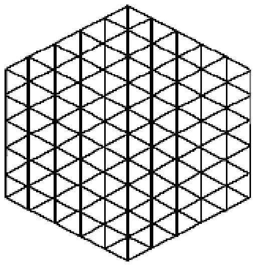

Figure 2: Hexagonal network of dimension six

Let G be the graph of a hexagonal network HX_n. By calculation, we obtain that G has $3n^2 - 3n + 1$ vertices and $9n^2 - 15n + 6$ edges. In G, there are three types of vertices as follows:

$$V_1 = \{u \in V(G) | d_G(u) = 3\}, \qquad |V_1| = 6$$
$$V_2 = \{u \in V(G) | d_G(u) = 4\}, \qquad |V_2| = 6n - 12$$
$$V_3 = \{u \in V(G) | d_G(u) = 6\}, \qquad |V_3| = 3n^2 - 9n + 7$$

Thus $\delta(G) = 3$ and hence $\delta_u = d_G(u) - \delta(G) + 1 = d_G(u) - 2$. Therefore there are three types of δ-vertices as given in Table 3.

$\delta_u \backslash u \in V(G)$	1	2	4
Number of vertices	6	$6n - 12$	$3n^2 - 9n + 7$

Table 3: δ-vertex partition of HX_n

By calculation, in G, there are five types of edges based on degrees of end vertices of each edge as follows:

$$E_1 = \{uv \in E(G) | d_G(u) = 3, d_G(v) = 4\}, \qquad |E_1| = 12$$
$$E_2 = \{uv \in E(G) | d_G(u) = 3, d_G(v) = 6\}, \qquad |E_2| = 6$$
$$E_3 = \{uv \in E(G) | d_G(u) = d_G(v) = 4\}, \qquad |E_3| = 6n - 18$$
$$E_4 = \{uv \in E(G) | d_G(u) = 4, d_G(v) = 6\}, \qquad |E_4| = 12n - 24$$
$$E_5 = \{uv \in E(G) | d_G(u) = d_G(v) = 6\}, \qquad |E_5| = 9n^2 - 33n + 30$$

Thus there are five types of δ-edges as given in Table 4.

$\delta_u, \delta_v \backslash uv \in E(G)$	(1, 2)	(1, 4)	(2, 2)	(2, 4)	(4, 4)
Number of edges	12	6	$6n - 18$	$12n - 24$	$9n^2 - 33n + 30$

Table 4: δ-edge partition of HX_n

Theorem 3.1. *The delta atom bond connectivity index of SL_n is given by*

$$\delta ABC\,(G) = \frac{9\sqrt{3}}{2\sqrt{2}}n^2 + \left(18 - \frac{33\sqrt{3}}{2}\right)\frac{1}{\sqrt{2}}n + 3\sqrt{3} - \frac{30}{\sqrt{2}} + \frac{30\sqrt{3}}{2\sqrt{2}}$$

Proof. Applying definition and δ-edge partition of SL_n, we conclude

$$\begin{aligned}\delta ABC(G) &= \sum_{uv\in E(G)} \sqrt{\frac{\delta_u+\delta_v-2}{\delta_u\delta_v}} \\ &= 12\left(\sqrt{\frac{1+2-2}{1\times 2}}\right)+6\left(\sqrt{\frac{1+4-2}{1\times 4}}\right)+(6n-18)\left(\sqrt{\frac{2+2-2}{2\times 2}}\right) \\ &+(12n-24)\left(\sqrt{\frac{2+4-2}{2\times 4}}\right)+\left(9n^2-33n+30\right)\left(\sqrt{\frac{4+4-2}{4\times 4}}\right)\end{aligned}$$

By solving the above equation, we get the desired result. □

Theorem 3.2. *The delta atom bond sum connectivity index of SL_n is given by*

$$\delta ABS(G) = \frac{9\sqrt{3}}{2}n^2+\left(\frac{6}{\sqrt{2}}+\frac{12\sqrt{2}}{\sqrt{3}}-\frac{33\sqrt{3}}{2}\right)n+\frac{12}{\sqrt{3}}+\frac{6\sqrt{3}}{\sqrt{5}}-\frac{18}{\sqrt{2}}-\frac{24\sqrt{2}}{\sqrt{3}}$$

Proof. Applying definition and δ-edge partition of SL_n, we conclude

$$\begin{aligned}\delta ABS(G) &= \sum_{uv\in E(G)} \sqrt{\frac{\delta_u+\delta_v-2}{\delta_u+\delta_v}} \\ &= 12\left(\sqrt{\frac{1+2-2}{1+2}}\right)+6\left(\sqrt{\frac{1+4-2}{1+4}}\right)+(6n-18)\left(\sqrt{\frac{2+2-2}{2+2}}\right) \\ &+(12n-24)\left(\sqrt{\frac{2+4-2}{2+4}}\right)+\left(9n^2-33n+30\right)\left(\sqrt{\frac{4+4-2}{4+4}}\right)\end{aligned}$$

By solving the above equation, we get the desired result. □

4. Results for Honeycomb Networks

If we recursively use hexagonal tiling in particular pattern, honeycomb networks are formed. These networks are very useful in Chemistry and also in Computer Graphics. A honeycomb network of dimension n is denoted by HC_n. A honeycomb network of dimension four is shown in Figure 3.

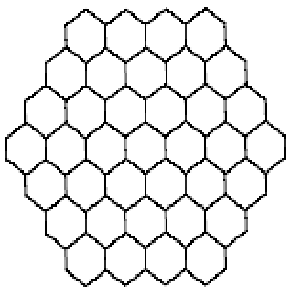

Figure 3: Honeycomb network of dimension four

Let G be the graph of a honeycomb network HC_n. By calculation, we obtain that G has $6n^2$ vertices and $9n^2 - 3n$ edges. In G, there are two types of vertices as follows:

$$V_1 = \{u \in V(G) | d_G(u) = 2\}, |V_1| = 6n$$
$$V_2 = \{u \in V(G) | d_G(u) = 3\}, |V_2| = 6n^2 - 6n$$

Therefore, we have $\delta(G) = 3$ and hence $\delta_u = d_G(u) - \delta(G) + 1 = d_G(u) - 2$. Thus there are two types of δ-vertices as given in Table 5.

$\delta_u \backslash u \in V(G)$	1	2
Number of vertices	$6n$	$6n^2 - 6$

Table 5: δ-vertex partition of HC_n

By calculation, in G, there are three types of edges based on degrees of end vertices of each edge as follows:

$$E_1 = \{uv \in E(G) | d_G(u) = d_G(v) = 2\}, \qquad |E_1| = 6$$
$$E_2 = \{uv \in E(G) | d_G(u) = 2, d_G(v) = 3\}, \qquad |E_2| = 12n - 12$$
$$E_3 = \{uv \in E(G) | d_G(u) = d_G(v) = 3\}, \qquad |E_3| = 9n^2 - 15n + 6$$

Hence there are 3 types of δ-edges as given in Table 6.

$\delta_u, \delta_v \backslash uv \in E(G)$	(1, 1)	(1, 2)	(2, 2)
Number of edges	6	$12n - 12$	$9n^2 - 15n + 6$

Table 6: δ-edge partition of HC_n

Theorem 4.1. *The delta atom bond connectivity index of HC_n is given by*

$$\delta ABC\,(G) = \frac{9}{\sqrt{2}}n^2 - \frac{3}{\sqrt{2}}n - \frac{6}{\sqrt{2}}$$

Proof. Applying definition and δ-edge partition of HC_n, we conclude

$$\begin{aligned}\delta ABC\,(G) &= \sum_{uv \in E(G)} \sqrt{\frac{\delta_u + \delta_v - 2}{\delta_u \delta_v}} \\ &= 6\left(\sqrt{\frac{1+1-2}{1 \times 1}}\right) + (12n - 12)\left(\sqrt{\frac{1+2-2}{1 \times 2}}\right) + \left(9n^2 - 15n + 6\right)\left(\sqrt{\frac{2+2-2}{2 \times 2}}\right)\end{aligned}$$

By solving the above equation, we get the desired result. □

Theorem 4.2. *The delta atom bond sum connectivity index of HC_n is given by*

$$\delta ABS\,(G) = \frac{9}{\sqrt{2}}n^2 + \left(\frac{12}{\sqrt{3}} - \frac{15}{\sqrt{2}}\right)n - \frac{12}{\sqrt{3}} + \frac{6}{\sqrt{2}}$$

Proof. Applying definition and δ-edge partition of HC_n, we conclude

$$\delta ABS\,(G) = \sum_{uv\in E(G)} \sqrt{\frac{\delta_u+\delta_v-2}{\delta_u+\delta_v}}$$

$$= 6\left(\sqrt{\frac{1+1-2}{1+1}}\right) + (12n-12)\left(\sqrt{\frac{1+2-2}{1+2}}\right) + \left(9n^2-15n+6\right)\left(\sqrt{\frac{2+2-2}{2+2}}\right)$$

By solving the above equation, we get the desired result. □

5. Results for Oxide Networks

The oxide networks are of vital importance in the study of silicate networks. An oxide network of dimension n is denoted by of OX_n. An oxide network of dimension five is shown in Figure 4.

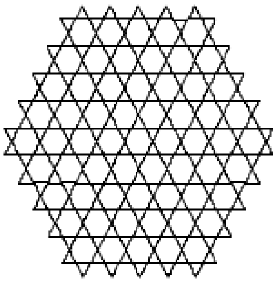

Figure 4: Oxide network of dimension 5

Let G be the graph of an oxide network OX_n. By calculation, we obtain that G has $9n^2+3n$ vertices and $18n^2$ edges. In G, there are two types of vertices as follows:

$$V_1 = \{u \in V(G) | d_G(u) = 2\}, \quad |V_1| = 6n$$
$$V_2 = \{u \in V(G) | d_G(u) = 4\}, \quad |V_2| = 9n^2-3n$$

Therefore, we have $\delta(G) = 2$ and hence $\delta_u = d_G(u) - \delta(G) + 1 = d_G(u) - 1$. Thus there are two types of δ-vertices as given in Table 7.

$\delta_u \backslash u \in V(G)$	1	3
Number of vertices	$6n$	$9n^2-3n$

Table 7: δ-vertex partition of OX_n

By calculation, in G, there are two types of edges based on degrees of end vertices of each edge as follows:

$$E_1 = \{uv \in E(G) | d_G(u) = 2, d_G(v) = 4\}, \qquad |E_1| = 12n$$
$$E_2 = \{uv \in E(G) | d_G(u) = d_G(v) = 4\}, \qquad |E_2| = 18n^2-12n$$

Therefore, we have $\delta(G) = 2$ and hence $\delta_u = d_G(u) - \delta(G) + 1 = d_G(u) - 1$. Thus there are two types of δ-edges as given in Table 8.

$\delta_u, \delta_v \backslash uv \in E(G)$	$(1, 3)$	$(3, 3)$
Number of edges	$6n$	$9n^2 - 3n$

Table 8: δ-vertex partition of OX_n

Theorem 5.1. *The delta atom bond connectivity index of OX_n is given by*

$$\delta ABC\,(G) = 6n^2 + \left(\frac{6\sqrt{2}}{\sqrt{3}} - 2\right) n$$

Proof. Applying definition and δ-edge partition of OX_n, we conclude

$$\begin{aligned}
\delta ABC\,(G) &= \sum_{uv \in E(G)} \sqrt{\frac{\delta_u + \delta_v - 2}{\delta_u \delta_v}} \\
&= 6n\left(\sqrt{\frac{1+3-2}{1\times 3}}\right) + (9n^2 - 3n)\left(\sqrt{\frac{3+3-2}{3\times 3}}\right)
\end{aligned}$$

By solving the above equation, we get the desired result. □

Theorem 5.2. *The delta atom bond sum connectivity index of OX_n is given by*

$$\delta ABS\,(G) = \frac{9\sqrt{2}}{\sqrt{3}} n^2 + \left(\frac{6}{\sqrt{2}} - \frac{3\sqrt{2}}{\sqrt{3}}\right) n$$

Proof. Applying definition and δ-edge partition of OX_n, we conclude

$$\begin{aligned}
\delta ABS\,(G) &= \sum_{uv \in E(G)} \sqrt{\frac{\delta_u + \delta_v - 2}{\delta_u + \delta_v}} \\
&= 6n\left(\sqrt{\frac{1+3-2}{1+3}}\right) + (9n^2 - 3n)\left(\sqrt{\frac{3+3-2}{3+3}}\right)
\end{aligned}$$

By solving the above equation, we get the desired result. □

6. Conclusion

In this study, we have introduced the delta atom bond connectivity index and delta atom bond sum connectivity index of a graph. Also these delta atom bond connectivity indices of certain networks are determined.

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