International Journal of Mathematics And its Applications
Volume 5, Issue 4–F (2017), 851–856.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Intuitionistic Fuzzy Replacement Problem with Change in Money Value

Research Article

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Abstract: Due to lack of accuracy or fluctuation in collected data, often it is too difficult to evaluate the precise value, especially in the continuous degradation element or system. To overcome this challenge, fuzzy set theory is employed to assess the performance. In this paper, the imprecise costs of the replacement problem, whose money value changes with time are taken as Triangular Intuitionistic Fuzzy Numbers (TIFN). A method is proposed to find the optimum replacement time and the effectiveness of the same is illustrated by an example.

Keywords: Intuitionistic Fuzzy Set, Triangular intuitionistic fuzzy number, intuitionistic fuzzy replacement problem.

1. Introduction

The fuzzy sets introduced by Zadeh. A [1] were generalized by Atanassov [2] to develop the intuitionistic fuzzy sets which includes non-membership function which is very useful to express vagueness more accurately. The basic arithmetic operations of generalized triangular intuitionistic fuzzy numbers and the notion of (α, β)-cut sets were defined by Seikh et al [5]. Nagoorgani et al [7] introduced a ranking technique for TIFN using (α, β)-cut, score function and accuracy function. Chiu and park [2] use fuzzy numbers in cash flow analysis and provides good survey of methods for ranking mutually exclusive fuzzy projects. Biswas and Pramanik [5] developed a method of finding the optimal replacement time of fuzzy equipment replacement problems with trapezoidal fuzzy numbers and triangular fuzzy numbers using Yager's ranking method.

The considerations in this paper are: a replacement problem where the capital cost, $\tilde{C}$, the running cost (to be paid at the beginning of the t^{th} year) $\tilde{R}_t$, are taken as TIFN. It is assumed that the running cost increases with time and the money has a value over time. $V = (1+r)^{-1}$ is the present worth of a rupee to be spent after one year, where r is the rate of interest.

2. Preliminaries

Definition 2.1 (Fuzzy set). *Let X be a classical set. A fuzzy set $\tilde{A}$ is defined by $\tilde{A} = \{(x, \mu_{\tilde{A}}(x))/x \in X\}$, where $\mu_{\tilde{A}}(x) : X \to [0,1]$ is called the membership function of $\tilde{A}$ and $\mu_{\tilde{A}}(x)$ is the degree of membership of x in $\tilde{A}$.*

Definition 2.2 (Intuitionistic fuzzy set). *Let X denote universe of discourse, then an intuitionistic fuzzy set $\tilde{A}^I$ in X is given by $\tilde{A}^I = \{(x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x))/x \in X\}$ where $\mu_{\tilde{A}}(x) : X \to [0,1]$ and $\nu_{\tilde{A}}(x) : X \to [0,1]$ are functions such that*

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$0 \leq \mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \leq 1, \ \forall \ x \in X$. *For each* $x \in X$, $\mu_{\tilde{A}}(x)$ *and* $\nu_{\tilde{A}}(x)$ *represent the degre of membership and the degree of non-membership respectively.*

Definition 2.3 (Intuitionistic fuzzy number)**.** *An intuitionistic fuzzy subset* $\tilde{A}^I = \{(x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x))/x \in X\}$ *of the real line R is called an intuitionistic fuzzy number if the following holds:*

(1). There exists $m \in R$, $\mu_{\tilde{A}}(m) = 1$ *and* $\nu_{\tilde{A}}(m) = 0$, *m is called the mean value of* $\tilde{A}^I$.

(2). $\mu_{\tilde{A}}$ *is a continuous mapping from R to the closed interval [0, 1] and* $\forall \ x \in R$, *the relation* $0 \leq \mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \leq 1$ *holds.*

The membership function and non-membership function of $\tilde{A}^I$ *are of the following form:*

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & -\infty < x \leq m - \alpha \\ f_1(x), & m - \alpha < x < m \\ 1, & x = m \\ h_1(x), & m < x < m + \beta \\ 0, & m + \beta \leq x < \infty \end{cases} \qquad \nu_{\tilde{A}}(x) = \begin{cases} 1, & -\infty < x \leq m - \alpha' \\ f_2(x), & m - \alpha' < x < m \ \& \ 0 \leq f_1(x) + f_2(x) \leq 1 \\ 0, & x = m \\ h_2(x), & m < x < m + \beta' \ \& \ 0 \leq h_1(x) + h_2(x) \leq 1 \\ 1, & m + \beta' \leq x < \infty \end{cases}$$

Here m is the mean value of $\bar{A}^I$, α *and* β *are called left and right spreads of membership function* $\mu_{\tilde{A}}(x)$ *respectively.* α', β' *represent left and right spreads of non-membership function* $\nu_{\tilde{A}}(x)$ *respectively.*

Definition 2.4 (Triangular intuitionistic fuzzy number)**.** *A triangular intuitionistic fuzzy number* $\tilde{a}$ *is an intuitionistic fuzzy subset in R with the following membership function* $\mu_{\tilde{a}}(x)$ *and non-membership function* $\nu_{\tilde{a}}(x)$.

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}; & a_1 \leq x \leq a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}; & a_2 \leq x \leq a_3 \\ 0; & otherwise \end{cases} \qquad \nu_{\tilde{a}'}(x) = \begin{cases} \dfrac{a_2 - x}{a_2 - a_1'}; & a_1' \leq x \leq a_2 \\ \dfrac{x - a_2}{a_3' - a_2}; & a_2 \leq x \leq a_3' \\ 1; & otherwise \end{cases}$$

Where $a_1' \leq a_1 \leq a_2 \leq a_3 \leq a_3'$ *and* $0 \leq \mu_{\tilde{a}'}(x) \leq 1$, $0 \leq \nu_{\tilde{a}'}(x) \leq 1$; $0 \leq \nu_{\tilde{a}'}(x) + \mu_{\tilde{a}'}(x) \leq 1$ *for all* $x \in X$. *Triangular intuitionistic fuzzy number* $\tilde{a}$ *is denoted by* $(a_1, a_2, a_3; a_1', a_2, a_3')$.

2.1. Operations on triangular intuitionistic fuzzy number

Let $\tilde{a} = (a_1, a_2, a_3; a_1', a_2, a_3')$ and $\tilde{b} = (b_1, b_2, b_3; b_1', b_2, b_3')$ be two triangular intuitionistic fuzzy numbers. The arithmetic operations on $\tilde{a}$ and $\tilde{b}$ is given below:

Addition: $(a_1, a_2, a_3; a_1', a_2, a_3') + (b_1, b_2, b_3; b_1', b_2, b_3') = (a_1 + b_1, a_2 + b_2, a_3 + b_3; a_1' + b_1', a_2 + b_2, a_3' + b_3')$

Subtraction: $(a_1, a_2, a_3; a_1', a_2, a_3') - (b_1, b_2, b_3; b_1', b_2, b_3') = (a_1 - b_3, a_2 - b_2, a_3 - b_1; a_1' - b_3', a_2 - b_2, a_3' - b_1')$

Multiplication: $(a_1, a_2, a_3; a_1', a_2, a_3') \times (b_1, b_2, b_3; b_1', b_2, b_3') = (a_1b_1, a_2b_2, a_3b_3; a_1'b_1', a_2b_2, a_3'b_3')$

Scalar Multiplication:

$$\begin{aligned} k(a_1, a_2, a_3; a_1', a_2, a_3') &= (ka_1, ka_2, ka_3; ka_1', ka_2, ka_3'), \quad \text{if } k > 0 \\ &= (ka_3, ka_2, ka_1; ka_3', ka_2, ka_1'), \quad \text{if } k < 0 \end{aligned}$$

Defuzzification: An accuracy function $H(\tilde{a}) = \frac{a_1 + 2a_2 + a_3 + a_1' + 2a_2 + a_3'}{8}$ is used to defuzzify the triangular intuitionistic fuzzy number.

3. Fuzzy Replacement Problem

The items whose maintenance cost increases with time and the money value decreases with constant rate are considered. Then the replacement policy will be

(1). Replace if the next period's cost is > the weighted average of previous costs.

(2). Do not replace if the next period's cost is < the weighted average of previous costs. Time is taken to be a discrete random variable. Let

$\tilde{C}$ = capital cost

$\tilde{R}_t$ = running cost at the beginning of the t^{th} year

r = rate of interest

$V = (1+r)^{-1}$ the present worth factor. (or) depreciation value

The table gives the present worth of the item at the end of the corresponding year. It is assumed that the item has no resale

YEAR	1	2	$\cdots$	$\cdots$	$\cdots$	n
Present worth	$\tilde{C}+\tilde{R}_1$	$\tilde{R}_2V$				$\tilde{R}_nV^{n-1}$
YEAR	$n+1$	$n+2$	$\cdots$	$\cdots$	$\cdots$	$2n$
Present worth	$(\tilde{C}+\tilde{R}_1)V^n$	$\tilde{R}_2V^{n+1}$				$\tilde{R}_nV^{2n-1}$
YEAR	$2n+1$	$2n+2$	$\cdots$	$\cdots$	$\cdots$	$3n$
Present worth	$(\tilde{C}+\tilde{R}_1)V^{2n}$	$\tilde{R}_2V^{2n+1}$				$\tilde{R}_nV^{3n-1}$

value at the time of replacement. The present worth of all discounted costs at the end of every n years will be given by

$$\begin{aligned}
\tilde{P}(n) &= (\tilde{C}+\tilde{R}_1)[1+V^n+V^{2n}+\cdots]+\tilde{R}_2V[1+V^n+V^{2n}+\cdots]+\cdots+\tilde{R}_nV^{n-1}[1+V^n+V^{2n}+\cdots] \\
&= (\tilde{C}+\tilde{R}_1+\tilde{R}_2V+\cdots+\tilde{R}_nV^{n-1})(1+V^n+V^{2n}+\cdots) \\
&= (\tilde{C}+\tilde{R}_1+\tilde{R}_2V+\cdots+\tilde{R}_nV^{n-1})\left(\frac{1}{1-V^n}\right) \quad [V<1] \\
&= \frac{\tilde{F}(n)}{1-V^n} \quad \text{where} \quad \tilde{F}(n)=\tilde{C}+\tilde{R}_1+\tilde{R}_2V+\cdots+\tilde{R}_nV^{n-1} \\
\tilde{P}(n+1) &= \frac{\tilde{F}(n+1)}{1-V^{n+1}}
\end{aligned}$$

$\tilde{P}(n)$ is minimum if $\tilde{P}(n+1)-\tilde{P}(n)>0>\tilde{P}(n)-\tilde{P}(n-1)$ i.e., if

$$\frac{\tilde{F}(n+1)}{1-V^{n+1}}-\frac{\tilde{F}(n)}{1-V^n}>0>\frac{\tilde{F}(n)}{1-V^n}-\frac{\tilde{F}(n-1)}{1-V^{n-1}}$$

Now

$$\begin{aligned}
\frac{\tilde{F}(n+1)}{1-V^{n+1}}-\frac{\tilde{F}(n)}{1-V^n} &= \frac{(1-V^n)\tilde{F}(n+1)-(1-V^{n+1})\tilde{F}(n)}{(1-V^{n+1})(1-V^n)} \\
&= \frac{\lfloor\tilde{F}(n+1)-\tilde{F}(n)\rfloor+\tilde{F}(n)V^{n+1}-\tilde{F}(n+1)V^n}{(1-V^{n+1})(1-V^n)}
\end{aligned}$$

But

$$\begin{aligned}
\tilde{F}(n+1) &= \tilde{F}(n)+\tilde{R}_{n+1}V^n \\
&= \frac{\tilde{R}_{n+1}V^n+\tilde{F}(n)V^{n+1}-V^n\lfloor\tilde{F}(n)+\tilde{R}_{n+1}V^n\rfloor}{(1-V^{n+1})(1-V^n)}
\end{aligned}$$

$$= \frac{\tilde{R}_{n+1}V^n[1-V^n] + \tilde{F}(n)V^n[V-1]}{(1-V^{n+1})(1-V^n)}$$
$$= \frac{V^n(1-V)\left[\tilde{R}_{n+1}\left(\frac{1-V^n}{1-V}\right) - \tilde{F}(n)\right]}{(1-V^{n+1})(1-V^n)}$$

So $\tilde{P}(n)$ is minimum if

$$\frac{V^n(1-V)\left[\tilde{R}_{n+1}\left(\frac{1-V^n}{1-V}\right) - \tilde{F}(n)\right]}{(1-V^{n+1})(1-V^n)} > 0 > \frac{V^{n-1}(1-V)\left[\tilde{R}_n\left(\frac{1-V^{n-1}}{1-V}\right) - \tilde{F}(n-1)\right]}{(1-V^n)(1-V^{n-1})}$$
$$\frac{V^n\left[\tilde{R}_{n+1}\left(\frac{1-V^n}{1-V}\right) - \tilde{F}(n)\right]}{(1-V^{n+1})} > 0 > \frac{V^{n-1}\left[\tilde{R}_n\left(\frac{1-V^{n-1}}{1-V}\right) - \tilde{F}(n-1)\right]}{(1-V^{n-1})}$$
$$\frac{V\left\{\tilde{R}_{n+1}\left[\frac{1-V^n}{1-V}\right] - \tilde{F}(n)\right\}}{1-V^{n+1}} > 0 > \frac{\left[\tilde{R}_n\left(\frac{1-V^{n-1}}{1-V}\right) - \tilde{F}(n-1)\right]}{1-V^{n-1}}$$
$$\tilde{R}_{n+1}\left[\frac{1-V^n}{1-V}\right] - \tilde{F}(n) > 0 > \tilde{R}_n\left[\frac{1-V^{n-1}}{1-V}\right] - [\tilde{F}(n) - \tilde{R}_nV^{n-1}$$
$$\tilde{R}_{n+1}\left[\frac{1-V^n}{1-V}\right] - \tilde{F}(n) > 0 > \tilde{R}_n\left[\frac{1-V^{n-1}}{1-V}\right] - \tilde{F}(n) + \tilde{R}_nV^{n-1}$$
$$\tilde{R}_{n+1}\left[\frac{1-V^n}{1-V}\right] > \tilde{F}(n) > \tilde{R}_n\left[\frac{1-V^{n-1}}{1-V} + V^{n-1}\right]$$
$$\tilde{R}_{n+1} > \tilde{F}(n)\left[\frac{1-V}{1-V^n}\right] > \tilde{R}_n$$
$$\tilde{R}_{n+1} > \frac{\tilde{F}(n)}{1+V+V^2+\cdots+V^{n-1}} > \tilde{R}_n$$
$$\tilde{R}_{n+1} > \frac{\tilde{F}(n)}{\sum_{r=0}^{n-1} V^r} > \tilde{R}_n$$

$\frac{\tilde{F}(n)}{\sum_{r=0}^{n-1} V^r}$ is the weighted average cost of previous n years with weights $0, 1, V, V^2, \ldots V^{n-1}$ respectively. The value of n satisfying the above relation will be the best replacement time.

4. Illustration

The triangular intuitionistic fuzzy capital cost of an equipment is $\tilde{C} = (2500, 3000, 3200; 2100, 3000, 3800)$. Assume that the present value of one rupee to be spent in a year's time is Re. 0.9. The running costs are given in the following table.

YEAR	RUNNING COST
1	(400, 500, 600; 300, 500, 800)
2	(400, 600, 800; 200, 600, 900)
3	(700, 800, 900; 500, 800, 1000)
4	(800, 1000, 1200; 600, 1000, 1500)
5	(1200, 1300, 1500; 900, 1300, 1600)
6	(1400, 1600, 1700; 1300, 1600, 1800)
7	(1800, 2000, 2200; 1500, 2000, 2400)

Find the optimum replacement time of the equipment.

Solution: The above is an intuitionistic fuzzy replacement problem where the money has a value over time.The costs are given as triangular intuitionistic fuzzy numbers.

year	$\tilde{R}_t$ (Rs. In thousands)	V^{t-1}	$\tilde{R}_t V^{t-1}$ (Rs. In thousands)	$\sum_{t=1}^{n} \tilde{R}_t V^{t-1}$ (Rs. In thousands)
1	(0.4,0.5,0.6;0.3,0.5,0.8)	1	(0.4,0.5,0.6;0.3,0.5,0.8)	(0.4,0.5,0.6;0.3,0.5,0.8)
2	(0.4,0.6,0.8;0.2,0.6,0.9)	0.9	(0.36,0.54,0.72;0.18,0.54,0.81)	(0.76,1.04,1.32;0.48,1.04,1.61)
3	(0.7,0.8,0.9;0.5,0.8,1.0)	0.81	(0.57,0.65,0.73;0.41,0.65,0.81)	(1.33,1.69,2.05;0.89,1.69,2.42)
4	(0.8,1.0,1.2;0.6,1.0,1.5)	0.73	(0.58,0.73,0.88;0.44,0.73,1.1)	(1.91,2.42,2.93;1.33,2.42,3.52)
5	(1.2,1.3,1.5;0.9,1.3,1.6)	0.66	(0.79,0.86,0.99;0.59,0.86,1.06)	(2.7,3.28,3.92;1.92,3.28,4.58)
6	(1.4,1.6,1.7;1.3,1.6,1.8)	0.59	(0.83,0.94,1.0;0.77,0.94,1.06)	(3.53,4.22,4.92;2.69,4.22,5.64)
7	(1.8,2.0,2.2;1.5,2.0,2.4)	0.53	(0.95,1.06,1.17;0.8,1.06,1.27)	(4.48,5.28,6.09;3.49,5.28,6.91)

Year	$\tilde{C} + \sum_{t=1}^{n} \tilde{R}_t V^{t-1} = \tilde{F}_n$ (Rs. In thousands)	$\sum_{t=1}^{n} V^{t-1}$	$\tilde{\omega}(n) = \frac{\tilde{F}_n}{\sum_{t=1}^{n} V^{t-1}}$ (Rs. In thousands)	$H(\tilde{a})$
1	(2.9,3.5,3.8;2.4,3.5,4.6)	1	(2.9,3.5,3.8;2.4,3.5,4.6)	3.463
2	(3.26,4.04,4.52;2.58,4.04,5.41)	1.9	(1.73,2.14,2.4;1.37,2.14,2.87)	2.116
3	(3.83,4.69,5.25;2.99,4.69,6.22)	2.71	(1.42,1.74,1.94;1.11,1.74,2.3)	1.716
4	(4.41,5.42,6.13;3.43,5.42,7.32)	3.44	(1.28,1.57,1.78;0.99,1.57,2.12)	1.556
5	(5.2,6.28,7.12;4.02,6.28,8.38)	4.1	(1.25,1.51,1.71;0.96,1.51,2.01)	1.496*
6.	(6.03,7.22,8.12;4.79,7.22,9.44)	4.69	(1.27,1.52,1.71;1.01,1.52,1.98)	1.506
7.	(6.98,8.28,9.29;5.59,8.28,10.71)	5.22	(1.33,1.57,1.77;1.06,1.57,2.03)	1.559

Where $H(\tilde{a}) = \frac{a_1+2a_2+a_3+a_1'+2a_2+a_3'}{8}$. $H(\tilde{a})$ is minimum at the fifth year. Therefore optimum replacement is at the end of the 5^{th} year.

5. Conclusion

The theory of intuitionistic fuzzy numbers is the generalization of fuzzy sets. Intuitionistic fuzzy numbers can be used to any activity demanding human expertise and knowledge which are inevitably imprecise and not totally reliable. To deal with this impreciseness the proposed method presents a procedure to find the optimum time of replacement for intuitionistic fuzzy replacement problem with present worth factor, all costs being assumed as TIFN. The proposed procedure is supported by an example. The procedure is very simple and more effective.

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