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Study of Retrial Chain with Non Continued Clients Along with Orbital Search

Research Article

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Abstract: In this paper, we expand the search approach to a syntactically complicated alone server retrial queueing specimen with non consistent clients. We account a retrial queueing prototype with circular search in which an identifying subscriber after some failed retrials allocates up further repetitions as well as abdicates the mechanism (Almasi, [1]). Let I_p be the liability that after the pth approach fails, a consumer will make the $(p+1)^{th}$ one. The set of chances $(p+1)^{th}$ is labeled the persistence exercise. We assume that the chance of a call restarts after downfall of a duplicate attempt, does not depend on the count of previous attempts (i.e., $I_2 = I_3 = \dots$) algebraic calculations in telephone networks display that this is a realistic accumulation in conduct to such networks. One consequential characteristic of the model below consideration is that, for frequent difficulties, the cases $I_2 < 1$ and $I_2 < 1$ yield centrally asymmetric acquisitions. The case $I_2 = 1$ can be examined in complete detail whiles the case $I_2 < 1$ is far more complex along with sealed form acquisition is attainable only for exponential service time distribution (Wang et. al., [2]). This paper is arranged as succeeds. In section 2, we explain the algebraic prototype: For the cases $I_2 = 1$, the simulation is examined in full assembly in section 3. In section 3.1, consistency clause is derived along with in 3.2, the bounding detachment of the mechanism state is acquired based on the supplementary-variable approach. The architecture of the active duration as well as its assertion in clauses of Laplace adapts have been consulted in 3.4 In section 3.5, we assign a direct procedure of algorithm for the first along with second circumstances attendant the busy time. In section 4, the case $I_2 < 1$ is counted along with the locked form solution is acquired for the exponential sustenance time separation in conditions of hyper geometric series. In section 4, we demonstrate many numerical examples to demonstrate the effect of the guidelines on the mechanism activity.

Keywords: Retrial chain, non continued clients, complex retrial.

1. Introduction

The concept of non continued and unreliable clients has been studied in the past by many mathematicians. For the specific case of $O|J|1$ retrial chain, Boxma, Perry, Stadje and Zacks [3] worked on the busy period and customer impatience. Impatience is a very natural and important concept in queuing models. There is a wide range of situations in which customers may become impatient when they do not receive service fast enough. One may think of customers at call centers, or of customers representing perishable goods, like blood samples which wait to be tested and become obsolete after a certain due date.

They explained the active duration as the time beginning at an epoch when an attaining consumer determines an empty procedure along with closing at the next withdrawal epoch at which the system is absent. Without decrease of collectivity we may accumulate that at duration x = 0 the system is empty. i.e., $B(O) = P(O) = O$ along with one primary consumer just attains at time $x = O$. Then a supposition active duration begins as well as completes at the first departure epoch at which the manner $\{(B(x), P(x)), x \geq 0\}$ acknowledgment to the state (0, 0) for the first time.

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They considered the M/G/1 + G model, in which the patience refers to the waiting (not sojourn) time of the arriving customer. They first derived an integral equation for the distribution of the busy period length, conditional on the initial workload in the system being with this they were able to solve this equation in the case of exponential patience, for a large class of service time distributions. Thus they obtained the Laplace Stieltjes transform (LST) of the distribution of the length of a busy period that starts with some workload v. Integration w.r.t. the service time distribution gives the transform of the unconditional busy period length. In the case of a discrete patience distribution, they followed another approach which is based on transform methods, the Wald martingale and stopping times. Again the LST of the busy period distribution is obtained. Farraj, Abdallah K., Scott L. Miller, and Khalid A. Qaraqe improved the idea in their paper Queue performance measures for cognitive radios in spectrum sharing systems [4].

This paper investigates the queuing system of a cognitive user who is trying to communicate, using spectrum sharing methods, over a channel reserved to a licensed user. For the cognitive user to use the channel, its transmission power should satisfy the outage probability requirement of the primary user. In this environment, the cognitive user is found to have an M/G/1 queue model, where the queue service rate is equal to the cognitive channel capacity. Some queue performance measures are investigated. These include the first two moments of the service time process, mean waiting and transit times, mean number of waiting and transit packets, mean duration of server busy period, and mean number of packets served during a busy period. They have also investigated the effects of changing some of the communication environment parameters on the queue behavior.

In earlier works, T. V. S. Somvanshi, Quazzafi Rabbani, and Sandeep Dixit [5] have explained the active duration as the time beginning at an epoch when an attaining consumer determines an empty procedure along with closing at the next withdrawal epoch at which the system is absent. In their paper Queuing Process with Two Server in Bulk- Service, they presented a two- server queuing Process fed by poisson arrivals and exponential service time distributions had been considered under the bulk-service discipline. Time-dependent probabilities for the queue length have been obtained in terms of Laplace transforms, from which different measures associated with the queuing process could be determined. The mean queue-length and the distributions of the length of busy periods for (i) at least one channel are busy and (ii) both channels being busy are obtained. In this paper the detonation has been used from Gross, Donald, et al, [6].

2. The Mathematical Model

We account an individual server queuing approximations to which primary consumer accomplishes according to a Poisson stream of rate μ. If the server is energetic at the duration of arrival of an atomic client, then with proneness $1 - I_1$ the client departs the arrangement without service along with probability $I_2 > 0$ forms a beginning of duplicate calls (Manuel et. al, [7]). Every such source creates a Poisson discipline of recurring calls with amount j?, when there are j buyers in the circularity. If the reiterated call determines the server clear, it is availed and leaves the system constantly. Otherwise, e.g., if the server is achieved at the duration of achievement of a repeated call, with possibility $1 - I_2$, the source leaves the process without application as well as with possibility I_2, it goes back to the orbit and retries for service. The service times are autonomous with average proneness function $(f)(E(0) - 0)$. Let $(s) = \int\limits_0^\infty e^{-sf} dB(f)$ be the Laplace-Stieltjes adjust of the service time detachment $E(f), \gamma_m = (-1)^m \gamma^m(0)$ be the m^{th} transience of the service time about the beginning, (Saaty, Thomas L., [8])

$i(f) = \frac{E'(f)}{1-E(f)}$ be the instantaneous application caliber allotted that the erased service duration is equivalent to x. Let ς be the epoch at which the n^{th} exercise accomplishment ensues. Currently after this service achievement, the server abandons for aspiration of consumers in the circularity with many perceived possibility p. With probability $1 - c$ the server stays idle.

In the latter case, the effect to follow depends on the competition between primary achievements along with retrial attempt. The look for age is acquired to be insufficient. The flows of primary arrivals, retrial of consumers along with service periods are acquired to be mutually autonomous.

At time t, let $P(x)$ be the abound of buyers in the circularity as well as $B(x)$ be the place of the server $B(x) = 1$ or 0 according as the server is active or liberate). The state area of the mechanism $E(x) = (B(x), P(x))$ is $S = \{0, 1\} \times N$. The adaptations between states are displayed in figure 1.

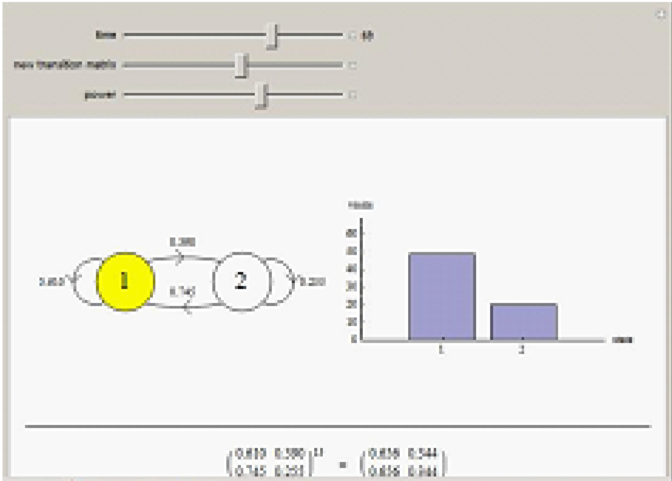

Figure 1. **Two-State, Discrete-Time Markov Chain**

3. The Case $I_2 = 1$

3.1. Stability Condition

We now examine the compulsory as well as enough provision for the process to be balanced. Let $P_1 = P(ip)$ be the enumerate of consumers in the circularity at the duration ip of the p$^{\text{th}}$ service completion.

Theorem 3.1. *$\{P_p, p \in N\}$ is positive recurrent if and only if $\varrho I_1 < 1$, where $\varrho = \mu\gamma_1$.*

Proof. $\{P_p, p \in N\}$ satisfies the equation $P_p = P_{p-1} - E_p + W_p$. Where W_p is the count of buyers attaining till the p$^{\text{th}}$ service time, with

$$E_p = \begin{cases} 1, & \text{if the i}^{\text{th}} \text{ consumer exercised behaves from the circularity} \\ 0, & \text{otherwise.} \end{cases}$$

The provisional dissolution of the Bernoulli random adaptable E_p is given by

$$c\left\{E_p = \frac{1}{P_{p-1} = b}\right\} = (1-c)\frac{bv}{(\mu + bv)} + c;\ b \neq 0,$$
$$c\left\{E_p = \frac{0}{P_{p-1} = b}\right\} = (1-c)\frac{\lambda}{(\mu + nv)}$$

The random variable W_p has distribution

$$m_b = C\{W_p = b\} = \int_0^\infty \frac{(\mu I_1 f)^b}{b!} e - \mu I_1 f dB(f)$$

With generating function

$$\sum_{b=0}^{\infty} m_b N^b = \gamma(\mu I_1 - \mu I_1) = m(N)$$

And mean value

$$GW_p = \sum_{b=0}^{\infty} bm_b = \mu I_1 \gamma_1 = \varrho I_1$$

due to the subsequence of random variables p from a markov succession. It is not difficult to see that $\{P_p;\ p \in N\}$ is clean as well as an oscillating. To confirm periodicity, we shall apply Foster's criterion, which accepts that a basic along with a recurrent Markov chain is periodic if there continues a non-negative behave $g(s), s \in N$ and $\epsilon > 0$ such that the mean drift.

$$\vartheta_s = G\left[g(P_{p+1}) - \frac{g(P_p)}{P_p} = s\right]$$

Is finite for all $s \in N$ and $\vartheta_s \leq -\epsilon$ for all $s \in N$, liberate may be a confined allotment. In our case, we accumulate the function $g(s) = s$. We then obtain

$$\vartheta_s = \begin{Bmatrix} \varrho I_1, & \text{if } s - 0 \\ \frac{-(1-c)sv}{\mu+sv} - c + \varrho I_1, & \text{if } s \geq 1. \end{Bmatrix}$$

Clearly, if $\varrho I_1 > 1$, then we have $\lim_{s\to\infty} \vartheta_s < 0$. Therefore, the embedded Markov chain $\{P_p, p \in N\}$ is periodic. To certify the constraint of the clause, we apply assumption 3.1 in Sennott et. al. (1983), which states that, $\{P_p, p \in N\}$ is non-ergodic if it appeases the Kaplan's provision, $\vartheta_q < +\infty (q \geq 0)$ and there is a q_0 such that $\vartheta_q < +0$, $(q \geq q_0)$. Furthermore, Kaplan's condition is satisfied because there exists an index m such that $C_{pq} = 0$ for $q < p - m : q > 0$ where $C = \{C_{pq}\}_{p,q} \in N$ is the one-step transition matrix affiliated with $\{P_p;\ p \in N\}$. This achieves the certainty. $\square$

3.2. Analysis of the Steady State Probabilities

In this area we inspect the consistent state refraction of our queueing system.

Theorem 3.2. *If $\varrho I_1 = \mu I_1 \gamma_1 < 1$ along with the mechanism is in the connected state, then the joint detachment of the server state along with queue length (Zahmati et. al, 2010)*

$$C_{0b}(f) = C\{B(x) = 0,\ P(x) = b\}$$
$$C_{1b}(f) = \frac{d}{dx} C\{B(x) = 1,\ \xi(x) < f,\ P(x) = b\}$$

has a partial generating function

$$C_0(N) = \sum_{b=0}^{\infty} N^b C_{0b} = \frac{\mu c C_{00}}{v} (N)^{\frac{-\mu c}{v}} \int_0^N x^{\frac{\mu c}{v} - 1} \varpi(N, x) dt \tag{1}$$

$$C_1(N, f) = \sum_{b=0}^{\infty} N^b C_{1n}(f) = \frac{\mu(1-N)}{[\gamma(\mu I_1 - \mu I_1 N) - N]} C_0(N)(1 - E(f)) e^{-\mu I_1 (1-N) f} \tag{2}$$

Where

$$r(x_1 x_2) = \exp\left\{\frac{\mu(1-c)}{v} \int_{x_1}^{x_2} \frac{1 - \gamma(\mu I_1 - \mu I_1 u)}{v - \gamma(\mu I_1 - \mu I_1 u)} du\right\} \tag{3}$$

$$C_{00} = \frac{\mu(1 - \mu I_1 \gamma_1) r(0,1)}{\mu c(1 + \mu\gamma_1 - \mu I_1 \gamma_1) \int_0^1 (x)^{\frac{\mu c}{\nu} - 1} r(0, x) dt} \tag{4}$$

If in the case $B(x) = 1$ we clumsiness the erased application duration, $\xi(t)$, then for the probabilities $C_{1b} = P\{B(x) = 1,\ P(x) = b\}$,

$$C_1(N) = \sum_{b=0}^{\infty} N^b C_{1b} = \frac{1 - \gamma(\mu I_1 - \mu I_1 N)}{I_1(\gamma(\mu I_1 - \mu I_1 N) - N)} C_0(N) \tag{5}$$

Proof. The set of algebraic equilibrium equations are acquired as

$$(\mu + bv)C_{0b} = [1 - (1 - \epsilon_{bo})] \int_0^\infty C_{1b}(f)i(f)dx \tag{6}$$

$$C'_{1b}(f) = -(\mu H_1 + i(f))C_{1b}(f) + \mu I_1 C_{1,b-1}(f) \tag{7}$$

$$C_{1b}(0) = \mu C_{0b} + (b+1)\nu C_{0,b+1} + c\int_0^\infty C_{1,b+1}(f)i(f)dx \tag{8}$$

For creating application $P_0(z)$ along with $P_1(z,x)$ these equations are converted to:

$$\mu C_0(N) + vNC_0(N) = (1-c)\int_0^\infty C_1(N,f)i(f)dx + \mu c C_{0_0} \tag{9}$$

$$\frac{\partial C_1(N,f)}{\partial x} = -(\mu I_1(1-N) + i(f))C_1(N,f) \tag{10}$$

$$(N - c\gamma(\mu I_1 - \mu I_1 N))C_1(N,0) + \mu c C_{0_0} = \mu N c'_0(N) + \mu N C_0(N) \tag{11}$$

Solving (10) yields

$$C_1(N,f) = C_1(N,f)(1 - E(f))e^{-\mu I_1(1-N)f} \tag{12}$$

Combining (9), (11) and (12), and aft.er some algebra we get

$$\begin{aligned} vN &= (N - \gamma(\mu I_1 - \mu I_1 N))C_0^{'}(N) + (\mu N - ((1-c)\mu c)\gamma(\mu I_1 - \mu I_1 N))C_0(N) \\ &= \mu c C_{0_0}(N - \gamma((\mu I_1 - \mu I_1 N)) \end{aligned} \tag{13}$$

Coefficient of $C_0(N)$ has two zeros $N1 = 0$ and $N2 = 1$. Choose an arbitrary point $z\varepsilon(0,1)$ Solving (13) for $N\varepsilon(0,z)$, we get

$$C_0(N) = \left[(\frac{N}{z})^{\frac{\mu\varrho}{v}} r(z,N)\right]^{-1} \left\{C_0(z) + \frac{\mu c C_{0_0}}{\mu(z)^{\frac{\lambda\rho}{v}}}\int_z^N x^{\frac{\mu c}{v}-1} r(z,x)dt\right\} \tag{14}$$

As $N \to 0 + C_0(0) < +\infty$ and $\left(\frac{N}{z}\right)^{-\frac{\mu\varrho}{v}}$ diverges. Thus we get

$$C_0(z) = \frac{\mu\varrho C_{0_0}}{\mu(N)^{\frac{\mu\rho}{v}}}\int_0^N (t)^{\frac{\lambda c}{\mu}-1} r(N,f)dt \tag{15}$$

On the difference hand, determining (13) for $N\varepsilon(z,1)$ and taking limit as $N \to 1-$; we get

$$C_0(N) = \frac{C_0(1)r(N,1)}{(N)^{\frac{\mu c}{v}}} - \frac{\mu\rho C_{0_0}}{v(N)^{\frac{\mu c}{v}}}\int_z^1 (f)^{\frac{\mu c}{\mu}-1} r(N,f)dt \tag{16}$$

For acquiring the association (16) it should be registered that

$$\lim_{N\to 1-}\left[\frac{1 - \gamma(\mu I_1 - \mu I_1 N)}{\gamma(\mu I_1 - \mu N) - N}\right] = \frac{+\mu I_1\gamma_1}{1 - \mu I_1\gamma_1} < \infty$$

Equating (15) and (16) we get

$$C_0(1) = \frac{\mu c C_{0_0}}{vr(0,1)}\int_0^1 (x)^{\frac{\mu c}{v}-1} r(0,x)dt \tag{17}$$

Next we can comprise acquisition of (13), as (1). Concurring the equations (11), (13), we get (2). Since $C_N(N) = \int_0^\infty C_1(N,f)dx$ we obtain (5). Now conducting the categorizing clause $Co(1) + C1(1) = 1$, we get

$$C_0(1) = \frac{1 - \mu I_1\gamma_1}{1 + \mu\gamma_1 - \mu I_1\gamma_1} \tag{18}$$

Using (17) and (18) we obtain (4). □

Corollary 3.3. *The sectional factorial circumstances* O_m^p *for* $p \in \{0, 1\}$, $m \in 0, 1$ *are given by*

$$O_0^0 = \frac{1 - \mu I_1 \gamma_1}{1 + \mu\gamma_1 - \mu I_1 \gamma_1} \tag{19}$$

$$O_0^1 = \frac{\mu\gamma_1}{1 + \mu\gamma_1 - \mu I_1 \gamma_1} \tag{20}$$

$$O_1^0 = \frac{\mu c C_{0_0}}{v} + \frac{\mu(\mu I_1 \gamma_1 - c)}{v(1 + \mu\gamma_1 - \mu I_1 \gamma_1)} \tag{21}$$

$$O_1^1 = \frac{\mu^2}{(1 - \mu I_1 \gamma_1)\left\{\frac{v I_1 \gamma_2 + I_1 \gamma_1^2 - c\gamma_1}{v(1 + \mu\gamma_1 - \mu I_1 \gamma_1)} + \frac{c\gamma_1}{v}\right\}} \tag{22}$$

3.3. Analysis of the Busy Period

Boxma, Onno, et al. (2010) and Somvanshi et.al (2012) explained the active duration is described as the time beginning at an epoch when an attaining consumer determines an empty procedure along with closing at the next withdrawal epoch at which the system is absent. Without decrease of collectivity we may accumulate that at duration $x = 0$ the system is empty. i.e., $B(O) = P(O) = O$ along with one primary consumer just attains at time $x = O$. Then a supposition active duration begins as well as completes at the first departure epoch at which the manner $\{(B(x),\ P(x), x \in 0\}$ acknowledgment to the state (0,0) for the first time. Let $C_0(x)$ be the detachment exercise of the active duration L along with $\pi_0^*(s) = G[e^{-sL}]$ is the Laplace-Stieltjes convert of L. Let us define the changeable taboo Probabilities of mechanism

$$C_0(x) = C\{B(x) = 0, P(x) = 0\}, x \geq 0 \tag{23}$$

$$C_b(x) = C\{L > x, B(x) = 1, P(x) = b\}, x \geq 0, b \geq 1 \tag{24}$$

$$R_b(f, x) = C\{L > x, P(x) = 1, P(x) = b, f \leq \xi(x) < f + dx\}, x \geq 0, f \geq 0, b \geq 0 \tag{25}$$

And, the boundary conditions are

$$C_0(0) = 0, R_b(f, 0) = \epsilon_{b0}\epsilon(f), b \geq 0 \tag{26}$$

Where ϵ_{b0} and $\epsilon(f)$ are Kronecker and Dirac functions, respectively. With the help of supplementary adjustable, we acquire the Kolmogorov equations that govern the dynamics of the procedure activity as:

$$\frac{d}{dt}C_0(x) = \int_0^\infty R_0(f, x)i(f)dx \tag{27}$$

$$\frac{d}{dt}C_b(x) = -(\mu + bv)C_b(x) + (1 - c)\int_0^\infty R_b(f, x)i(f)dx, b \geq 1 \tag{28}$$

$$\frac{\partial R_b(f, x)}{\partial t} + \frac{\partial R_b(f, x)}{\partial t} = -(\mu I_1 + i(f))R_b(f, x) + \mu I_1(1 - \epsilon_{b0})R_{b-1}(f, x), b \geq 0 \tag{29}$$

$$R_b(0, x) = \mu(1 - \epsilon_{b0})C_b(x) + (b + 1)\nu C_{b+1}(x) + c\int_0^\infty R_{b+1}(f, x)i(f)dx, b \geq 0 \tag{30}$$

To determine the above equations, we instant create Laplace transforms along with generating functions:

$$C_b(s) = \int_0^\infty e^{-sx}C_b(x)dt, b \geq 1 \tag{31}$$

$$R_b(f, s) = \int_0^\infty e^{-sx}R_b(f, x)dt, b \geq 0 \tag{32}$$

$$C(s, N) = \sum_{b=1}^\infty N^b C_b(s) \tag{33}$$

$$R(x, s, N) = \sum_{b=0}^\infty N^b R_b(f, s) \tag{34}$$

Then equations (27)-(30) become

$$\pi_0(s) = \int_0^\infty R_0(f,s)i(f)dx \tag{35}$$

$$\nu N\frac{\delta C(s,N)}{\partial N} + (s+\mu)C(s,N) = (1-c)\int_0^\infty (R(f,s,N) - R_0(f,s))i(f)dx \tag{36}$$

$$\frac{\partial R(f,s,N)}{\partial x} + (s+\mu I_1(1-N)+i(f))Q(f,s,N) = \partial(f) \tag{37}$$

$$R(0,s,N) = \frac{v}{(1-c)}\frac{\partial C(s;N)}{\partial N} + \frac{c(s+\mu(1-c)N}{(1-c)N}C(s,N) \tag{38}$$

Solving (36), we get:

$$R(f,s,N) = (1+R(0,s,N))(1-E(f))e^{-(s+\mu HI_1(1-N))f} \tag{39}$$

Combining (33), (35), (37) and (38), we get:

$$\begin{aligned}\nu N(N-\gamma(s+\mu I_1(1-N)))\frac{\partial C(s,N)}{\partial N} &+ ((s+\mu)N - ((1-c)\mu N\\ &+ (s+\mu)c)\gamma(s+\gamma I_1(1-N))C(s,N) = (1-c)N(\gamma(s+\mu I_1(1-N)) - \pi_0^*(s))\end{aligned} \tag{40}$$

Let

$$h(N,s,f) = N - \gamma(s+\gamma I_1(1-N)) \tag{41}$$

For $R_e(s) > 0$, $|N| \leq 1$ and $|f| \leq 1$. Next for each stabilized amount of $(s,f)h$ has a genuine zero $\pi_\infty^*(\frac{s}{\mu I_1})$ in the unit disc $|N| \leq 1$, where $\pi_\infty^*(\frac{s}{\mu I_1})$ is the Laplace convert of the span of the active duration in the standard $O|J|(1|\infty$ consecution (without retrials) with endurance function I_1. Thus, co-operative of $\frac{\partial C(s,N)}{\partial N}$ has the two zeros$N_1 = 0$ and $N_2 = \pi_\infty(\frac{s}{\mu I_1})$. Choose an arbitrary number $\bar{N} \in (N_1, N_2)$. Solve (40) for $N\varepsilon[\bar{N}, N_2]$;

$$C(s,N) = [(\frac{N}{\bar{N}})^{\frac{(s+\mu)}{\nu}c}e^{\frac{\mu(1-c)(N-\bar{N})}{\nu}}r(N-\bar{N})]^{-1}\{C(s,\bar{N}) + \int_{\bar{N}}^N R(u)(\frac{u}{\bar{N}})^{\frac{(s+\mu)c}{\nu}}e^{\frac{\mu(1-c)(u-\bar{N})}{\nu}}r^*(\bar{N},u)du\} \tag{42}$$

Where

$$R(u) = \frac{(1-c)u\gamma(s+\mu I_1(1-u)) - (1-c)u\pi_0^*(s)}{\nu u(u-\gamma(s+\mu I_1(1-u)))} \tag{43}$$

And

$$r^*(x_1,x_2) = \exp\left\{\frac{1-c}{\nu}\int_{x_1}^{x_2}\frac{s+\mu(1-u)}{u-\gamma(s+\mu I_1(1-u))}du\right\} \tag{44}$$

As $N - +N2$, $[r*(z,Z)] - l$ diverges and $C(s,N2)$ is finite. Thus,

$$C(s,\bar{N}) = \int_{N_2}^{\bar{N}} R(u)(\frac{u}{\bar{N}})^{\frac{(s+\mu)}{\nu}}e^{\frac{\mu(1-c)(u-\bar{N})}{\nu}}r^*(\bar{N},u)du \tag{45}$$

In the future position, determine (40) for $N \in [N_1, \bar{N}]$ along with acquiring bound as $N \in [N_1, \bar{N}]$ we get:

$$C(s,\bar{N}) = \int_{N_1}^{\bar{N}} R(u)(\frac{u}{\bar{N}})^{\frac{(s+\mu c)}{\mu}}e^{\frac{\mu(1-c)(\mu-\bar{N})}{\mu}}r^-(\bar{N},u)du \tag{46}$$

Equating the above assertions for $C(s,\bar{N})$ along with consequent many algorithms, we get:

$$\pi_0^*(s) = \frac{\int_0^{\pi_\infty^*(s/\mu I_1)}\left\{\frac{u^{\frac{(s+\mu)c}{\nu}}e^{\frac{(1-c)u}{\nu}}\gamma(s+\mu I_1(1-u))r^*(0,u)}{\nu-\gamma(s+\mu I_1(1-u))}\right\}}{\int_0^{\pi_\infty^*(s/\mu I_1)}\left\{\frac{u^{\frac{(s+\mu)c}{\nu}}e^{\frac{(1-c)u}{\nu}}r^*(0,u)}{s-\gamma(s+\mu I_1(1-u))}\right\}} \tag{47}$$

Note: that the domain of $\pi_0^*(s) > 0$. Taking limit as $s \to 0$, we acquire undetermined expressions. These expressions behave complicated to be determined applying L'Hospital rule.

3.4. Calculation of the First and Second Moment of the Busy Period

In the conclusive area, Farraj et. al. (2011) approached even though he have accumulated the expression for $G[e^{-sL}]$, the first bases of L cannot be added from it. A direct way to find the first moment of $G(L)$ of the active duration is allowed by the concept of regenerative mechanism as follow: The bounding possibilities Cpq can be acknowledged as

$$C_{pq} = \frac{G[Q_{pq}]}{\lambda^{-1} + G(L)} \tag{48}$$

where Q_{pq} is the portion of duration in a regenerative cycle always which the technique is in the state (p, q). Since $G[Q_{0_0}] = \frac{1}{\lambda}$ we get $G(L) = \mu^{-1}(C_{0_0}^{-1} - 1)$ in this area we acquire an obstructed configuration expression for applying the approach $G(L^2)$ chosen. First we define

$$z(N) = C(0, N) = \sum_0^\infty N^b \int_0^\infty C_b(x)dt \tag{49}$$

$$z(N) = \int_0^\infty R(f, 0, N)dx = \sum_{b=0}^\infty N^b \int_0^\infty \int_0^\infty R_b(f, x)dtdx \tag{50}$$

Then

$$z(1) = \int_0^\infty C[L > x, C(x) = 0]dt \tag{51}$$

And

$$y(1) = \int_0^\infty C[L > x, C(x) = 1]dt \tag{52}$$

Putting $s = 0$ in (40), we get,

$$vN(N - \gamma(\mu I_{1N}))z'(N) + (\mu N - (1 - c)\mu N + \mu c)\gamma(\mu I_1 - \mu I_{1N})z(N) = (1 - c)N[\gamma((\mu I_1 - \mu I_{1N}) - 1] \tag{53}$$

Apocalypse of the above differential equation is acquired by conducting the same approach that we have conducted for determining (13), then

$$z(N) = \frac{-1}{\mu} + \frac{c}{v}(N)^{\frac{-\mu c}{v}} \int_0^N (x)^{\frac{-\mu c}{v} - 1} r(N, x)dt \tag{54}$$

where $r(N, x)$ is as in (3) and

$$N(1) = \frac{-1}{\mu} + \frac{c}{vr(0, 1)} \int_0^1 (u)^{\frac{-\mu c}{v} - 1} r(0, x)du \tag{55}$$

On the other hand, putting $s = 0$ in (37), we get

$$R(f, 0, N) = (1 + R(0, 0N))(1 - E(f))e^{-\mu I(1-N)f} \tag{56}$$

So that,

$$y(N) = \frac{(1 + R(0, 0, N)((1 - \beta(\mu I_1 - \mu I_{1N}))}{\mu I_1 - \mu I_{1N}} \tag{57}$$

Putting $s = 0$ in (38), and as $N \to 1$, we get

$$R(0, 0, 1) = \frac{\mu}{(1 - \mu I\gamma_1)(z(1) + I_1\gamma_1)} \tag{58}$$

Collecting limit as $N \to 1$ in (57) we claim another expression for $R(O, 0, 1)$ along with equating the above two assertions for $R(O, 0, 1)$, we acquire:

$$y(1) = \frac{(1 - \mu N(1))}{1 - \mu I_1\gamma_1}\gamma_1 \tag{59}$$

Now,

$$G[L] = z(1) + y(1) = \frac{\gamma_1}{1 - \mu I_1 \gamma_1} + \frac{(1 + \mu\gamma_1 - \mu I_1 \gamma_1)}{1 - \mu I_1 \gamma_1} z(1) \tag{60}$$

Now, in order to compute $[L^2]$, we define

$$\phi(s, N) = \frac{\partial C(s, N)}{\partial s} \quad \text{and} \quad c(N) = \phi(s, N) \tag{61}$$

Differentiating (40) with respect to s, along with setting $s = 0$, we acquire after some approximation

$$B^{'} + \frac{\mu N - ((1-c)\mu N + \mu c)\gamma(\mu I_1 \gamma_1)}{vN(N - \gamma(\lambda I_1 - \mu I_1 N))} c(N) = h(N) \tag{62}$$

Where

$$\begin{aligned} h(N) = \frac{1}{\nu N(N - \gamma(\mu I_1 - \mu I_{1N}))} \Bigg\{ & (1-c)N\gamma^{'}(\mu I_1 - \mu I_{1N}) + \frac{(1-\varrho)N\gamma_1}{1-\varrho} - \frac{(1-\varrho)N(1 - \gamma(\mu I_1 - \mu I_{1N}))\gamma^{'}(\mu I_1 - \mu I_{1N})}{N - \gamma(\mu I_1 - \mu I_{1N})} \\ & + \frac{(1-c)N(1 + \mu\gamma_1 - \varrho)}{1-\varrho} z(1) - \Bigg(N - ((1-c)\mu N + \mu c)\gamma^{'}(\mu I_1 - \mu I_{1z}) - c\gamma(\mu I_1 - \mu I_{1N}) \\ & + \frac{(\mu N - ((1-c)\mu N + \mu c)\gamma(\mu I_1 - \mu I_{1N}))\gamma^{'}(\mu I_1 - \mu I_{1N})}{N - \gamma(\mu I_1 - \mu I_{1N})} \Bigg) z(N) \Bigg\} \end{aligned} \tag{63}$$

Solving (62), we get:

$$a(N) = (N^{\frac{\mu c}{v}} r(1, N))^{-1} \{ \int_1^N x^{\frac{\mu c}{v}} h(x) r(1, x) dt \} \tag{64}$$

and

$$a(1) = \frac{1}{r(0,1)} \int_0^1 h(x) x^{\frac{\mu c}{v}} r(0, x) dt \tag{65}$$

Now we consider the case $B(x) - 1$ and define

$$\phi(f, s, N) = \frac{\partial R(f, s, N)}{\partial N} \quad \text{and} \quad d(N) = \int_0^\infty \phi 1(f, 0, N) dx \tag{66}$$

Differentiating (37) with respect to s and setting $s = 0$, yields:

$$\frac{\partial \phi_1(f, 0, N)}{\partial x} = -R(f, 0, N) - (\mu I_1 - \mu I_1 N + I(f))\phi_1(N, 0, f) \tag{67}$$

Solving (67) and after some algebra, we get:

$$\phi_1(f, 0, N) = (1 - E(f)) e^{-(\mu I_1 - \mu I_1 N) f} \phi_1(0, 0, N) - f(1 + R(0, 0, N)) \tag{68}$$

Connecting the above assertion with respect to x we determine that

$$d(N) = \phi_1(0, 0, 1) \frac{1 - \gamma(\mu I_1 - \mu I_1 N)}{\mu I_1 - \mu I_1 N} - (1 + R(0, 0, N) \int_0^\infty e^{-(\mu I_1 - \mu I_1 N) f} f(1 - E(f)) dx \tag{69}$$

Thus

$$d(1) = \gamma_1 \phi_1(0, 0, 1) - \frac{\gamma_2}{2}(1 + R(0, 0, 1)) \tag{70}$$

$R(0, 0, 1)$ is given by (58). To obtain $\phi_1(0, 0, 1)$ differentiate (38) with respect to s and set $(s, N) = (0, 1)$. These yields:

$$\psi_1(0, 0, 1) = \frac{1}{(1-c)}(\mu N(1) + cz(1) + va^{'}(1)) \tag{71}$$

From (62)

$$a^{'}(1) = h(1) + \frac{\mu}{v}\frac{\mu I_1\gamma_1}{(1-\mu I_1\gamma_1)}a(1) \tag{72}$$

Thus,

$$\psi_1(0,0,1) = \frac{1}{(1-c)}(cz(1) + vh(1) + \frac{\mu}{1-\mu I_1\gamma_1}c(1)) \tag{73}$$

Using (58), (69) and (73) we get

$$d(1) = \frac{-\gamma_2}{2} + (\frac{c\gamma_1}{1-c} - \frac{\mu\gamma_2}{2(1-c)})z(1) - \frac{v\gamma_2}{2(1-c)}z^{'}(1) + \frac{\mu\gamma_1}{(1-c)(1-cI_1)}c(1) + \frac{\mu\gamma_1}{(1-c)}h(1) \tag{74}$$

From (53)

$$z'(1) = \frac{(1-c)\varrho I_1}{v(1-\varrho I_1)} - \frac{\lambda(c-\varrho I_1)}{v(c-\varrho I_1)}z(1) \tag{75}$$

From (63)

$$h(1) = \frac{1}{2\mu(1-cI_1)^2}\left\{\begin{array}{c} ((\mu I_1)^2(1-c)\gamma_2 - 2\mu(1-c)\gamma_1 - 2(1-cI_1)(1-cI_1)1 - c\varrho I_1) \\ -2\mu^2(1-c)I_1\gamma_2)z(1) - 2(1-c)(1+\mu\gamma_1-\varrho I_1)z^{'}(1) \\ -2\mu I_1(1-c)(1-\varrho I_1)(\gamma_1^2\gamma_2) + (1-p)(G[L]-\gamma_1)2(1-cI_1) - (\mu I_1)^2\gamma_2 \end{array}\right\} \tag{76}$$

Now

$$G[L^2] = -1(B(1)+d(1)) = 2\int_0^\infty xC\{L>x\}dt \tag{77}$$

Using (60), (75), (76) and (77) we get:

$$\begin{aligned} G[L^2] = {} & \frac{1}{\nu(1-\varrho I_1)^3}\left\{\begin{array}{c} [\mu\nu\gamma_2 - 2\mu\gamma_1(c-\varrho I_1)(1+\mu\gamma_1-\varrho I_1)]a(1) + 2\gamma_1\varrho I_1(1-c)(1-cI_1) \\ +2\gamma_1^2\varrho I_1[\mu(1-c) - \nu\varrho I_1(1-\varrho I_1)] + \nu\gamma_2[1-3(\varrho I_1)^2(1-\varrho I_1)] \end{array}\right\} \\ & - 2\left[1 + \frac{\mu\gamma_1}{(1-c)(1-\varrho I_1)}\right]c(1) \end{aligned} \tag{78}$$

4. The Case $I_2 < 1$

The case $I_2 < 1$ is far additional complex along with locked form acquisition is attainable only for exponential application duration detachment. We acquire the bounding detachment of the procedure state as well as factorial circumstances in clauses of hyper geometric categories. We also accumulate many critical activity allocations in the consistent state and their algebraic affects for several examples.

4.1. Limiting Distribution of the System State

Initial we begin many denotation (Gross, Donald, et al,2013)

$$A_q^p(f) = ({}_2A_2\left[p, p+\frac{\mu}{v}; D+E+q, D-E+q; f\right]$$

Where $({}_2F_2$ is the hyper geometric series given by

$$({}_2A_2[z,y;a,k;N] = \sum_{m=0}^{\infty}\frac{(1)_m(y)_m N^m}{(c)_m(k)_m m!}$$

Theorem 4.1. *For a $O|O|1$ retrial sequence with no consistent client along with circular look for in the coherent state, the bordering possibilities $\{C_{pq}\}(p,q) \in S$ are given by*

$$C_{0b} = C_{00}(\mu I_1)^b(1-c)\prod_{p=0}^{b-1}\left\{\frac{\mu+pv}{\mu vc+(p+1)v[v+(1-I_2)(\mu+(p+1)v)]}\right\} \tag{79}$$

$$C_{1b} = C_{00}\left(\frac{\mu}{v}\right)((\mu I_1))^n\prod_{p=0}^{b-1}\left\{\frac{\mu+(i+1)v}{\mu vc+(i+1)v[v+(1-H_2)(\mu+(i+1)v)]}\right\} \tag{80}$$

Where C_{00} is given by

$$C_{0_0}^{-1} = 1+\frac{(1-c)\mu^2 I_1}{\mu vc+v[1-I_2(\mu+v)]}F_1^1(r)+\frac{\mu}{v}A_0^1(r) \tag{81}$$

Acknowledging fractional developing reacts $C_0(N)$ and are $C_1(N)$

$$C_0(N) = C_{0_0}\left\{1+\frac{(1-c)\mu^2 I_1 z}{\mu vc+v[v+(1-I_2)(\mu+v)]}\right\} \tag{82}$$

And

$$C_1(N) = C_{0_0}\frac{\mu}{v}A_0^1(rN) \tag{83}$$

Where $D = 1+\frac{\mu}{2v}+\frac{v}{2v(1-I_2)}$; $E = \frac{\sqrt{(\mu+v-\mu I_2)^2-4\mu vc(1-I_2)}}{2v(1-I_2)}$ and $r = \frac{\mu I_1}{v(1-I_2)}$.

Proof. Hence the exercise period is exponentially allocated, the discipline $F(x) = (B(x), P(x))$ is a Markov process with state space $\{0,1\}\times z_+$, where z_+ is the set of non-negative integers. The mechanism of calculation equilibrium equations for the possibilities C_{0b} and C_{1b} continues

$$(\mu+bv)C_{0b} = v[1-c+c\epsilon_{bo}]C_{1n} \tag{84}$$

$$[\mu I_1+v+bv(1-I_2)]C_{1b} = \lambda C_{0b0}+(b+1)vC_{0,b+1}+\lambda I_1 C_{1,b+1}+[vc+(b+1)v(1-I_2)]C_{1b} \tag{85}$$

Disclaim possibilities C_{1n}, n with the help of equation (84) as well as rewrite the approaching equation as

$$[\mu vc+(b+1)v(v+(1-I_2)(\mu+(b+1)v))]C_{0,b+1}-\mu I_1(\mu+bv)C_{0b} = [\mu vc+bv(v+(1-I_2)(\mu+bv))]C_{0n}-\mu I_1(\mu+(b+1)v)C_{0,b-1} \tag{86}$$

Applying (80) along with (84), we accumulate (81). Affecting equation (80) also (81), we capture the disclosure for $C_0(N)$ further $C_1(N)$ as in (83) and (84). Accosting the arranging condition $C_0(1)+C_1(1)+1$, we access the proneness C_{0_0} as in (82). ☐

Corollary 4.2. *The partial factorial moments*

$$O_m^p = \sum_{q=0}^{\infty} C_{pq} \tag{87}$$

$$O_m^c = \sum_{q=m}^{\infty} q(q-1)\dots(q-m+1)C_{pq}; p\in\{0,1\}, m\geq 1. \tag{88}$$

Are given by

$$O_0^0 = C_{0_0}\left\{1+\frac{(1-c)\mu^2 I_1}{\mu vc+v[v+(1-I_2)(\mu+v)]}A_1^1(r)\right\} \tag{89}$$

$$O_m^0 = \frac{C_{0_0}(1-c)\mu^2 I_1 r^{m-1}}{(\mu vc + v[v + (1-I_2)(\mu+v)])} \frac{(1)_{m-1}(1-\frac{\mu}{v})_{m-1}}{(D+E+1)_{m-1}(D-E+1)_{m-1}}$$
$$\left\{ mA_m^m(r) + \frac{m(m+\frac{\mu}{v})r}{(D+E+m)(D-E+m)} A_{m+1}^{m+1}(r) \right\} \tag{90}$$
$$O_0^1 = C_{0_0}\frac{\mu}{v}A_0^1(r) \tag{91}$$
$$O_m^1 = C_{0_0}\frac{\mu}{v}\frac{(1)_m(1+\frac{\mu}{v})_m}{(D+E)_m(D-E)_m} r^m A_m^{m+1}(r) \tag{92}$$

Proof. To acquire the above assertions, we operate the consequent acclaimed connections for the hyper geometric consecution

$$\frac{d}{dz^n}({}_2A_2[z,y;a,k;mN] = m\frac{zy}{ak^b}({}_2A_2[z+1,y+1;a+1,k+1;mN] \tag{93}$$

And

$$\frac{d^n}{dz^n}({}_2A_2[z,y;a,k;mN] = \frac{m^b(z)_b(y)_b}{(a)_b(k)_b}({}_2A_2[z+b,y+b;a+b,k+b;mN] \tag{94}$$

□

5. Conclusion

In this area we examine the consequence of the guidelines c, ϱ, I_1 along with I_2 on C_{0_0}, GP and GL through tables and graphs. In workbench 3.2, we account the case $I_2 = 1$ (i.e. $O|J|1$ case) along with a $O|G_2|$ retrial chain with $v = 0.5$, $\psi = 1$ as well as $I_1 = 0.6C_{0_0}$, GP and GL also GL are accounted for threesome asymmetric figures of pH_1(0.25, 0.5 and 0.75). As assumed, P_{0_0} elevations while EN along with EL compact with the addition of c. We apply the equations (4), Corollary 3.3 along with (60) for the approximation of these values. We aim C_{0_0} along with GP for the case $I_2 < 1$ (i.e. $O|O|1$) in the section 3.4. We account the case $v = 0.5$ as well as arrange the exercise rate $\psi = 1$ so that $\varrho = \mu = 0.4, 0.7$ also 1 in fig. 1. It is determined that C_{0_0} increases along with GP ends their values for the $O|O|1$ retrial chains with non-persistent clients $(c = 0)$ to the acknowledging one for the approved $O|O|1$ consequences with no constant clients $(c = 1)$.

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