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Some Remarks on Fuzzy Hausdorf spaces

Research Article

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Abstract: In this paper the concept of fuzzy Hausdorf space are studied. Several properties and examples of fuzzy Hausdorf space are given to illustrate the concept and relations between fuzzy hausdorf space and some fuzzy topological spaces are discussed.

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1. Introduction

The theory of fuzzy sets was initiated by L.A.Zadeh in his classical paper [14] in the year 1965 as an attempt to develop a mathematically precise framework in which to treat systems or phenomena which cannot themselves be characterized precisely. The potential of fuzzy notion was realized by the researchers and has successfully been applied for investigations in all the branches of Science and Technology. The paper of C.L.Chang [2] in 1968 paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. The concept of fuzzy Hausdorf spaces have been studied in Rekha Srivastava, S.N.Lal and Arun K. Srivastava in [4]. In this paper, we introduce the concept of Hausdorf spaces in fuzzy setting and investigate several characterizations of fuzzy Hausdorf spaces and the relations of fuzzy Hausdorf spaces and some fuzzy topological spaces are studied.

2. Preliminaries

Now we introduce some basic notions and results used in the sequel. In this work by (X, T) or simply by X, we will denote a fuzzy topological space due to Chang [2].

Definition 2.1 ([2]). *Let λ and μ be any two fuzzy sets in a fuzzy topological space (X, T). Then we define:*

- $\lambda \vee \mu : X \to [0,1]$ *as follows:* $(\lambda \vee \mu)(x) = max\,\{\lambda(x),\, \mu(x)\}$;
- $\lambda \wedge \mu : X \to [0,1]$ *as follows:* $(\lambda \wedge \mu)(x) = min\,\{\lambda(x),\, \mu(x)\}$;

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- $\mu = \lambda^c \Leftrightarrow \mu(x) = 1 - \lambda(x)$.

For a family $\{\lambda_i / i \in I\}$ *of fuzzy sets in* (X, T)*, the union* $\psi = \vee_i \lambda_i$ *and intersection* $\delta = \wedge_i \lambda_i$ *are defined respectively as* $\psi(x) = sup_i\{\lambda_i(x), x \in X\}$ *and* $\delta(x) = inf_i\{\lambda_i(x), x \in X\}$.

Definition 2.2 ([1])**.** *Let* (X, T) *be a fuzzy topological space. For a fuzzy set* λ *of* X*, the interior and the closure of* λ *are defined respectively as* $int(\lambda) = \vee\{\mu/\mu \leq \lambda, \mu \in T\}$ *and* $cl(\lambda) = \wedge\{\mu/\lambda \leq \mu, 1 - \mu \in T\}$.

Definition 2.3 ([12])**.** *A fuzzy set* λ *in a fuzzy topological space* (X, T) *is called fuzzy dense if there exists no fuzzy closed set* μ *in* (X, T) *such that* $\lambda < \mu < 1$ *That is* $cl(\lambda) = 1$.

Definition 2.4 ([7])**.** *A fuzzy set* λ *in a fuzzy topological space* (X, T) *is called fuzzy nowhere dense if there exists no non-zero fuzzy open set* μ *in* (X, T) *such that* $\mu < cl(\lambda)$ *That is, int* $cl(\lambda) = 0$.

Definition 2.5 ([8])**.** *Let* (X, T) *be a fuzzy topological space. A fuzzy set* λ *in* (X, T) *is called fuzzy first category set if* $\lambda = \bigvee_{i=1}^{\infty} \lambda_i$*, where* λ_i*'s are fuzzy nowhere dense sets in* (X, T)*. A fuzzy set which is not fuzzy first category set is called a fuzzy second category set in* (X, T).

Definition 2.6 ([8])**.** *A fuzzy topological space* (X, T) *is called fuzzy first category if* $1 = \vee_{i=1}^{\infty}(\lambda_i)$ *where* λ_i*'s are fuzzy nowhere dense sets in* (X, T)*. A topological space which is not of fuzzy first category, is said to be of fuzzy second category.*

Definition 2.7 ([9])**.** *Let (X,T) be a fuzzy topological space. Then* (X, T) *is called a fuzzy Baire space if* $int(\vee_{i=1}^{\infty}(\lambda_i)) = 0$*, where*$(\lambda_i)$*'s are fuzzy nowhere dense sets in* (X, T).

Definition 2.8 ([10])**.** *A fuzzy topological space* (X, T) *is said to be a fuzzy Quasi-maximal space if for every fuzzy dense set* λ *in* (X, T) *with* $int(\lambda) \neq 0$ *(the null set),* $int(\lambda)$ *is also fuzzy dense in* (X, T) .

Definition 2.9 ([10])**.** *A fuzzy topological space* (X, T) *is called a fuzzy submaximal space if for each fuzzy set* λ *in* (X, T) *such that* $cl(\lambda) = 1$*, then* $\lambda \in T$ *in* (X, T).

Definition 2.10 ([11])**.** *A fuzzy topological space* (X, T) *is called a fuzzy nodec space if every non-zero fuzzy nowhere dense set* λ *is fuzzy closed in* (X, T)*. That is, if* λ *is a fuzzy nowhere dense set in* (X, T)*, then* $1 - \lambda \in T$.

3. Fuzzy Hausdorf Space

Developed by the concept of fuzzy housdorf space studied in [4] we shall now define:

Definition 3.1. *A fuzzy topological space* (X, T) *is said to be a fuzzy Hausdorf space if whenever* λ, μ *in* (X, T) *and* $\lambda \neq \mu$ *we can find the fuzzy open sets* γ *and* δ *such that* $\lambda \leq \gamma, \mu \leq \delta$ *and* $\gamma \wedge \delta = 0$.

Example 3.2. *Let* $X = \{a, b, c\}$*. The fuzzy sets* λ *and* μ *are defined on* X *as follows:* $\lambda : X \to [0, 1]$ *defined as* $\lambda(a) = 1; \lambda(b) = 0; \lambda(c) = 0$.

$\mu : X \to [0, 1]$ *defined as* $\mu(a) = 0; \mu(b) = 1; \mu(c) = 1$*. Then* $T = \{0, \lambda, \mu, 1\}$ *is a fuzzy topology on* X*. Now consider the following fuzzy sets defined on* X *as follows:*

$\alpha : X \to [0, 1]$ *defined as* $\alpha(a) = 0.5; \alpha(b) = 0; \alpha(c) = 0$.

$\beta : X \to [0, 1]$ *defined as* $\beta(a) = 0; \beta(b) = 0.4; \beta(c) = 0.3$.

$\delta : X \to [0, 1]$ *defined as* $\delta(a) = 0; \delta(b) = 0; \delta(c) = 0.1$.

The fuzzy sets α and β in (X,T) and $\alpha \neq \beta$ then $\alpha \leq \lambda$ and $\beta \leq \mu$ implies that $\lambda \wedge \mu = 0$, where the fuzzy sets λ and μ are fuzzy open sets in (X,T).The fuzzy sets α and δ in (X,T) and $\alpha \neq \delta$ then $\alpha \leq \lambda$ and $\delta \leq \mu$ implies that $\lambda \wedge \mu = 0$, where the fuzzy sets λ and μ are fuzzy open sets in (X,T). The fuzzy sets α and $1-\lambda$ in (X,T) and $\alpha \neq 1-\lambda$ then $\alpha \leq \lambda$ and $1-\lambda \leq \mu$ implies that $\lambda \wedge \mu = 0$, where the fuzzy sets λ and μ are fuzzy open sets in (X,T). The fuzzy sets β and $1-\mu$ in (X,T) and $\beta \neq 1-\mu$ then $\beta \leq \mu$ and $1-\mu \leq \lambda$ implies that $\lambda \wedge \mu = 0$, where the fuzzy sets λ and μ are fuzzy open sets in (X,T). The fuzzy sets δ and $1-\mu$ in (X,T) and $\delta \neq 1-\mu$ then $\delta \leq \mu$ and $1-\mu \leq \lambda$ implies that $\lambda \wedge \mu = 0$, where the fuzzy sets λ and μ are fuzzy open sets in (X,T). The fuzzy sets $1-\lambda$ and $1-\mu$ in (X,T) and $1-\lambda \neq 1-\mu$ then $1-\lambda \leq \mu$ and $1-\mu \leq \lambda$ implies that $\lambda \wedge \mu = 0$, where the fuzzy sets λ and μ are fuzzy open sets in (X,T). Therefore any two distinct fuzzy sets in (X,T) which contains susequently two disjoint fuzzy open sets in (X,T). Hence (X,T) is a fuzzy Hausdorf space.

Example 3.3. *Let $X = \{a,b,c\}$. The fuzzy sets λ, μ, γ and δ are defined on X as follows:*
$\lambda : X \to [0,1]$ *defined as* $\lambda(a) = 0.5; \lambda(b) = 0$.
$\mu : X \to [0,1]$ *defined as* $\mu(a) = 0; \mu(b) = 0.4$.
$\gamma : X \to [0,1]$ *defined as* $\gamma(a) = 1; \gamma(b) = 0$.
$\delta : X \to [0,1]$ *defined as* $\delta(a) = 0; \delta(b) = 1$.
Then $T = \{0, \lambda, \mu, \gamma, \delta, (\lambda \vee \mu), (\lambda \vee \delta), (\mu \vee \gamma), 1\}$ is a fuzzy topology on X. The fuzzy sets $1-\lambda$ and $1-\gamma$ in (X,T) and $1-\lambda \neq 1-\gamma$ then $1-\lambda \leq (\lambda \vee \delta)$ and $1-\gamma \leq \delta$ implies that $(\lambda \vee \delta) \wedge \delta \neq 0$, where the fuzzy sets $\lambda \vee \delta$ and δ are fuzzy open sets in (X,T). Hence (X,T) is not a fuzzy Hausdorf space.

Proposition 3.4. *The distinct fuzzy sets $\lambda \leq \alpha$ and $\mu \leq \beta$, where $\alpha, \beta \in T$ in a fuzzy Hausdorf space, then $\alpha \wedge \beta$ is fuzzy nowhere dense set in (X,T).*

Proof. The distinct fuzzy sets $\lambda \leq \alpha$ and $\mu \leq \beta$, where $\alpha, \beta \in T$ then by definition of fuzzy Hausdorf space $\alpha \wedge \beta = 0$ implies that $int\ cl(\alpha \wedge \beta) = 0$. Hence $\alpha \wedge \beta$ is fuzzy nowhere dense set in (X,T). □

Theorem 3.5. *A fuzzy set λ is fuzzy nowhere dense in a fuzzy topological space (X,T) then $int(\lambda) = 0$ in (X,T).*

Proposition 3.6. *The distinct fuzzy sets $\lambda \leq \alpha$ and $\mu \leq \beta$, where $\alpha, \beta \in T$ in a fuzzy Hausdorf space, then $int(\alpha \wedge \beta) = 0$ in (X,T).*

Proof. The distinct fuzzy sets $\lambda \leq \alpha$ and $\mu \leq \beta$, where $\alpha, \beta \in T$. By Proposition 3.4, $\alpha \wedge \beta$ is fuzzy nowhere dense set and by Theorem 3.5, $\alpha \wedge \beta$ is empty interior. That is $int(\alpha \wedge \beta) = 0$. □

Proposition 3.7. *The distinct fuzzy sets $\lambda \leq \alpha$ and $\mu \leq \beta$, where $\alpha, \beta \in T$ in a fuzzy Hausdorf space, then $\alpha \wedge \beta$ is not a fuzzy regularopen set in (X,T).*

Proof. The distinct fuzzy sets $\lambda \leq \alpha$ and $\mu \leq \beta$, where $\alpha, \beta \in T$ in a fuzzy Hausdorf space, then by Proposition 3.4, $intcl(\alpha \wedge \beta) = 0$ implies that $intcl(\alpha \wedge \beta) = 0 \neq \lambda$. Hense $\alpha \wedge \beta$ is not a fuzzy regularopen set in (X,T). □

Theorem 3.8. *A fuzzy set λ is fuzzy nowhere dense in a fuzzy topological space (X,T) then λ is fuzzy semi-closed in (X,T).*

Lemma 3.9 ([1]). *In a fuzzy topological space (X,T),*

(1). every fuzzy open set is fuzzy α-open.

(2). every fuzzy α-open set is both fuzzy semiopen and fuzzy preopen.

(3). every fuzzy semiopen set is fuzzy semi-preopen.

(4). every fuzzy preopen set is fuzzy semi-preopen.

Proposition 3.10. *The distinct fuzzy sets $\lambda \leq \alpha$ and $\mu \leq \beta$, where $\alpha, \beta \in T$ in a fuzzy Hausdorf space, then $1-(\alpha \wedge \beta)$ in (X, T) is*

(1). is fuzzy semiopen.

(2). is fuzzy semi-preopen.

Proof. The distinct fuzzy sets $\lambda \leq \alpha$ and $\mu \leq \beta$, where $\alpha, \beta \in T$ in a fuzzy Hausdorf space, by Proposition 3.10, $(\alpha \wedge \beta)$ is fuzzy semi-closed. Then $1-(\alpha \wedge \beta)$ is fuzzy open. Now by Lemma 3.9 (3), $1-(\alpha \wedge \beta)$ is fuzzy semi-open, and by Lemma 3.9 (4), $1-(\alpha \wedge \beta)$ fuzzy semi-preopen. □

Proposition 3.11. *The distinct fuzzy sets $\lambda \leq \alpha$ and $\mu \leq \beta$, where $\alpha, \beta \in T$ in a fuzzy Hausdorf space, then $\alpha \wedge \beta$ is not a fuzzy regular-closed set in (X, T).*

Proof. The distinct fuzzy sets $\lambda \leq \alpha$ and $\mu \leq \beta$, where $\alpha, \beta \in T$ in a fuzzy Hausdorf space, then by Proposition 3.6, $int(\alpha \wedge \beta) = 0$ implies that $clint(\alpha \wedge \beta) = 0 \neq \lambda$. Hense $\alpha \wedge \beta$ is not a fuzzy regular-closed in (X, T). □

Proposition 3.12. *The distinct fuzzy sets $\lambda \leq \alpha$ and $\mu \leq \beta$, where $\alpha, \beta \in T$ in a fuzzy Hausdorf space, then $\alpha \wedge \beta$ is fuzzy semi-closed set in (X, T).*

Proof. The distinct fuzzy sets $\lambda \leq \alpha$ and $\mu \leq \beta$, where $\alpha, \beta \in T$ in a fuzzy Hausdorf space, then by Proposition 3.4, $intcl(\alpha \wedge \beta) = 0$ implies that $intcl(\alpha \wedge \beta) = 0 \leq \lambda \wedge \beta$. Hense $\alpha \wedge \beta$ is fuzzy semi-closed in (X, T). □

Proposition 3.13. *The distinct fuzzy sets $\lambda \leq \alpha$ and $\mu \leq \beta$, where $\alpha, \beta \in T$ in a fuzzy Hausdorf space, then $\alpha \wedge \beta$ is fuzzy α-closed set in (X, T).*

Proof. The distinct fuzzy sets $\lambda \leq \alpha$ and $\mu \leq \beta$, where $\alpha, \beta \in T$ in a fuzzy Hausdorf space, then by Proposition 3.4, $intcl(\alpha \wedge \beta) = 0$ implies that $clintcl(\alpha \wedge \beta) = 0 \leq \lambda \wedge \beta$. Hense $\alpha \wedge \beta$ is fuzzy α-closed in (X, T). □

Proposition 3.14. *The distinct fuzzy sets $\lambda \leq \alpha$ and $\mu \leq \beta$, where $\alpha, \beta \in T$ in a fuzzy Hausdorf space, then $\alpha \wedge \beta$ is fuzzy pre-closed set in (X, T).*

Proof. The distinct fuzzy sets $\lambda \leq \alpha$ and $\mu \leq \beta$, where $\alpha, \beta \in T$ in a fuzzy Hausdorf space, then by Proposition 3.6, $int(\alpha \wedge \beta) = 0$ implies that $clint(\alpha \wedge \beta) = 0 \leq \lambda$. Hense $\alpha \wedge \beta$ is fuzzy pre-closed in (X, T). □

Proposition 3.15. *The distinct fuzzy sets $\lambda \leq \alpha$ and $\mu \leq \beta$, where $\alpha, \beta \in T$ in a fuzzy Hausdorf space, then $\alpha \wedge \beta$ is fuzzy semi-preclosed set in (X, T).*

Proof. The distinct fuzzy sets $\lambda \leq \alpha$ and $\mu \leq \beta$, where $\alpha, \beta \in T$ in a fuzzy Hausdorf space, then by Proposition 3.6, $intcl(\alpha \wedge \beta) = 0$ implies that $intclint(\alpha \wedge \beta) = intcl(0) = 0 \leq \lambda$. Hense $\alpha \wedge \beta$ is fuzzy semi-preclosed in (X, T). □

Lemma 3.16 ([1]). *In a fuzzy topological space (X, T),*

(1). every fuzzy closed set is fuzzy Fg-closed.

(2). every fuzzy semi-closed set is fuzzy Fsg-closed.

(3). every fuzzy α-closed set is $F\alpha g$-closed.

(4). every fuzzy pre-closed set is Fpg-closed.

(5). every fuzzy semi-pre-closed set is $Fspg$-closed.

Proposition 3.17. *The distinct fuzzy sets $\lambda \leq \alpha$ and $\mu \leq \beta$, where $\alpha, \beta \in T$ in a fuzzy Hausdorf space, then $(\alpha \wedge \beta)$ is*

(1). fuzzy semi-closed in (X, T).

(2). Fsg-closed.

(3). $F\alpha g$-closed.

(4). Fpg-closed.

(5). $Fspg$-closed.

Proof. The distinct fuzzy sets $\lambda \leq \alpha$ and $\mu \leq \beta$, where $\alpha, \beta \in T$ in a fuzzy Hausdorf space, then

(1). by Proposition 3.12, $(\alpha \wedge \beta)$ is fuzzy semi-closed in (X, T).

(2). Now $(\alpha \wedge \beta)$ is fuzzy semi-closed in (X, T) by Lemma 3.16 (2), $(\alpha \wedge \beta)$ is Fsg-closed.

(3). by Proposition 3.13, $(\alpha \wedge \beta)$ is fuzzy α-closed and by Lemma 3.16 (3), $(\alpha \wedge \beta)$ $F\alpha g$-closed.

(4). by Proposition 3.14, $(\alpha \wedge \beta)$ is fuzzy pre-closed and by Lemma 3.16 (4), $(\alpha \wedge \beta)$ Fpg-closed.

(5). by Proposition 3.15, $(\alpha \wedge \beta)$ is fuzzy semi-pre-closed and by Lemma 3.16 (5), $(\alpha \wedge \beta)$ $Fspg$-closed. □

Proposition 3.18. *The distinct fuzzy sets $\lambda \leq \alpha$ and $\mu \leq \beta$, where $\alpha, \beta \in T$ in a fuzzy Hausdorf space, then $cl\ int[1 - (\alpha \wedge \beta)] = 1$.*

Proof. The distinct fuzzy sets $\lambda \leq \alpha$ and $\mu \leq \beta$, where $\alpha, \beta \in T$ in a fuzzy Hausdorf space. Then $\alpha \wedge \beta = 0$ implies that $cl(\alpha \wedge \beta) = 0$ implies that $1 - cl(\alpha \wedge \beta) = 1$ implies that $int(1 - \alpha \wedge \beta) = 1$ therefore $cl\ int(1 - \alpha \wedge \beta) = 1$. □

4. Fuzzy Hausdorf Space and Some Fuzzy Topological Spaces

Proposition 4.1. *A fuzzy Hausdorf space, then the space is a fuzzy Quasi-maximal space.*

Proof. Let (X, T) be a fuzzy Hausdorf space, The distinct fuzzy sets $\lambda \leq \alpha$ and $\mu \leq \beta$ in (X, T), and $\alpha, \beta \in T$, By Proposition 3.6, $int(\alpha \wedge \beta) = 0$. Now $cl(1 - \alpha \wedge \beta) = 1$ and $int(1 - \alpha \wedge \beta) \neq 0$, then $clint(1 - \alpha \wedge \beta)$ implies that $clint[1 - (\alpha \wedge \beta)] = clint[1 - 0] = clint(1) = 1$. Therefore the fuzzy dense set $1 - \alpha \wedge \beta$ empty interior and interior of $1 - \alpha \wedge \beta$ is fuzzy dense in (X, T). Hence (X, T) is fuzzy Quasi-maximal space. □

Proposition 4.2. *A fuzzy Hausdorf space need not be fuzzy Quasi-regular space.*

Proof. In Example 3.2, (X, T) is fuzzy Hausdorf space and the fuzzy open set λ and μ is non-empty set in (X, T), then $cl(\lambda) \nleq \mu$ and $cl(\mu) \nleq \lambda$. Hence (X, T) is not of fuzzy Quasi-regular space. □

Proposition 4.3. *A fuzzy submaximal space need not be fuzzy Hausdorf space. Consider the following example.*

Example 4.4. *Let $X = \{a, b, c\}$. The fuzzy sets λ, μ and γ are defined on X as follows:*

$\lambda : X \to [0, 1]$ *defined as* $\lambda(a) = 0.9; \lambda(b) = 0.8; \lambda(c) = 0.8$

$\mu : X \to [0, 1]$ *defined as* $\mu(a) = 0.9; \mu(b) = 0.8; \mu(c) = 0.7$

$\gamma : X \to [0, 1]$ *defined as* $\gamma(a) = 0.8; \gamma(b) = 0.7; \gamma(c) = 0.7$

Then $T = \{0, \lambda, \mu, \gamma, 1\}$ is a fuzzy topology on X. The fuzzy dense sets λ, μ and γ in (X, T) are fuzzy open. Hence (X, T) is fuzzy submaximal space but not a fuzzy Hausdorf space. Since $\mu \leq \lambda, \gamma \leq \mu$ and $\lambda \neq \mu$ but $\lambda \wedge \mu \neq 0$ same as $\gamma \leq \lambda, 1 - \lambda \leq \gamma$ and $\lambda \neq \gamma$ but $\lambda \wedge \gamma \neq 0$.

Proposition 4.5. *A fuzzy Hyperconnected space need not be fuzzy Hausdorf space.*

Proof. In Example 3.3, the fuzzy open sets in λ, μ, γ in (X, T) are fuzzy dense in (X, T), hence (X, T) is fuzzy Hyperconnected space but not of fuzzy Hausdorf space (by Example 3.3). □

References

[1] K.K.Azad, *On fuzzy semi continuity, Fuzzy almost continuity and Fuzzy weakly continuity*, J. Math. Anal. Appl., 82(1981), 14-32.

[2] C.L.Chang, *Fuzzy Topological Spaces*, J.Math. Anal. Appl., 24(1968), 182-190.

[3] M.S.El.Naschie, *On the certification of heterotic strings, M theory and $\in^{\infty}$ theory*, Chaos. Solitions and Fractals, (2000), 2397-2408.

[4] Rekha Srivastava, S.N.Lal and Arun K.Srivastava, *Fuzzy Hausdorff topological space*, J. Math. Anal. Appl., 81(1981), 497-506.

[5] M.K.Singal and N.Rajvanshi, *Fuzzy α-sets and α-continuous functions*, Fuzzy Sets and Systems, 45(5)(1992), 383-390.

[6] X.Tang, *Spatial Object modeling in fuzzy topological spaces with applications to land cover chang in China*, Ph.D. Dissertation, University of Twente, Enschede, The Netherlands, (2004).

[7] G.Thangaraj and S.Anjalmose, *Fuzzy D-Baire Spaces*, Annals of Fuzzy Mathematics and Informatics, 7(1)(2014), 99-108.

[8] G.Thangaraj and S.Anjalmose, *Fuzzy D'-Baire Spaces*, Annals of Fuzzy Mathematics and Informatics, 10(4)(2015), 583-589.

[9] G.Thangaraj and S.Anjalmose, *On Fuzzy Baire Spaces*, J. of Fuzzy Math., 21(3)(2013), 667-676.

[10] G.Thangaraj and S.Anjalmose, *Fuzzy Quasi-maximal Spaces*, International Journal of Mathematics and its Applications, 3(4-E)(2015), 55-61.

[11] G.Thangaraj and S.Anjalmose, *Some remarks on Fuzzy Baire Spaces*, Scientia Magna, 9(1)(2013), 1-6.

[12] G.Thangaraj and G.Balasubramanian, *On Somewhat Fuzzy Continuous functions*, J. Fuzzy Math., 11(3)(2003), 725-736.

[13] Valentingregori and Hans-Petera, *Kunzi α-Fuzzy compactness in I-topological spaces*, IJMS, 41(2003), 2609-2617.

[14] L.A.Zadeh, *Fuzzy Sets*, Information and Control, 8(1965), 338-353.