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# A Comparison of Triangular Fuzzy Numbers and Intuitionistic Triangular Fuzzy Numbers Using Apex Base Least Cost Method 

Research Article

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#### Abstract

The transportation model is a special class of the linear programming problem. In real world problems, optimization techniques are useful for solving problems like, project schedules, assignment problems and network flow analysis. The objective is to minimize the transportation cost of some commodities through a capacitated network when the supply and demand of nodes and the capacity and cost of edges are represented as fuzzy numbers. Here, we are proposed for solving fuzzy transportation problem, where fuzzy demand and supply all are in the form of triangular fuzzy numbers and Intuitionistic triangular fuzzy numbers. In this paper a new method is ABLC for finding an optimal solution for a wide range of fuzzy transportation problem. This method is very easy to understand and use compared to other methods. A numerical example is solved using the proposed algorithm.


Keywords: Triangular Fuzzy number, Intuitionistic triangular fuzzy number, Initial Basic Feasible Solution, Apex and Base Angles, Least adjust cost method.
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## 1. Introduction and Preliminaries

A Transportation problem is one of the earliest and most important applications of linear programming problem in which a commodity is to be transported from various sources of supply to various destinations of demand in such a way that the total transportation cost is minimum. In this paper a new method named as Apex Base Least Cost Method to solve a fuzzy transportation problem with triangular and intuitionistic triangular fuzzy numbers. This fuzzy numbers based on apex and base angles. This method provides the correct ordering of triangular and intuitionistic triangular fuzzy numbers, And this proposed method is illustrated with the help of numerical example.

Definition 1.1. A fuzzy set is characterized by a membership function mapping element of a domain, space, or the universe of discourse $X$ to the unit interval [ 0,1 .

Definition 1.2. A real fuzzy number $\tilde{a}$ is a fuzzy subset of the real number $R$ with membership function $\mu$ following conditions, satisfying the conditions,
(1). $\mu(A)$ is continuous from $R$ to $[0,1]$
(2). $\mu(A)$ is strictly increasing $\mathcal{B}$ continuous on $\left[a_{1}, a_{2}\right]$

[^0](3). $\mu(A)$ is strictly decreasing $\mathfrak{\xi}$ continuous on $\left[a_{2}, a_{4}\right]$

Definition 1.3. $A$ fuzzy number $A=\left(a_{1}, a_{2}, a_{3}\right)$ is triangular with membership function

$$
\mu_{A}(X)= \begin{cases}\frac{x-a_{1}}{a_{2}-a_{1}}, & a_{1} \leq x \leq a_{2} \\ 1, & x=a_{2} \\ \frac{a_{3}-x}{a_{3}-a_{2}}, & a_{2} \leq x \leq a_{3} \\ 0, & \text { otherwise }\end{cases}
$$

Definition 1.4. A triangular intuitionistic fuzzy number $\check{A}$ is an intuitionistic fuzzy subset in $R$ with the following membership function $\mu_{A}(X)$ and non membership function $\nu_{A}(X)$

$$
\begin{aligned}
& \mu_{A}(X)= \begin{cases}\frac{x-a_{1}}{a_{2}-a_{1}}, & a_{1} \leq x \leq a_{2} \\
\frac{a_{3}-x}{a_{3}-a_{2}}, & a_{2} \leq x \leq a_{3} \\
0, & \text { otherwise }\end{cases} \\
& \nu_{A}(X)= \begin{cases}\frac{x-a_{1}}{a_{2}-a_{1}^{\prime}}, & a_{1} \leq x \leq a_{2} \\
\frac{a_{3}-x}{a_{3}-a_{2}}, & a_{2} \leq x \leq a_{3} \\
0, & \text { otherwise }\end{cases}
\end{aligned}
$$

Where $a_{1}^{\prime} \leq a_{1} \leq a_{2} \leq a_{3} \leq a_{3}^{\prime}$ and $\mu_{A}(X), \nu_{A}(X) \leq 0.5$ for $\mu_{A}(X)=\nu_{A}(X) \forall x \in R$. This is denoted by $\check{A}=$ $\left(a_{1} a_{2} a_{3} ; a_{1}^{\prime} a_{2} a_{3}^{\prime}\right)$.

Definition 1.5. We define a accuracy function

$$
\begin{aligned}
\text { Triangular Fuzzy Number } a & =\frac{\left(a_{1}+2 a_{2}+a_{3}\right)}{4} \\
\text { Intuitionistic Triangular Fuzzy Number } a & =\frac{\left(a_{1}+2 a_{2}+a_{3}\right) ;\left(a_{1}^{\prime}+2 a_{2}+a_{3}^{\prime}\right)}{8}
\end{aligned}
$$

## 2. Apex Base Least Cost Method

To find an initial basic feasible solution in which transportation costs are represented as triangular and intuitionistic triangular fuzzy numbers.

## Algorithm

Step 1: Construct the fuzzy transportation table from the given fuzzy transportation problem.

Step 2: First We Convert the fuzzy transportation table into balanced one, if it is not.

Step 3: Select the row or column parallel to the least demand or supply.

Step 4: Select a cell in a marked row/ or column with minimum cost. If tie occurs, choose the cell with the least average value if it's fuzzy supply and demand.

Step 5: Find the adjust row cost and adjust column cost for the selected cell in the selected row/or column and choose the adjust cell with minimum cost. Allocate the parallel minimum of its supply and demand.

Step 6: After performing Step5 delete the row or column (where supply or column becomes zero) for further calculation.

Step 7: Repeat Step4 to Step 6 until all the demands are satisfied and all the supplies are exhausted.

Step 8: This allotment yields a fuzzy initial basic feasible solution to the given fuzzy transportation problem. i.e) Total fuzzy cost $=\sum_{i=1}^{m} \sum_{j=1}^{n} C_{i j} X_{i j}$.

## 3. Numerical Example

Example 3.1. Solve the fuzzy transportation problem

|  | $D_{1}$ | $D_{2}$ | $D_{3}$ | $D_{4}$ |
| :---: | :---: | :---: | :---: | :---: |
| $S_{1}$ | $(3,7,11)$ | $(13,18,23)$ | $(6,13,20)$ | $(15,20,25)$ |
| $S_{2}$ | $(16,19,24)$ | $(3,5,7)$ | $(5,7,10)$ | $(20,23,26)$ |
| $S_{3}$ | $(11,14,17)$ | $(7,9,11)$ | $(2,3,4)$ | $(5,7,8)$ |

Here the sources are $(7,9,11),(6,8,11),(9,11,13)$ and destinations are $(2,4,5),(3,5,7),(6,7,9),(10,12,14)$ respectively.

Solution. The fuzzy transportation problem is given by

|  | $D_{1}$ | $D_{2}$ | $D_{3}$ | $D_{4}$ | Supply |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $S_{1}$ | $(3,7,11)$ | $(13,18,23)$ | $(6,13,20)$ | $(15,20,25)$ | $(7,9,11)$ |
| $S_{2}$ | $(16,19,24)$ | $(3,5,7)$ | $(5,7,10)$ | $(20,23,26)$ | $(6,8,11)$ |
| $S_{3}$ | $(11,14,17)$ | $(7,9,11)$ | $(2,3,4)$ | $(5,7,8)$ | $(9,11,13)$ |
| Demand | $(2,4,5)$ | $(3,5,7)$ | $(6,7,9)$ | $(10,12,14)$ |  |

Step 1: Given transportation table is balanced one.

|  | $D_{1}$ | $D_{2}$ | $D_{3}$ | $D_{4}$ | S |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $S_{1}$ | $(3,7,11)$ | $(13,18,23)$ <br> $\mathbf{( 3 , 5 , 7 )}$ | $(6,13,20)$ | $(15,20,25)$ | $(7,9,11)$ <br> $\mathbf{( 4 , 4 , 4 )}$ |
| $S_{2}$ | $(16,19,24)$ | $(3,5,7)$ | $(5,7,10)$ | $(20,23,26)$ | $(6,8,11)$ |
| $S_{3}$ | $(11,14,17)$ | $(7,9,11)$ | $(2,3,4)$ | $(5,7,8)$ | $(9,11,13)$ |
| D | $(3,4,5)$ | $(3,5,7) \uparrow$ | $(6,7,9)$ | $(10,12,14)$ |  |

Delete column $D_{2}$ as for demand is exhausted and adjust supply as $\{(7,9,11)-(3,5,5)\}=(4,4,4)$. Delete Row $S_{1}$ as for source is exhausted and adjust demand $\{(6,7,9)-(4,4,4)\}=(2,3,5)$. Finally

|  | $D_{1}$ | $D_{2}$ | $D_{3}$ | $D_{4}$ | S |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $S_{1}$ | $(3,7,11)$ | $(13,18,23)$ <br> $\mathbf{( 3 , 5 , 7 )}$ | $(6,13,20)$ <br> $\mathbf{( 4 , 4 , 4 )}$ | $(15,20,25)$ | $(7,9,11)$ |
| $S_{2}$ | $(16,19,24)$ <br> $\mathbf{( 3 , 4 , 5 )}$ | $(3,5,7)$ | $(5,7,10)$ | $(20,23,26)$ <br> $\mathbf{( 3 , 4 , 6})$ | $(6,8,11)$ |
| $S_{3}$ | $(11,14,17)$ | $(7,9,11)$ | $(2,3,4)$ <br> $\mathbf{( 2 , 3 , 5 )}$ | $(5,7,8)$ <br> $\mathbf{( 7 , 8 , 8 )}$ | $(9,11,13)$ |
| D | $(2,4,5)$ | $(3,5,7)$ | $(6,7,9)$ | $(10,12,14)$ |  |

The total optimal cost is $=(13,18,23)(3,5,7)+(6,13,20)(4,4,4)+(16,19,24)(3,4,5)+(20,23,26)(3,4,6)$

$$
\begin{aligned}
& +(2,3,4)(2,3,5)+(5,7,8)(7,8,8) \\
= & (210,375,531) \\
= & \mathbf{R s} .372 .5
\end{aligned}
$$

Example 3.2. Solve the Fuzzy transportation problem with Intuitionistic Triangular fuzzy number

|  | $D_{1}$ | $D_{2}$ | $D_{3}$ | $D_{4}$ |
| :---: | :---: | :---: | :---: | :---: |
| $S_{1}$ | $(-1,0,1 ; 2,3,4)$ | $(0,1,2 ; 3,4,5)$ | $(8,9,10 ; 11,12,13)$ | $(4,5,6 ; 7,8,9)$ |
| $S_{2}$ | $(-2,-1,0 ; 1,2,3)$ | $(-3,-2,-1 ; 0,1,2)$ | $(2,4,5 ; 6,7,8)$ | $(-3,-1,0 ; 1,2,4)$ |
| $S_{3}$ | $(2,3,4, ; 5,6,7)$ | $(3,6,7 ; 8,9,10)$ | $(11,12,14 ; 15,16,17)$ | $(5,6,8 ; 9,10,11)$ |

Here the sources are (1,3,5;6,7,8), (-2,-1,0;1,2,4), (5,6,7;10,12,13) and destinations are (4,5,6;7,8,9), (1,2,3;5,6,7), $(0,1,2 ; 3,4,5),(-1,0,1 ; 2,3,4)$ respectively.

Solution. The fuzzy transportation problem is given by

|  | $D_{1}$ | $D_{2}$ | $D_{3}$ | $D_{4}$ | S |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $S_{1}$ | $(-1,0,1 ; 2,3,4)$ | $(0,1,2 ; 3,4,5)$ | $(8,9,10 ; 11,12,13)$ | $(4,5,6 ; 7,8,9)$ | $(1,3,5 ; 6,7,8)$ |
| $S_{2}$ | $(-2,-1,0 ; 1,2,3)$ | $(-3,-2,-1 ; 0,1,2)$ | $(2,4,5 ; 6,7,8)$ | $(-3,-1,0 ; 1,2,4)$ | $(-2,-1,0 ; 1,2,4)$ |
| $S_{3}$ | $(2,3,4 ; 5,6,7)$ | $(3,6,7 ; 8,9,10)$ | $(11,12,14 ; 15,16,17)$ | $(5,6,8 ; 9,10,11)$ | $(5,6,7 ; 10,12,13)$ |
| D | $(4,5,6 ; 7,8,9)$ | $(1,2,3 ; 5,6,7)$ | $(0,1,2 ; 3,4,5)$ | $(-1,0,1 ; 2,3,4)$ |  |

## Step 1:

|  | $D_{1}$ | $D_{2}$ | $D_{3}$ | $D_{4}$ | S |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $S_{1}$ | $(-1,0,1 ; 2,3,4)$ | $(0,1,2 ; 3,4,5)$ | $(8,9,10 ; 11,12,13)$ | $(4,5,6 ; 7,8,9)$ | $(1,3,5 ; 6,7,8)$ |
| $S_{2}$ | $(-2,-1,0 ; 1,2,3)$ <br> $(\mathbf{- 2 , 1 , 0} ; \mathbf{1 , 2 , 4})$ | $(-3,-2,-1 ; 0,1,2)$ | $(2,4,5 ; 6,7,8)$ | $(-3,-1,0 ; 1,2,4)$ | $(-2,-1,0 ; 1,2,4)$ <br> $\leftarrow$ |
| $S_{3}$ | $(2,3,4 ; 5,6,7)$ | $(3,6,7 ; 8,9,10)$ | $(11,12,14 ; 15,16,17)$ | $(5,6,8 ; 9,10,11)$ | $(5,6,7 ; 10,12,13)$ |
| D | $(4,5,6 ; 7,8,9)$ <br> $(\mathbf{6 , 6 , 6} ; \mathbf{6 , 6}, \mathbf{5})$ | $(1,2,3 ; 5,6,7)$ | $(0,1,2 ; 3,4,5)$ | $(-1,0,1 ; 2,3,4)$ |  |

Delete row $S_{2}$. And Adjust demand $\{(4,5,6 ; 7,8,9)-(-2,-1,0 ; 1,2,4)\}=(6,6,6 ; 6,6,5)$

|  | $D_{1}$ | $D_{2}$ | $D_{3}$ | $D_{4}$ | S |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $S_{1}$ | $(0,1,2 ; 3,4,5)$ | $(0,1,2 ; 3,4,5)$ | $(8,9,10 ; 11,12,13)$ | $(4,5,6 ; 7,8,9)$ | $(1,3,5 ; 6,7,8)$ |
| $S_{3}$ | $(2,3,4 ; 5,6,7)$ | $(3,6,7 ; 8,9,10)$ | $(11,12,14 ; 15,16,17)$ | $(5,6,8 ; 9,10,11)$ <br> $(\mathbf{- 1 , 0 , 1 ; 2 , 3}, \mathbf{4})$ | $(5,6,7 ; 10,12,13)$ <br> $(\mathbf{6 , 6 , 6} ; \mathbf{8 , 9}, \mathbf{9})$ |
| D | $(4,5,6 ; 7,8,9)$ | $(1,2,3 ; 5,6,7)$ | $(0,1,2 ; 3,4,5)$ | $(-1,0,1 ; 2,3,4) \uparrow$ |  |

Delete column $D_{4}$, as demand is exhausted. The adjust supply $\{(5,6,7 ; 10,12,13)-(-1,0,1 ; 2,3,4)\}=(6,6,6 ; 8,9,9)$. Finally

|  | $D_{1}$ | $D_{2}$ | $D_{3}$ | $D_{4}$ | S |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $S_{1}$ | $\begin{gathered} (-1,0,1 ; 2,3,4) \\ (\mathbf{0}, \mathbf{1}, \mathbf{2} ; \mathbf{1}, \mathbf{1}, \mathbf{1}) \end{gathered}$ | $\begin{aligned} & (0,1,2 ; 3,4,5) \\ & (\mathbf{1}, \mathbf{2}, \mathbf{3} ; \mathbf{5}, \mathbf{6}, \mathbf{7}) \end{aligned}$ | $\begin{gathered} (8,9,10 ; 11,12,13) \\ (\mathbf{- 2 , 0}, \mathbf{2} ; \mathbf{2}, \mathbf{3}, \mathbf{5}) \end{gathered}$ | (4,5,6;7,8,9) | (1,3,5;6,7,8) |
| $S_{2}$ | $\begin{gathered} (-2,-1,0 ; 1,2,3) \\ (-\mathbf{2}, \mathbf{- 1}, \mathbf{0} ; \mathbf{1}, \mathbf{2}, 4) \end{gathered}$ | (-3,-2,-1;0,1,2) | (2,4,5;6,7,8) | $(-3,-1,0 ; 1,2,4)$ | $(-2,-1,0 ; 1,2,4)$ |
| $S_{3}$ | $\begin{aligned} & (2,3,4 ; 5,6,7) \\ & (6,5,4 ; 5,5,4) \end{aligned}$ | (3,6,7;8,9,10) | $\begin{gathered} (11,12,14 ; 15,16,17) \\ (\mathbf{0}, \mathbf{1}, \mathbf{2 ; 3}, \mathbf{4}, \mathbf{5}) \end{gathered}$ | $\begin{gathered} (5,6,8 ; 9,10,11) \\ (\mathbf{- 1 , 0 , 1} ; \mathbf{2}, \mathbf{3}, \mathbf{4}) \\ \hline \end{gathered}$ | $(5,6,7 ; 10,12,13)$ |
| D | (4,5,6;7,8,9) | $(1,2,3 ; 5,6,7)$ | (0,1,2;3,4,5) | (-1,0,1;2,3,4) |  |

The total optimal cost is $=(-1,0,1 ; 2,3,4)(0,1,2 ; 1,1,1)+(0,1,2 ; 3,4,5)(1,2,3 ; 5,6,7)+(8,9,10 ; 11,12,13)(-2,0,2 ; 2,3,5)$

$$
\begin{aligned}
& +(-2,-1,0 ; 1,2,3)(-2,-1,0 ; 1,2,4)+(2,3,4, ; 5,6,7)(6,5,4 ; 5,5,4) \\
& +(11,12,14 ; 15,16,17)(0,1,2 ; 3,4,5)+(5,6,8 ; 9,10,11)(1,0,1 ; 2,3,4) \\
& =(3.28 .60 ; 106,153,208) \\
& =\text { Rs. } 87.125
\end{aligned}
$$

## 4. Conclusion

This methods provides an optimal solution is less number of iterations, directly for the given fuzzy transportation problem. And fuzzy numbers have initial basic feasible solution. As this method requires less number of times and is very easy to understand and apply. So it will be very helpful for decision makers who are dealing with logistic and supply sequence problem.

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