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Solving Fuzzy Assignment Problem Using Centroid Ranking Method

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Abstract: Assignment problem has various applications in the real world because of their wide applicability in industry, commerce, management science, etc. Traditional classical assignment problems cannot be successfully used for real life problem; hence the use of fuzzy assignment problem is more appropriate. Fuzzy sets were introduced by Lofti A. Zadeh in 1965 as an extension of representing impreciseness or vagueness in day to day life. In this paper, Triangular Fuzzy Assignment Problem has been defuzzified into crisp values using Centroid Ranking method and Hungarian method has been applied to find an optimal solution.The proposed method is illustrated by numerical examples.

Keywords: Fuzzy Assignment Problem, Triangular Fuzzy Number, Triangular Fuzzy Assignment Problem, Centroid Ranking Method.
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1. Introduction

The term Assignment Problem (AP) was first appeared in Votaw and Orden (1952). APs are widely applied in manufacturing and service systems. An Assignment Problem is a special type of linear programming problem where the objective is to assign n number of jobs to n number of persons at a minimum cost (time). Zadeh (1965) has introduced the concept of fuzzy sets to deal with imprecision, vagueness in real life situations. Since then, tremendous efforts have been spent; significant advances have been made on the development of numerous methodologies and their applications to various decision problems. Fuzzy assignment problems have received great attention in recent years. Hungarian method proposed by Kuhn (1955) is widely used for the solution to APs. Chen (1985) proposed a fuzzy assignment model that did not consider the differences of individuals and also proved some theorems. Wang (1987) solved a similar model by graph theory. Lin and Wen (2004) investigated a fuzzy AP in which the cost depends on the quality of the job. Dubois and Fortemps (1999) proposed a flexible AP, which combines with fuzzy theory, multiple criteria decision-making and constraint-directed methodology. Huang and Xu (2005) proposed a solution procedure for the APs with restriction of qualification. Mukherjee and Basu (2010) proposed a new method for solving fuzzy APs. Kumar (2009) proposed a method to solve the fuzzy APs, occurring in real life situations. Kumar and Gupta (2012) proposed two new methods for solving fuzzy APs and fuzzy travelling salesman problems. Kumar and Gupta (2011) proposed methods for solving fuzzy APs with different membership functions. K. Kalaiarasi [1] proposed a fuzzy assignment model with TFN using Robust Ranking technique. Jatinder Pal Singh [2] proposed method to solve fuzzy assignment problem using FHM and Operations for Subtraction and Division on TFN proposed by Gani and Assarudeen (2012). In this paper the fuzzy assignment problem has been converted into crisp assignment problem using Centroid Ranking Method and Hungarian assignment has been applied to find an optimal solution.

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2. Preliminaries

Definition 2.1 (Fuzzy Set)**.** *The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. A function $\mu_{\breve{a}}$ such that the value assigned to the universal set X fall within a specified range i.e. $\mu_{\breve{a}} : X \to [0,1]$. The assigned value indicates the membership grade of the element in the set A. The function $\mu_{\breve{a}(x)}$ is called the membership function and the set $\hat{A} = \{(x, \mu_{\breve{a}(x)}) : x \in X\}$ defined by $\mu_{\breve{a}(x)}$ for each $x \in X$ is called a fuzzy set.*

Definition 2.2 (Triangular fuzzy number (TFNs))**.** *A fuzzy number $\hat{a}$ on R is said to be a triangular fuzzy number (TFN) or linear fuzzy number if its membership function $\hat{a} : R \longrightarrow [0,1]$ has the following characteristics.*

$$\mu_{\hat{a}(x)} = \begin{cases} (x-a_1)/(a_2-a_1) & \textit{if } a_1 \le x \le a_2 \\ (a_3-x)/(a_3-a_2) & \textit{if } a_2 \le x \le a_3 \\ 0 & \textit{otherwise} \end{cases} \tag{1}$$

We denote this triangular fuzzy number by $\hat{a} = (a_1, a_2, a_3)$ We use F(R) to denote the set of all triangular fuzzy numbers. Also if $m = a_2$, represents the modal value or midpoint, $\alpha = (a_2 - a_1)$ represents the left spread and $\beta = (a_3 - a_2)$ represents the right spread of the triangular fuzzy number $\hat{a} = (a_1, a_2, a_3)$ then the triangular fuzzy number $\hat{a}$ can be represented by the triple $\hat{a} = (\alpha, m, \beta,)$ i.e., $\hat{a} = (a_1, a_2, a_3) = (\alpha, m, \beta)$.

2.1. Defuzzification

Defuzzification is the process of finding singleton value (crisp value) which represents the average value of the TFNs. Here Method of magnitude is used to defuzzify the TFNs because of its simplicity and accuracy.

Robust ranking technique

For a convex fuzzy number $\tilde{a}$, the Robust's Ranking Index is defined by,

$$R(\tilde{a}) = \int_0^1 (0.5)(a_\alpha^L, a_\alpha^U)\mathrm{d}(\alpha) \tag{2}$$

where $(a_\alpha^L, a_\alpha^U) = \{(b-a)\alpha + a, c - (c-b)\alpha)\}$ which is the α-level cut of the fuzzy number $\tilde{a}$.

Method of magnitude

A triangular fuzzy number $\breve{a} \in F(R)$ can also be represented as a pair $\breve{a}= (\underline{a}, \bar{a})$ of functions $(\underline{a}(r), \bar{a}(r))$ for $0 \le r \le 1$ which satisfies the following requirements:

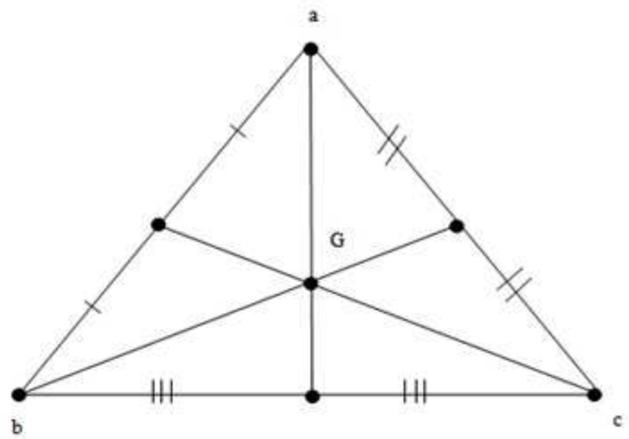
(1) $\underline{a}(r)$ is a bounded monotonic increasing left continuous function.

(2) $\bar{a}(r)$ is a bounded monotonic decreasing left continuous function.

(3) $\underline{a}(r) \le \bar{a}(r)$ for $0 \le r \le 1$.

Definition 2.3. *For an arbitrary triangular fuzzy number $\breve{a} = (\underline{a}, \bar{a})$ the number $a_0 = (\frac{\underline{a}(1)+\bar{a}(1)}{2})$ is said to be a location index number of $\breve{a}$. The two non-decreasing left continuous functions $a_* = (a_0 - \underline{a})$ and $a^* = (\bar{a} - a_0)$ are called the left fuzziness index function and the right fuzziness index functions respectively. Hence every triangular fuzzy number $\breve{a} = (a_1, a_2, a_3)$ can also be represented by $\breve{a} = (a_0, a_*, a^*)$.*

Definition 2.4. *The centroid of a triangle is the point where the three medians of the triangle intersect. The medians are the segments that connected a vertex to the midpoint of the opposite side.*



The Centroid of triangle abc

3. Ranking Functions and Fuzzy Assignment Problem

3.1. Ranking of triangular Fuzzy Numbers

Many different approaches for the ranking of fuzzy numbers have been proposed in the literature. Abbasbandy and Hajjari proposed a new ranking method based on the left and the right spreads at some α -levels of fuzzy numbers. For an arbitrary triangular fuzzy number $\breve{a} = (a_1, a_2, a_3) = (a_0, a_*, a^*)$ with parametric form $\breve{a} = (\underline{a}(r), \bar{a}(r))$ we define the magnitude of the triangular fuzzy number by $\breve{a}$ by

$$Mag(\breve{a}) = \frac{1}{2}\int_0^1 (\overline{a} + \underline{a} + a_0)(f(r))\mathrm{d}r = \frac{1}{2}\int_0^1 (a^* + 3a_0 + a_*)(f(r))\mathrm{d}r \tag{3}$$

[Since $\bar{a} + \underline{a} + a_0 = a^* + a_0 + a_0 - a_* + a_0 = a^* + 3a_0 - a_*$)]. Where the function $f(r)$ is a non-negative and increasing function on $[0, 1]$ with $f(0) = 0$, $f(1) = 1$ and

$$\int_0^1 (f(r))\mathrm{d}r = \frac{1}{2} \tag{4}$$

The function $f(r)$ can be considered as a weighting function. In real life applications, $f(r)$ can be chosen by the decision maker according to the situation. In this paper, for convenience we use $f(r) = r$. Hence

$$Mag(\breve{a}) = \left(\frac{a^* + 3a_0 - a_*}{4}\right) = \left(\frac{\bar{a} + \underline{a} + a_0}{4}\right)$$

The magnitude of a triangular fuzzy number synthetically reflects the information on every membership degree, and meaning of this magnitude is visual and natural. Mag($\breve{a}$) is used to rank fuzzy numbers.

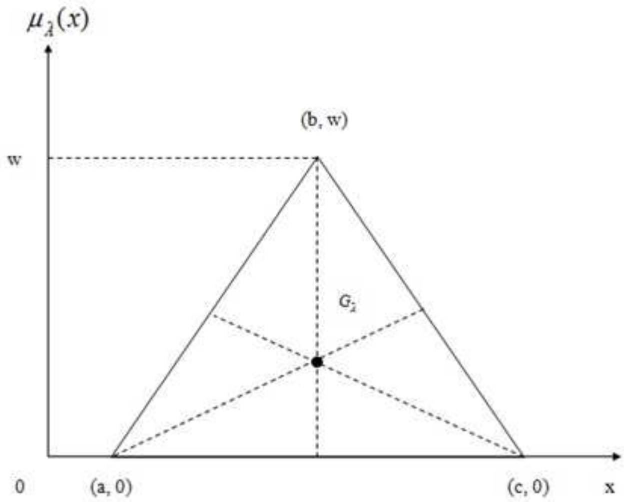
Theorem 3.1. *For any two fuzzy numbers, $A = \langle s_1, l_1, r_1\rangle$ and $B = \langle s_2, l_2, r_2\rangle$ and $A \leq B$ if and only if $s_1 \leq s_2$ and $s_1 - l_1 \leq s_2 - l_2$ and $s_1 + r_1 \leq s_2 + r_2$.*

Properties 3.2. *For any two triangular fuzzy number $\breve{a} = (a_0, a_*, a^*)$ and $\breve{b} = (b_0, b_*, b^*)$*

(1) $Mag(\breve{a}) \geq Mag(\breve{b})$ if and only if $\breve{a} \geq \breve{b}$.

(2) $Mag(\breve{a}) \leq Mag(\breve{b})$ if and only if $\breve{a} \leq \breve{b}$.

(3) $Mag(\breve{a}) = Mag(\breve{b})$ if and only if $\breve{a} \approx \breve{b}$.



Centroid Ranking Function

3.2. Centroid of a Generalized Triangular

The centroid of a triangle fuzzy number $\tilde{A} = (a, b, c; w)$ as $G_{\tilde{A}} = \left(\frac{a+b+c}{3}, \frac{w}{3}\right)$. The ranking function of the generalized triangle fuzzy number $\tilde{A} = (a, b, c; w)$ which maps the set of all fuzzy numbers to a set of real numbers is defined as $R(\tilde{A}) = \left(\frac{a+b+c}{3}\right)\left(\frac{w}{3}\right)$

3.3. Assignment Problem

The assignment problem can be stated in the form of $n \times n$ cost matrix $[C_{ij}]$ of real numbers as given in the following table:

Jobs $\rightarrow$ Persons $\downarrow$	1	2	3	..j..	N
1	$C_1 1$	$C_1 2$	$C_1 3$	$..C_1 j..$	$C_1 n$
2	$C_2 1$	$C_2 2$	$C_2 3$	$..C_2 j..$	$C_2 n$
-	-	-	-	-	-
-	-	-	-	-	-
i	$C_i 1$	$C_i 2$	$C_i 3$	$..C_i j..$	$C_i n$
-	-	-	-	-	-
N	$C_n 1$	$C_n 2$	$C_n 3$	$..C_n j..$	$C_n n$

Mathematically assignment problem can be stated as

$$\begin{aligned} &Minimize \;\; z = \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}x_{ij} \\ &Subjectto \;\; \sum_{i=1}^{n} x_{ij} = 1, \qquad i = 1, 2, 3, \ldots, n \\ &\qquad\qquad\;\; \sum_{j=1}^{n} x_{ij} = 1, \qquad j = 1, 2, 3, \ldots, n, \\ &\qquad\qquad\qquad\qquad\qquad\quad x_{ij} \in \{0, 1\} \end{aligned} \tag{5}$$

where

$$x_{ij} = \begin{cases} 1 & \text{if the } i^th \text{ person is assigned the } j^{th} \text{ job} \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

is the decision variable denoting the assignment of the person i to job j, C_{ij} is the cost of assignment the j^{th} job to the i^{th} person. The objective is to minimize the total cost of assigning all the jobs to the available persons (One job to one person).

When the costs $\tilde{C}_{ij}$ are fuzzy numbers, then the fuzzy assignment problem becomes

$$Y(\tilde{z}) = \sum_{i=1}^{n} \sum_{j=1}^{n} Y(\tilde{C}_{ij}) x_{ij} \tag{7}$$

Subject to the same conditions (5). For an unbalanced problem add dummy rows / columns then follow the same procedure.

3.4. Fuzzy Assignment Problem

The generalized fuzzy assignment problem can be represented in the form of nxn fuzzy cost matrix $\tilde{C}_{ij}$ as given below:

Jobs → Persons ↓	1	2	3	..j..	N
1	$\tilde{C}_1 1$	$\tilde{C}_1 2$	$\tilde{C}_1 3$	$..\tilde{C}_1 j..$	$\tilde{C}_1 n$
2	$\tilde{C}_2 1$	$\tilde{C}_2 2$	$\tilde{C}_2 3$	$..\tilde{C}_2 j..$	$\tilde{C}_2 n$
-	-	-	-	-	-
-	-	-	-	-	-
i	$\tilde{C}_i 1$	$\tilde{C}_i 2$	$\tilde{C}_i 3$	$..\tilde{C}_i j..$	$\tilde{C}_i n$
-	-	-	-	-	-
N	$\tilde{C}_n 1$	$\tilde{C}_n 2$	$\tilde{C}_n 3$	$..\tilde{C}_n j..$	$\tilde{C}_n n$

The cost or time $[\tilde{C}_{ij}]$ are generalized trapezoidal fuzzy numbers $\tilde{C}_{ij} = [C_{ij}^{(1)}, C_{ij}^{(2)}, C_{ij}^{(3)}, C_{ij}^{(4)}; w_{ij}]$. The goal is, effective way of assigning the j^{th} job to the i^{th} resource (all jobs to available resources) by minimizing the total cost with minimum time.

4. Algorithms

4.1. Hungarian Assignment Algorithm

Various steps of the computational procedure for obtaining an optimal solution may be summarized as follows:

Step 1: If the number of rows are not equal to the number of columns and vice versa, then a dummy row or dummy column must be added with zero cost elements.

Step 2: Find the smallest cost in each row of the cost matrix; subtract this smallest cost element from each element in that row. Therefore, there will be at least one zero in each row of this new matrix which is called the first reduced cost matrix.

Step 3: In the reduced cost matrix, find the smallest element in each column. Subtract the smallest cost element from each element in that column. As a result, there would be at least one zero in each row and column of the second reduced cost matrix.

Step 4: Determine an optimum assignment as follows:

(i) Examine the rows successively until a row with exactly one zeros is found. Box around the zero element as an assigned cell and cross out all other zero in its column. Proceed in this manner until all the rows have been examined. If there are more than one zero in any row, then do not consider that row and pass on to the next row.

(ii) Repeat the procedure for the columns of the reduced cost matrix. If there is no single zero in any row or column of the reduced matrix, then arbitrarily choose a row or column having the minimum number of zeroes. Arbitrarily, choose zero in the row or column and cross the remaining zeroes in that row or column.

Repeat steps (i) and (ii) until all zeros are either assigned or crossed out.

Step 5: An optimal assignment is found, if the number of assigned cells equals the number of rows (and columns). If a zero cell is arbitrarily chosen, there may be an alternate optimum. If no optimum solution is found (some rows or columns without an assignment), then go to next step.

Step 6: Draw the minimum number of horizontal and / or vertical lines through all the zeros as follow:

(i) Mark (✓) to those rows where no assignment has been made.

(ii) Mark (✓) to those columns which have zeros in the marked rows.

(iii) Mark (✓) rows (not already marked) which have assignments in marked columns.

(iv) The process may be repeated until no more rows or columns can be checked.

(v) Draw straight lines through all unmarked rows and marked columns.

Step 7: If the minimum number of lines passing through all the zeros is equal to the number of rows or columns, the optimum solution is attained by an arbitrary allocation in the positions of the zeroes not crossed in step 3. Otherwise we shall move to the next step.

Step 8: Revise the cost matrix as follows:

(i) Find the elements that are covered by a line. Choose the smallest of these elements and subtract this element from all the uncrossed elements and add the same at the point of intersection of the two lines.

(ii) Other elements crossed by the lines remain unchanged.

Step 9: Go to Step 4 and repeat the procedure till an optimum solution is attained.

4.2. Algorithm to solve fuzzy assignment problem

Step 1: First test whether the given fuzzy cost matrix of a fuzzy assignment problem is a balanced one or not. If not change this unbalanced assignment problem into balanced one by adding the number of dummy row(s) / column(s) and the values for the entries are zero. If it is a balanced one (i.e. number of persons are equal to the number of works) then go to Step 2.

Step 2: Defuzzify the fuzzy cost by using Centroid ranking method.

Step 3: Apply Hungarian Algorithm to determine the best combination to produce the lowest total costs, where each machine should be assigned to only one job and each job requires only one machine.

5. Numerical Example

Example 5.1. *Here we are going to solve fuzzy Assignment problem using Centroid Ranking Technique: To allocate 4 jobs to 4 different machines, the fuzzy assignment cost C_ij is given below:*

$$\begin{bmatrix} (1,5,9) & (3,7,11) & (7,11,15) & (2,6,10) \\ (4,8,12) & (1,5,9) & (4,9,13) & (2,6,10) \\ (0,4,8) & (3,7,11) & (6,10,14) & (3,7,11) \\ (6,10,14) & (0,4,8) & (4,8,12) & (-1,3,7) \end{bmatrix} \tag{8}$$

Solution. The given Triangular fuzzy cost matrix is balanced one. Now we calculate $R(1,5,9)$ by applying Centroid Ranking Technique of a Triangular fuzzy number $R(\tilde{A}) = \left(\frac{a+b+c}{3}\right)\left(\frac{w}{3}\right)$ and the problem is done by taking the value of ω as 1.

$$R(1,5,9) = \left(\frac{1+5+9}{3}\right)\left(\frac{1}{3}\right) = 1.67$$
$$R(3,7,11) = \left(\frac{3+7+11}{3}\right)\left(\frac{1}{3}\right) = 2.33$$

Similarly, $R(7,11,15) = 3.67$, $R(2,6,10) = 2$, $R(4,8,12) = 2.67$, $R(1,5,9) = 1.67$, $R(4,9,13) = 2.89$, $R(2,6,10) = 2$, $R(0,4,8) = 1.33$, $R(3,7,11) = 2.33$, $R(6,10,14) = 3.33$, $R(3,7,11) = 2.33$, $R(6,10,14) = 3.33$, $R(0,4,8) = 1.33$, $R(4,8,12) = 2.67$, $R(-1,3,7) = 1$. The rank of triangular fuzzy cost table is:

	I	II	III	IV
A	1.67	2.33	3.67	2
B	2.67	1.67	2.89	2
C	1.33	2.33	3.33	2.33
D	3.33	1.33	2.67	1

Proceeding by Hungarian method,the optimal allocations are : Therefore

	I	II	III	IV
A	**0**	0.21	0	0.21
B	1.45	**0**	0	0.66
C	0	0.55	**0**	0.88
D	2.45	0	0.12	**0**

The fuzzy optimal total cost

$$\tilde{a}_{11} + \tilde{a}_{22} + \tilde{a}_{33} + \tilde{a}_{44} = (1,5,9) + (1,5,9) + (6,10,14) + (-1,3,7) = (7,23,39).$$

Now, by using Centroid Ranking Technique of a Triangular fuzzy number then $R(7,23,39) = 7.67$.

Result: We will compare the assignment cost which has been found out in Example 5.1 with the assignment cost calculated by existing methods (Kalaiarasi [1], Jatinder Pal Singh [2], Selvi [3])

	Existing methods			Proposed Method
	Kalaiarasi [1]	Jatinder Pal Singh [2]	Selvi [3]	
Example 5.1	Assignment cost=23	Assignment cost=22.75	Assignment cost=8.5	Assignment cost=7.67

6. Conclusion

In this paper, the fuzzy costs of a Triangular Fuzzy Assignment Problem has been defuzzified into crisp values by using Centroid Ranking method and solved by using Hungarian method. By comparing the results of the proposed method and existing methods, it is shown that the proposed method has given better results than the existing methods

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