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Wheel Related Intersection Cordial Labeling of Graphs

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Abstract: An intersection cordial labeling of a graph G with vertex set V is an injection f from V to the power set of $\{1, 2, \ldots, n\}$ such that if each edge uv is assigned the label 1 if $f(u) \cap f(v) \neq \emptyset$ and 0 otherwise; Then the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. If a graph has an intersection cordial labeling, then it is called intersection cordial graph. In this paper, we proved the wheel related graphs are intersection cordial.

MSC: 05C78.

Keywords: Cordial labeling, Intersection cordial labeling, Intersection cordial graphs.

1. Introduction

By a graph, we mean a finite, undirected graph without loops and multiple edges and for terms not defined here, we refer to Harary [5]. Here we consider only connected graph. Let $X = \{1, 2, \ldots, n\}$ be a set and $\wp(X)$ be the collection of all subsets of X, called the power set of X. If A is a subset of B, we denote it by $A \subset B$, otherwise by $A \not\subset B$. Note that $\wp(X)$ contains 2^n subsets. Graph labeling [4] is a strong communication between Algebra [3] and structure of graphs [5]. By combining the set theory concept in Algebra and Cordial labeling concept in Graph labeling, we introduced a new concept called intersection cordial labeling [8]. In this paper, we prove some wheel related graphs are intersection cordial. A vertex labeling [4] of a graph G is an assignment f of labels to the vertices of G that induces each edge uv a label depending on the vertex label $f(u)$ and $f(v)$. Graceful and harmonious labeling are two well known labelings. Cordial labeling is a variation of both graceful and harmonious labeling [1].

Definition 1.1. *For $p \geq 4$, the wheel on p vertices, denoted by W_p, is defined to the graph $K_1 + C_{p-1}$. Note that $q = 2(p-1)$. If we remove anyone of the rim edge of W_p, then we get the fan graph F_p.*

Definition 1.2. *Let $G = (V, E)$ be a graph. A mapping $f : V(G) \to \{0, 1\}$ is called binary vertex labeling of G and $f(v)$ is called the label of the vertex v of G under f.*

Let $v_f(0)$ and $v_f(1)$ be the number of vertices of G having labels 0 and 1 respectively under f and $e_f(0)$ and $e_f(1)$ be the number of edges having labels 0 and 1 respectively under f^*. The concept on cordial labeling was introduced by cahit [1].

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Definition 1.3 ([2]). *For an edge $e = uv$, the induced edge labeling $f^* : E(G) \to \{0, 1\}$ is given by $f^*(e) = |f(u) - f(v)|$. A binary vertex labeling of a graph G is called a cordial labeling, if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is cordial, if it admits cordial labeling.*

Definition 1.4 ([8]). *Let $X = \{1, 2, \ldots, n\}$ be a set. Let $G = (V, E)$ be a simple (p, q)-graph and $f : V \to \{0, 1\}$ be an injection. Also, let $2^{n-1} < p \leq 2^n$. For each edge uv, assign label 1 if either $f(u) \subset f(v)$ or $f(v) \subset f(u)$ otherwise assign 0 if $f(u)$ is not a subset of $f(v)$ and $f(v)$ is not a subset of $f(u)$, f is called a subset cordial labeling if $|e_f(0) - e_f(1)| \leq 1$. A graph is called a subset cordial graph if it has a subset cordial labeling [8].*

Motivated by the above definition, we defined intersection cordial labeling [6].

Definition 1.5 ([6]). *Let $X = \{1, 2, \ldots, n\}$ be a set. Let $G = (V, E)$ be a simple (p, q)-graph and $f : V \to \{0, 1\}$ be an injection. Also, let $2^{n-1} < p \leq 2^n$. For each edge uv, assign label 1 if $f(u) \cap f(v) \neq \emptyset$ and 0 if $f(u) \cap f(v) = \emptyset$. f is called a intersection cordial labeling if $|e_f(0) - e_f(1)| \leq 1$. A graph is called an intersection cordial graph if it has an intersection cordial labeling. Here $\rho(X)$ denotes the power set of X.*

The following results have been proved in [6] and it will be used proving some results in this paper.

Theorem 1.6 ([6]). *The star graph $K_{1,q}$ is intersection cordial.*

Theorem 1.7 ([6]). *The path P_{2^n} is intersection cordial.*

Theorem 1.8 ([6]). *The Cycle C_{2^n} is intersection cordial.*

In the paper [7], we have proved some star related graph such as bistar, splitting graph of star, splitting graph of bistar, identification of a star with a graph and identification of bipartite graph with a graph are intersection cordial.

2. Main Results

In this section, we will prove some wheel related graphs are intersection cordial. First, we will prove the wheel graph is intersection cordial.

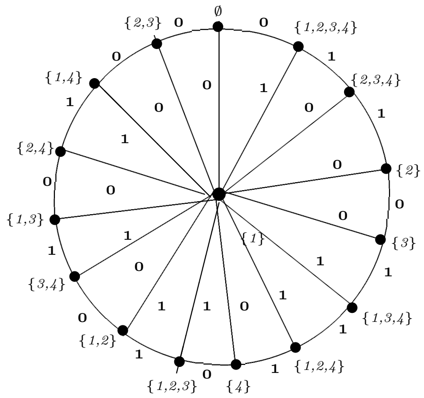
Theorem 2.1. *Wheel graph $W_{1,2^n-1}$ is intersection cordial for $n \geq 3$.*

Proof. Let $X = \{1, 2, \ldots, n\}$. Note that the wheel graph is the combined form of star and cycle graphs. Now, we label the subsets of X to the vertices of $W_{1,2^n-1}$ as in the Theorem 1.6. We note that, the cycle part of wheel $W_{1,2^n-1}$ containing $2^n - 1$ vertices, but in the Theorem 1.8, the cycle containing 2^n vertices. So the contributions of 0's and 1's to the rim edges will have the changes. We see that the label $\{1\}$ was assigned to the vertex of cycle as in the Theorem 1.8, now it is assigned to the centre of the wheel. Note that, this assignment reduces the contribution of 0 to $e_f(0)$. They are given in following table. Note that $W_{1,2^n-1}$ has $2^{n+1} - 2$ edges.

No.	Edges	$e_f(0)$	$e_f(1)$
(i)	The edges in the star part of wheel	2^{n-1}	$2^{n-1} - 1$
(ii)	The edges in the cycle part of wheel	$2^{n-1} - 1$	2^{n-1}
	Total	$2^n - 1$	$2^n - 1$

Thus $e_f(0) = e_f(1) = 2^n - 1$ and so $| e_f(0) - e_f(1) |= 0$. Hence $W_{1,2^n-1}$ is intersection cordial. □

Example 2.2. *Consider the graph $W_{1,15}$.*



Here $e_f(0) = 15$, $e_f(1) = 15$. *Thus* $|e_f(0) - e_f(1)| = 0$ *and so* $W_{1,2^{n-1}}$ *is intersection cordial.*

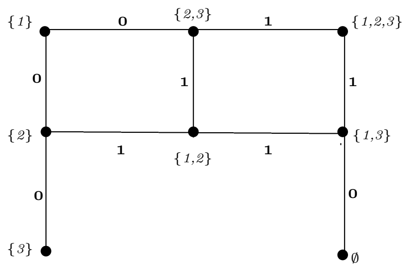
Corollary 2.3. *The fan graph* F_n *is intersection cordial.*

Proof. If we remove anyone of the rim edges of W_n, then we get the fan graph. By Theorem 2.1, it follows that $|e_f(0) - e_f(1)| \leq 1$. Then F_n is intersection cordial. □

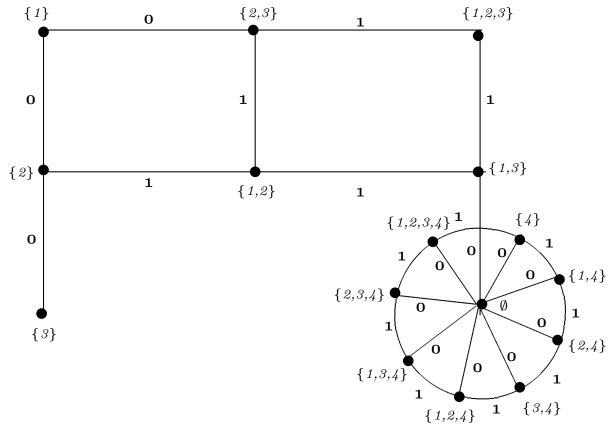
Theorem 2.4. *Let* $X = \{1, 2, \ldots, n\}$ *and* G *be any intersection cordial* $(2^n, q)$*- graph. Then the graph* $W = G * W_{1,2^n}$ *obtained by identifying the central vertex of* $W_{1,2^n}$ *with the vertex that labeled* $\emptyset$ *in* G *is also intersection cordial.*

Proof. Since G is intersection cordial, $| e_{f_G}(0) - e_{f_G}(1) | \leq 1$. Already 2^n subsets of $\{1, 2, \ldots, n\}$ have been labeled to the vertices of G. Now, we construct 2^n subsets by adding a new element $n+1$ to each subset of X and by replacing $\emptyset$ with $\{n+1\}$. Assign these 2^n subsets to the vertices of $W_{1,2^n}$ in any order and identifying the central vertex of $W_{1,2^n}$ labeled with $\emptyset$ in G. Since the labeling of central vertex is $\emptyset$, all the spokes of $W_{1,2^n}$ get the label 0 and it contributes 2^n to $e_{f_W}(0)$. Also, we see that all the labeling of rim vertices having the element $n+1$ in common. So, the rim edges get the label 1 and it contributes 2^n to $e_{f_W}(1)$. Hence $| e_{f_G}(0) - e_{f_G}(1) | \leq 1$ implies $| e_{f_W}(0) - e_{f_W}(1) | \leq 1$. Thus W is intersection cordial. □

Example 2.5. *Let* $X = \{1, 2, 3\}$. *Consider the following intersection cordial graph* G.



We see that $e_{f_G}(0) = 4$ *and* $e_{f_G}(1) = 5$. *Then* $| e_{f_G}(0) - e_{f_G}(1) | = 1$. *Now, we identifying the central vertex of* $W_{1,8}$ *to the vertex of* G *labeled by* $\emptyset$. *Then intersection cordiality of* $W = G * W_{1,8}$ *is shown below. For* W, *we take the set* $\{1, 2, 3, 4\}$.



Now, we see that $e_{f_W}(0) = 12$ *and* $e_{f_W}(1) = 13$. *Then* $\mid e_{f_W}(0) - e_{f_W}(1) \mid = 1$. *Thus* W *is intersection cordial.*

Theorem 2.6. *The wheel corona* $W_{2^n}^+ = (C_{2^n-1} + K_1)^+$ *is intersection cordial.*

Proof. The graph $W_{2^n}^+$ is obtained by adding pendent edges to each of the rim vertices of W_{2^n}. This graph has $2^{n+1} - 1$ vertices and $3.(2^n - 1)$ edges. Let $X = \{1, 2, \ldots, n, n+1\}$. Now, we label $2^{n+1} - 1$ subsets of X to the vertices of $W_{2^n}^+$. First, we label $\emptyset$ to the central vertex of W_{2^n}. Then we see that the $2^n - 1$ spokes of W_{2^n} get the label 0, since the intersection of any subset with $\emptyset$ is $\emptyset$.

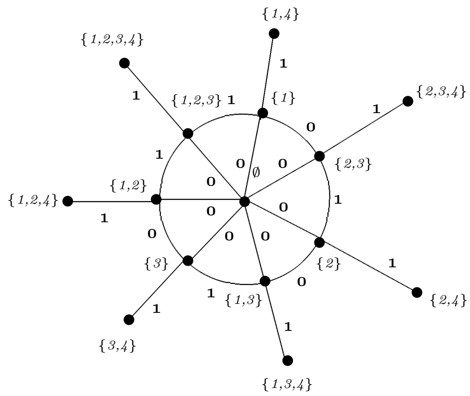
Next, we label the rim vertices of W_{2^n-1} as follows. Starting any rim vertex and label 1-element subset of $\{1, 2, \ldots, n\}$. Then label the next vertex by its complement. After exhausting 1-element subsets and their complements, then we label 2-elements subsets of $\{1, 2, \ldots, n\}$ and their complements alternatively. Continuing this process, until all the subsets of $\{1, 2, \ldots, n\}$ are exhausted except the set $\{1, 2, \ldots, n\}$ whose complement is $\emptyset$ which has already been labeled to the central vertex. We also label the set $\{1, 2, \ldots, n\}$ to the last remaining vertex of W_{2^n-1}. We see that, the rim edges got the labels 0 and 1 alternatively and the last edge whose end vertices are labeled by the subsets $\{1, 2, \ldots, n\}$ and 1, got the label 1.

Next, we label the pendent vertices of $W_{2^n}^+$ by $A \cup \{n+1\}$ where $A(\neq \emptyset)$ is the subset of $\{1, 2, \ldots, n\}$ which was labeled to corresponding end vertex in C_{2^n-1} of W_{2^n-1}. We see that $(A \cup \{n+1\}) \cap A = A \neq \emptyset$. So all the pendent edges of $W_{2^n-1}^+$ get the label 1. The edge contribution is shown in the following table.

No.	Edges	$e_f(0)$	$e_f(1)$
(i)	Spokes of W_{2^n-1}	$2^n - 1$	0
(ii)	Rim edges of W_{2^n-1}	$2^{n-1} - 1$	2^{n-1}
(iii)	Pendent edges of $W_{2^n-1}^+$	0	$2^n - 1$
	Total	$3.2^{n-1} - 2$	$3.2^{n-1} - 1$

Then $\mid e_f(0) - e_f(1) \mid = 1$. Hence W_{2^n+-1} is intersection cordial. □

Example 2.7. *Consider the graph* $W_8^+ = (C_7 + K_1)^+$ *and let* $X = \{1, 2, 3, 4\}$.



Here $e_f(0) = 10, e_f(1) = 11$ *and so* $\mid e_f(0) - e_f(1) \mid = 1$.

Conclusion 2.8. *We are also trying to prove the following wheel related graphs are intersection cordial.*

(1). Wheel with two centres

(2). Gear graph

(3). Flower graph

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