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Q(a) Balance Edge-Magic of Few Connected Graphs

S. Vimala[1]*

1 Department of Mathematics, Mother Teresa Women's University, Kodaikanal, Tamil Nadu, India.

Abstract: In this paper Q(a)-BEM label is discussed the following connected graphs such as Cycle graph, Ladder graph and Gear graph. If G is a (p, q)- graph in which the edges are labeled $1, 2, 3, \ldots, q$ so that the vertex sums are constant mod p, then G is called an Edge-Magic graph (in short, EM graph). Our purpose of this work is to show that in Q(a)-BEM graphs.

Keywords: Edge magic, balance edge-magic graph, Q(a) balance edge-magic graph, Cycle graph, Ladder graph, Gear graph.

1. Introduction and Preliminaries

Labeling provides many marvellous results in all science and social sciences. All graphs in this paper are connected. The graph G has vertex set $V(G)$ and edge set $E(G)$. A labeling (or valuation) of a graph is a map that carries graph elements to numbers (usually to the positive or non- negative integers). Edge magic graph introduced by Sin Min Lee, Eric Seah and S.K Tan in 1992. Various author discussed in edge magic graphs like Edge magic $(p, 3p-1)$-graphs, Zykov sums of graphs, cubic multigraphs, Edge-magicnessof the composition of a cycle with a null graph and many graphs in several years. In 2007 Sin-Min Lee and Thomas Wong and Sheng-Ping Bill Lo [3] introduced two types of magic labeling such as $Q(a)$-Balance Edge-Magic Graphs(BEM) and $Q(a)$-Balance Super Edge-Magic Graphs(BSEM) of some graphs and proved several conjectures. The labeling to be edge-magic if the sum of all labels associated with an edge equals a constant independent of the choice of edge, and vertex-magic if the same property holds for vertices. In this paper, magic labeling, strong $Q(a)$ BEM of cycle, Ladder and Gear graphs.

A graph G is a (p, q)-graph in which the edges are labeled by $1, 2, 3, \ldots, q$ so that the vertex sum are constant, mod p, than G is called an edge-magic graph (for simplicity we denote EM). A (p, q)-graph G in which the edges are labeled by $Q(a)$ so that the vertex sums mod p is a constant, is called $Q(a)$ Balance Edge-Magic(in short, $Q(a)$-BEM). For $a \geq 1$, then we denote

$$Q(a) = \begin{cases} \pm a, \pm(a+1), \pm(a+2), \cdots \pm\left((a-1)+\frac{q}{2}\right) & \text{if q is even} \\ 0, \pm a, \pm(a+1), \pm(a+2), \cdots \pm\left((a-1)+\frac{q-1}{2}\right) & \text{if q is odd} \end{cases}$$

A cycle graph C_n, sometimes simply known as an n-cycle is a graph on n nodes containing a single cycle through all nodes. Cycle graphs (as well as disjoint unions of cycle graphs) are two-regular. The n-ladder graph can be defined as $p_2 * p_n$, where p_n is a path graph. It is therefore equivalent to the $2 * n$ grid graph. The ladder graph is named for its resemblance to a ladder consisting of two rails and n rings between them. The gear graph, also sometimes known as a bipartite wheel graph,

* *E-mail: @mail.com*

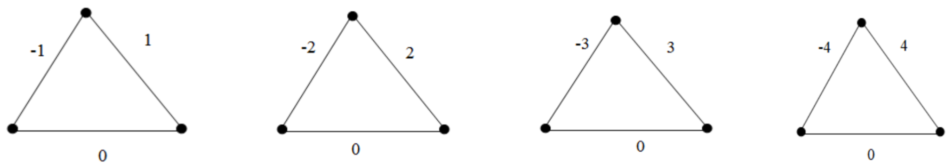
is a wheel graph with a graph vertex added between each pair of adjacent graph vertices of the outer cycle. It has $2n+1$ vertices and $3n$ edges. The Windmill graph $D_n^{(m)}$ is the graph obtained by taking m copies of the complete graph K_n with a vertex in common (Gallian 2011, p-16). The case $n = 3$, the corresponds to the Dutch Windmill graph $D_n^{(m)}$.

2. Main Results of Q(a)-Balance Edge-Magic Labeling

In this chapter discuss about $Q(a)$-Balance Edge Magic (BEM) of Cycle graph, Ladder graph and Gear graph.

Theorem 2.1. *Cycle graph is $Q(a)$-BEM for $a = 1, 2, 3, 4$.*

Proof. If $n = 3$, It suffices to show that $Q(a)$-BEM for $a = 1, 2, 3, 4$. For $a \geq 1$, we denote as, here q is Odd, figure shows that is strong $Q(a)$-BEM.



Q(1)-BEM Q(2)-BEM Q(3)-BEM Q(4)-BEM

For the above pictures

Q(1)- BEM labeling for Cycle graph = { 0,1, -1}

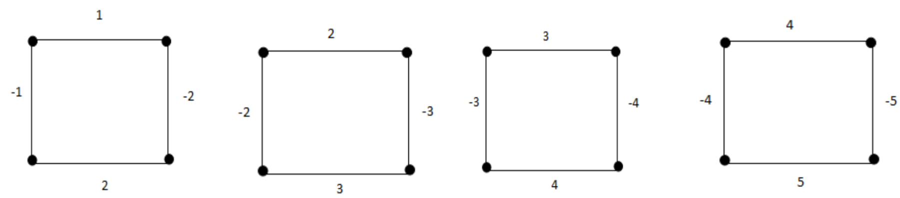
Q(2)- BEM labeling for Cycle graph = { 0,2, -2}

Q(3)- BEM labeling for Cycle graph = { 0, 3,-3}

Q(4)- BEM labeling for Cycle graph = { 0, 4,-4} □

Theorem 2.2. *Ladder graph is strong Q(a)-BEM for $a = 1, 2, 3, 4$.*

Proof. If $n = 2$, It suffices to show that is strong Q(a)-BEM for $a = 1, 2, 3, 4$. Here q is even, figure shows that G is strong Q(a)-BEM



Q(1)-BEM Q(2)-BEM Q(3)-BEM Q(4)-BEM

From the above picture

Q(1)- BEM labeling for Ladder graph = { 1, -1, 2, 2}

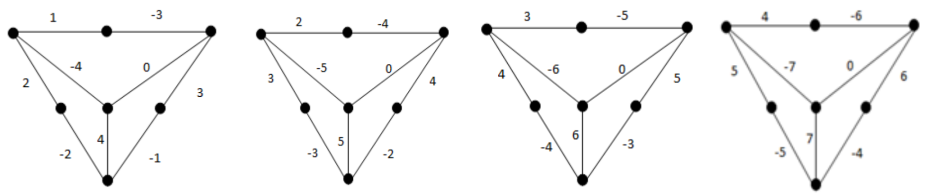
Q(2)- BEM labeling for Ladder graph = { 2, -2, 3,3}

Q(3)- BEM labeling for Ladder graph = { 3, -3, 4,4}

Q(4)- BEM labeling for Ladder graph = { 4, -4, 5,5} □

Theorem 2.3. *Gear graph is strong Q(a)-BEM for $a = 1, 2, 3, 4$.*

Proof. If $n = 7$, It suffices to show that is Q(a)-BEM for $a = 1, 2, 3, 4$. Here q is odd,



Q(1)-BEM Q(2)-BEM Q(3)-BEM Q(4)-BEM

Q(1)- BEM labeling for gear graph $= \{0, 1, -1, 2, -2, 3, -3, 4, -4\}$

Q(2)- BEM labeling for gear graph $= \{0, 2, -2, 3, -3, 4, -4, 5, -5\}$

Q(3)- BEM labeling for gear graph $= \{0, 3, -3, 4, -4, 5, -5, 6, -6\}$

Q(4)- BEM labeling for gear graph $= \{0, 4, -4, 5, -5, 6, -6, 7, -7\}$ □

Theorem 2.4. *Windmill graph $D_3^{(4)}$ (W_9) is strong Q(a) BEM.*

Proof. Q is even, so for $a = 1, 2, 3, 4$

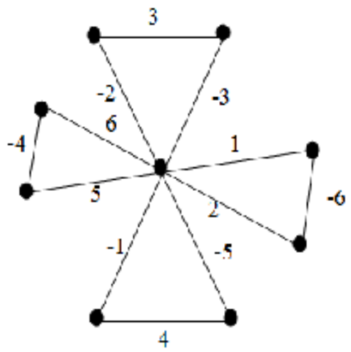


Figure 1: Q(1)-BEM for labelingof Windmill graph $\{1, -1, 2, -2, 3, -3, 4, -4, 5, -5, 6, -6\}$

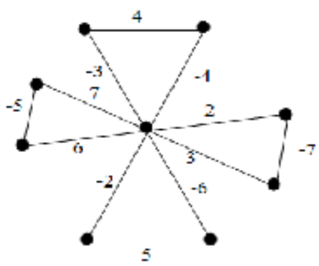


Figure 2: Q(2)-BEM for labeling of Windmill graph $\{2, -2, 3, -3, 4, -4, 5, -5, 6, -6, 7, -7\}$

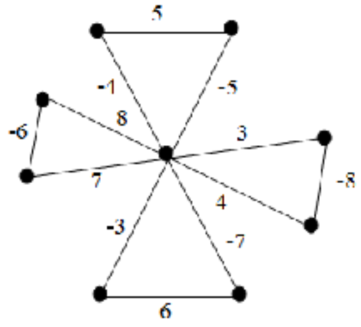


Figure 3: Q(3)-BEM for labeling of Windmill graph $\{3, -3, 4, -4, 5, -5, 6, -6, 7, -7, 8, -8\}$

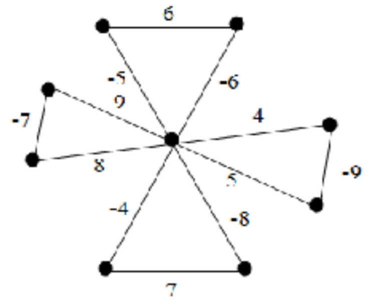


Figure 4: Q(4)-BEM for labeling of Windmill graph $\{4, -4, 5, -5, 6, -6, 7, -7, 8, -8, 9, -9\}$

□

3. Conclusion

This results has been found about Q(a)-Balance Edge magic labeling. Propose work will be proceed in P(a) methods.

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