Int. J. Math. And Appl., 6(3)(2018), 27–30 (Special Issue)
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Divisor Degree Index of Graphs

S. P. Kanniga Devi[1,*] and K. Nagarajan[2]

1 Research Scholar, Sri S.R.N.M.College, Sattur, Tamil Nadu, India.

2 Department of Mathematics, Kalasalingam University, Srivilliputhur, Virudhunagar, Tamil Nadu, India.

Abstract: In this paper, we introduce the concepts of divisor degree of a vertex v and divisor degree index dd (G) of a simple graph G. We also introduce maximum divisor degree and minimum divisor degree of a simple graph G. Also, we find divisor degree of some standard graphs and establish the relation between degree and divisor degree index of graphs.

MSC: 05C07.

Keywords: Degree, Divisor degree index, Maximum divisor degree and Minimum divisor degree.

1. Introduction

By a graph, we mean a finite, undirected, non-trivial, connected graph without loops and multiple edges. The order and size of a graph are denoted by n and m respectively. For terms not defined here we refer to Harary [1]. In this paper we have used the following notations.

(1). $[x]$ denotes integral part of real number x.

(2). $\sum\limits_{i\sim k}$ means summation over all pair of adjacent vertices v_i and v_k.

We introduced the concept of divisor degree matrix $DD(G)$ of a simple graph G and obtain eigenvalues of $DD(G)$ [2]. We also introduced divisor degree energy (DDE) of graphs denoted by $E_{DD}(G)$ and found DDE of some standard graphs [2]. Motivated by divisor degree energy of a graph [2], in this paper we introduce divisor degree of a vertex v and divisor degree index $dd(G)$ of a simple and connected graph G.

Definition 1.1. *Let G be a simple graph with n vertices and m edges. Let d_i and d_k be the degrees of v_i and v_k respectively. Define*

(1). $dd(v_i) = \begin{cases} \sum\limits_{i\sim k}\left(\left[\frac{d_i}{d_k}\right]+\left[\frac{d_k}{d_i}\right]\right), & \text{if } v_i \text{ and } v_k \text{ are adjacent} \\ 1, & \text{if } d_i = d_k; v_i \text{ and } v_k \text{ are adjacent} \\ 0, & \text{otherwise} \end{cases}$ *is called divisor degree of a vertex v_i.*

(2). $dd(G) = \sum\limits_{i=1}^{n} dd(v_i)$ *is called divisor degree index of a graph G.*

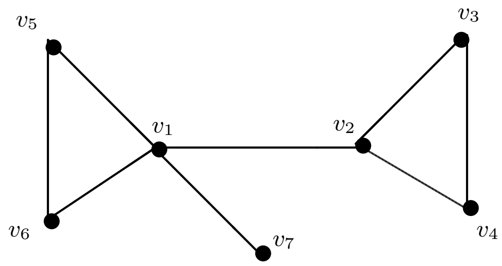
* E-mail: spkmat10@gmail.com

Definition 1.2. *For any graph G, we define as follows:*

(1). $\delta_{dd}(G) = \min\{dd(v)/v\epsilon V(G)\}$ *is called minimum divisor degree of G.*

(2). $\Delta_{dd}(G) = max\{dd(v)/v\epsilon V(G)\}$ *is said to be maximum divisor degree of G.*

Example 1.3. *Consider the graph G*



(1). $dd(v_1) = \left(\left[\frac{d_1}{d_2}\right] + \left[\frac{d_2}{d_1}\right]\right) + \left(\left[\frac{d_1}{d_5}\right] + \left[\frac{d_5}{d_1}\right]\right) + \left(\left[\frac{d_1}{d_6}\right] + \left[\frac{d_6}{d_1}\right]\right) + \left(\left[\frac{d_1}{d_7}\right] + \left[\frac{d_7}{d_1}\right]\right)$

$= 1 + 2 + 2 + 4 = 9$

$dd(v_2) = \left(\left[\frac{d_2}{d_1}\right] + \left[\frac{d_1}{d_2}\right]\right) + \left(\left[\frac{d_2}{d_3}\right] + \left[\frac{d_3}{d_2}\right]\right) + \left(\left[\frac{d_2}{d_4}\right] + \left[\frac{d_4}{d_2}\right]\right)$

$= 1 + 1 + 1 = 3$

Similarly, $dd(v_3) = 2, dd(v_4) = 2, dd(v_5) = 3, dd(v_6) = 3$ *and* $dd(v_7) = 4$.

(2). $dd(G) = \sum_{i=1}^{7} dd(v_i) = 9 + 3 + 2 + 2 + 3 + 3 + 4 = 26.$

(3). $\delta_{dd}(G) = \min\{9, 3, 2, 2, 3, 3, 4\} = 2.$

(4). $\Delta_{dd}(G) = \max\{9, 3, 2, 2, 3, 3, 4\} = 9.$

2. Divisor Degree Index of Some Standard Graphs

We observe that in first theorem of graph theory, we prove for regular graph for the divisor degree index. Then, we have the following results.

Result 2.1.

(1). $dd(C_n) = 2m$.

(2). $dd(K_n) = 2m$.

Theorem 2.2. *If P_n is a path graph of order n, then $dd(P_n) = 2n + 2$, $n \geq 3$.*

Proof. Let $P_n : v_1, v_2, ..., v_n$ be a path graph. Note that $d(v_1) = d(v_n) = 1$ and $d(v_i) = 2$, for $i = 2, 3, ..., (n-1)$. Clearly $dd(v_1) = dd(v_n) = 2$ and $dd(v_2) = dd(v_{n-1}) = 3$. Also $dd(v_j) = 2$, for $j = 3, 4, ..., n-2$. Therefore,

$$\begin{aligned} dd(P_n) &= \sum_{k=1}^{n} dd(v_k) \\ &= 2 + 3 + 2(n-4) + 3 + 2 \\ &= 2n + 2. \end{aligned}$$

□

Theorem 2.3. *Let K_{n_1,n_2} be a complete bipartite graph. Then $dd(K_{n_1,n_2}) = 2n_1n_2\left[\frac{n_2}{n_1}\right], (n_1 \leq n_2)$.*

Proof. Without loss of generality we partition the vertex set of the complete bipartite graph K_{n_1,n_2} into disjoint sets $A = \{u_1, u_2, ..., u_{n_1}\}$ and $B = \{v_1, v_2, ..., v_{n_2}\}$ such that no two vertices in either sets are adjacent to each other. Note that $d(u_i) = n_2$, for $i = 1, 2, ..., n_1$ and $d(v_j) = n_1$, for $j = 1, 2, ..., n_2$. Then

$$\sum_{i=1}^{n_1} d(u_i) + \sum_{j=1}^{n_2} d(v_j) = 2n_1n_2$$

Clearly, for $i = 1, 2, ..., n_1$

$$dd(u_i) = \sum_{i\sim j}\left(\left[\frac{d_i}{d_j}\right] + \left[\frac{d_j}{d_i}\right]\right) = n_2\left[\frac{n_2}{n_1}\right]$$

Also, for $j = 1, 2, ..., n_2$

$$dd(v_j) = \sum_{j\sim i}\left(\left[\frac{d_i}{d_j}\right] + \left[\frac{d_j}{d_i}\right]\right) = n_1\left[\frac{n_2}{n_1}\right]$$

Therefore,

$$dd(K_{n_1,n_2}) = \sum_{i=1}^{n_1} d(u_i) + \sum_{j=1}^{n_2} d(v_j) = 2n_1n_2\left[\frac{n_2}{n_1}\right].$$

□

Corollary 2.4. *For a star graph K_{1,n_2}, $dd(K_{1,n_2}) = 2n_2\left[\frac{n_2}{n_1}\right]$.*

Theorem 2.5. *If W_n is a wheel graph of order n, then $dd(W_n) = m\left(1 + \left[\frac{m}{6}\right]\right)$, where m is the number of edges in G.*

Proof. Let $v_1, v_2, ..., v_n$ be n vertices and $m = 2(n-1)$. Note that $d(v_i) = 3$ for $i = 1, 2, ..., (n-1)$ and $d(v_n) = n-1$. Clearly $dd(v_i) = 2 + \left[\frac{n-1}{3}\right]$ for $i = 1, 2, ..., (n-1)$ and $d(v_n) = (n-1)\left[\frac{n-1}{3}\right]$. Hence,

$$\begin{aligned}
dd(W_n) &= \sum_{i=1}^{n-1} dd(v_i) + dd(v_n) \\
&= (n-1)\left(2 + \left[\frac{n-1}{3}\right]\right) + (n-1)\left[\frac{n-1}{3}\right] \\
&= 2(n-1)\left(1 + \left[\frac{n-1}{3}\right]\right) \\
&= m\left(1 + \left[\frac{m}{6}\right]\right).
\end{aligned}$$

□

3. Relation Between Degree and Divisor Degree Index of Graphs

Theorem 3.1. *Let G be a graph with n vertices and m edges. Then $d_i \leq dd(v_i)$, where d_i is a degree of a vertex v_i, $i = 1, 2, ..., n$.*

Proof. **Case 1:** Let $d_i > d_j$ or $d_j > d_i$, where v_i is adjacent to v_j. Then $\left[\frac{d_i}{d_j}\right] + \left[\frac{d_j}{d_i}\right] > 1$ and so

$$\sum_{i\sim j}\left(\left[\frac{d_i}{d_j}\right] + \left[\frac{d_j}{d_i}\right]\right) > 2d_i > d_i.$$

Hence, $d_i < dd(v_i)$.

Case 2: Let $d_i = d_j$. Then $\left[\frac{d_i}{d_j}\right] + \left[\frac{d_j}{d_i}\right] = 1$ and so

$$\sum_{i\sim j}\left(\left[\frac{d_i}{d_j}\right] + \left[\frac{d_j}{d_i}\right]\right) = d_i.$$

Hence, $d_i = dd(v_i)$.

□

Theorem 3.2. *Let G be a simple and connected graph with n vertices and m edges. Then $dd(G) \geq 2m$ with equality holds if G is regular.*

Proof. By Theorem 3.1, we have $d_i \leq dd(v_i)$. Then

$$\sum_{i=1}^{n} d_i \leq \sum_{i=1}^{n} dd(v_i)$$

Hence

$$dd(G) \geq 2m$$

If G is a regular graph, then $d_i = d_j$ where d_i and d_j are degrees of v_i and v_j respectively. Then $\left[\frac{d_i}{d_j}\right] + \left[\frac{d_j}{d_i}\right] = 1$ and so

$$\sum_{i \sim j} \left(\left[\frac{d_i}{d_j}\right] + \left[\frac{d_j}{d_i}\right] \right) = n - 1$$

Therefore,

$$\sum_{i=1}^{n} \left(\sum_{i \sim j} \left(\left[\frac{d_i}{d_j}\right] + \left[\frac{d_j}{d_i}\right] \right) \right) = n(n-1) = 2m.$$

□

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