Int. J. Math. And Appl., 6(3)(2018), 37–44 (Special Issue)

ISSN: 2347-1557

Available Online: http://ijmaa.in/

International Journal of *Mathematics And its Applications*

Some Results in Fuzzy Soft β-Continuity

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Abstract: β-continuous functions, fuzzy β-continuous functions and soft β-continuous functions have been already investigated by topologists. In this paper the concept of a fuzzy soft β-continuous function is introduced and its relationship with the existing concept in the literature of fuzzy soft topology is discussed.

Keywords: Fuzzy soft sets, Fuzzy soft topology, Fuzzy soft mapping, Fuzzy soft β-continuous.

1. Introduction

In the year 1998, Thakur S.S and Singh S [12] discussed the concept of fuzzy sets semi-pre open sets and fuzzy semi-pre continuity between fuzzy topological spaces. In 2015, Metin AKDAG and Alkan Ozkan [8] have been introduced the concept of on soft preopen sets and soft pre separation axioms between soft topological spaces. In this paper we introduce the notion of fuzzy soft β-continuity and some results along with examples have been discussed. Throughout this paper X and Y denote the initial sets. E and K denote the parameter spaces.

2. Preliminaries

Definition 2.1. *A pair (F, E) is called a soft set [9] over X where F is a mapping given by $F : E \to 2^X$ and 2^X is the power set of X.*

Definition 2.2. *A fuzzy set [15] of on X is a mapping $f : X \to I^X$ where $I = [0, 1]$.*

Definition 2.3. *A pair $\widetilde{\lambda} = (\lambda, E)$ is called a fuzzy soft set [13] over (X, E) where $\lambda : E \to I^X$ is a mapping, I^X is the collection of all fuzzy subsets of X. $FS(X, E)$ denotes the collection of all fuzzy soft sets over (X, E). We denote $\widetilde{\lambda}$ by $\widetilde{\lambda} = \{(e, \lambda(e)) : e \in E\}$ where $\lambda(e)$ is a fuzzy subset of X for each e in E.*

Definition 2.4 ([13]). *For any two fuzzy soft sets $\widetilde{\lambda}$ and $\widetilde{\mu}$ over a common universe X and a common parameter space E, $\widetilde{\lambda}$ is a fuzzy soft subset of $\widetilde{\mu}$ if $\lambda(e) \le \mu(e)$ for all $e \in E$. If $\widetilde{\lambda}$ is a fuzzy soft subset of $\widetilde{\mu}$ then we write $\widetilde{\lambda} \subseteq \widetilde{\mu}$ and $\widetilde{\mu}$ contains $\widetilde{\lambda}$. Two fuzzy soft sets $\widetilde{\lambda}$ and $\widetilde{\mu}$ over (X, E) are soft equal if $\widetilde{\lambda} \subseteq \widetilde{\mu}$ and $\widetilde{\mu} \subseteq \widetilde{\lambda}$. That is $\widetilde{\lambda} = \widetilde{\mu}$ if and only if $\lambda(e) = \mu(e)$ for all $e \in E$. We use the following notations: $\overline{0}(x) = 0$, for all x in X and $\overline{1}(x) = 1$, for all x in X.*

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Definition 2.5 ([13]). *A fuzzy soft set $\widetilde{\varphi}_X$ over (X, E) is said to be a null fuzzy soft set if for all $e \in E$, $\varphi_X(e) = \overline{0}$ and $\widetilde{\varphi}_X = (\varphi_X, E)$.*

Definition 2.6 ([13]). *A fuzzy soft set $\tilde{1}_X$ over (X, E) is said to be absolute fuzzy soft set if for all $e \in E$, $1_X(e) = \overline{1}$ and $\tilde{1}_X = (1_X, E)$.*

Definition 2.7 ([14]). *The union of two fuzzy soft sets $\widetilde{\lambda}$ and $\widetilde{\mu}$ over (X, E) is defined as $\widetilde{\lambda}\widetilde{\cup}\widetilde{\mu} = (\lambda\widetilde{\cup}\mu, E)$ where $(\lambda\widetilde{\cup}\mu)(e) = \lambda(e) \cup \mu(e) =$ the union of fuzzy sets $\lambda(e)$and$\mu(e)$ for all $e \in E$.*

Definition 2.8 ([14]). *The intersection of two fuzzy soft sets $\widetilde{\lambda}$ and $\widetilde{\mu}$ over (X, E) is defined as $\widetilde{\lambda}\widetilde{\cap}\widetilde{\mu} = (\lambda\widetilde{\cap}\mu, E)$ where $(\lambda\widetilde{\cap}\mu)(e) = \lambda(e) \cap \mu(e) =$ the intersection of fuzzy sets $\lambda(e)$ and $\mu(e)$ for all $e \in E$.*
The arbitrary union and arbitrary intersection of fuzzy soft sets over (X,E) are defined as $\widetilde{\cup}\left\{\widetilde{\lambda}_\alpha : \alpha \in \Delta\right\} = (\widetilde{\cup}\{\lambda_\alpha : \alpha \in \Delta\}, E)$ and $\widetilde{\cap}\left\{\widetilde{\lambda}_\alpha : \alpha \in \Delta\right\} = (\widetilde{\cap}\{\lambda_\alpha : \alpha \in \Delta\}, E)$ where $\left(\widetilde{\cup}\{\lambda_\alpha : \alpha \in \Delta\}\right)(e) = \cup\{\lambda_\alpha(e) : \alpha \in \Delta\} =$ the union of fuzzy sets $\lambda_\alpha(e), \alpha \in \Delta$ and $\left(\widetilde{\cap}\{\lambda_\alpha : \alpha \in \Delta\}\right)(e) = \cap\{\lambda(e) : \alpha \in \Delta\} =$ the intersection of fuzzy sets $\lambda_\alpha(e), \alpha \in \Delta$, for all $e \in E$.

Definition 2.9 ([14]). *The complement of a fuzzy soft set (λ, E) over (X, E), denoted by $(\lambda, E)^C$ is defined as $(\lambda, E)^C = (\lambda^C, E)$ where $\lambda^C : E \to I^X$ is a mapping given by $\lambda^C(e) = 1 - \lambda(e)$ for every e in E.*

Definition 2.10 ([14]). *A fuzzy soft topology $\widetilde{\tau}$ on (X, E) is a family of fuzzy soft sets over (X, E) satisfying the following axioms.*

(1). $\widetilde{\varphi}_X, \tilde{1}_X$ belong to $\widetilde{\tau}$,

(2). Arbitrary union of fuzzy soft sets in $\widetilde{\tau}$, belongs to $\widetilde{\tau}$,

(3). The intersection of any two fuzzy soft sets in $\widetilde{\tau}$, belongs to $\widetilde{\tau}$.

Members of $\widetilde{\tau}$ are called fuzzy soft open sets in $(X, \widetilde{\tau}, E)$. A fuzzy soft set $\widetilde{\lambda}$ over (X, E) is fuzzy soft closed in $(X, \widetilde{\tau}, E)$ if $(\widetilde{\lambda})^C \in \widetilde{\tau}$. The fuzzy soft interior of $\widetilde{\lambda}$ in $(X, \widetilde{\tau}, E)$ is the union of all fuzzy soft open sets $\widetilde{\mu} \subseteq \widetilde{\lambda}$ denoted by $\widetilde{fs}\ int(\widetilde{\lambda}) = \widetilde{\cup}\{\widetilde{\mu} : \widetilde{\mu} \subseteq \widetilde{\lambda}, \widetilde{\mu} \in \widetilde{\tau}\}$. The fuzzy soft closure of $\widetilde{\lambda}$ in $(X, \widetilde{\tau}, E)$ is the intersection of all fuzzy soft closed sets $\widetilde{\eta}$, $\widetilde{\lambda} \subseteq \widetilde{\eta}$ denoted by $\widetilde{fs}\ cl(\widetilde{\lambda}) = \widetilde{\cap}\{\widetilde{\eta} : \widetilde{\lambda} \subseteq \widetilde{\eta}, (\widetilde{\eta})^C \in \widetilde{\tau}\}$.

Definition 2.11 ([1]). *Let $(X, \widetilde{\tau}, E)$ be a fuzzy soft topological space and let $\widetilde{\lambda}$ be a fuzzy soft set over (X, E). Then $\widetilde{\lambda}$ is fuzzy soft semi-open if $\widetilde{\lambda} \subseteq \widetilde{fs}\ cl(\widetilde{fs}\ int(\widetilde{\lambda}))$ and fuzzy soft semi closed if $\widetilde{fs}\ int(\widetilde{fs}\ cl(\widetilde{\lambda})) \subseteq \widetilde{\lambda}$.*

Definition 2.12 ([1]). *Let $(X, \widetilde{\tau}, E)$ be a fuzzy soft topological space and let $\widetilde{\lambda}$ be a fuzzy soft set over (X, E). Then $\widetilde{\lambda}$ is fuzzy soft pre-open if $\widetilde{\lambda} \subseteq \widetilde{fs}\ int(\widetilde{fs}\ cl(\widetilde{\lambda}))$ and fuzzy soft pre-closed if $\widetilde{fs}\ cl(\widetilde{fs}\ int(\widetilde{\lambda})) \subseteq \widetilde{\lambda}$.*

Definition 2.13 ([1]). *Let $(X, \widetilde{\tau}, E)$ be a fuzzy soft topological space and let $\widetilde{\lambda}$ be a fuzzy soft set over (X, E). Then $\widetilde{\lambda}$ is fuzzy soft α-open if $\widetilde{\lambda} \subseteq \widetilde{fs}\ Int(\widetilde{fs}\ Cl(\widetilde{fs}Int(\widetilde{\lambda})))$ and fuzzy soft α-closed if $\widetilde{\lambda} \supseteq \widetilde{fs}\ Cl(\widetilde{fs}Int(\widetilde{fs}\ Cl(\widetilde{\lambda})))$.*
The classes of all fuzzy soft α-open, fuzzy soft pre-open, fuzzy soft semi-open and, fuzzy soft semi-pre-open sets over (X, E) are denoted as $\widetilde{FS}\alpha(X)$, $\widetilde{FS}SO(X)$, $\widetilde{FS}PO(X)$ and $\widetilde{FS}SP(X)$ respectively.
The fuzzy soft pre-interior, fuzzy soft pre-closure, fuzzy soft semi-interior, fuzzy soft semi-closure and fuzzy soft α-interior,fuzzy soft α-closure, fuzzy soft semi-pre-interior, fuzzy soft semi-pre-closure of X are denoted by $\widetilde{fs}PCl(\widetilde{\lambda})$, $\widetilde{fs}PInt(\widetilde{\lambda})$, $\widetilde{fs}SInt(\widetilde{\lambda})$, $\widetilde{fs}SCl(\widetilde{\lambda})$, $\widetilde{fs}\alpha Cl(\widetilde{\lambda})$, $\widetilde{fs}\alpha Int(\widetilde{\lambda})$, $\widetilde{fs}SPInt\left(\widetilde{\lambda}\right)$, $\widetilde{fs}SPCl\left(\widetilde{\lambda}\right)$ respectively.

Definition 2.14 ([1]). *Let $(X, \widetilde{\tau}, E)$ be a fuzzy soft topological space and let $\widetilde{\lambda}$ be a fuzzy soft set over (X, E). Then its fuzzy soft pre-closure and fuzzy soft pre-interior are defined as:*

$$\widetilde{fs}PCl(\widetilde{\lambda}) = \cap\{\widetilde{\mu} | \widetilde{\mu} \supseteq \widetilde{\lambda}, \widetilde{\mu} \in \widetilde{FS}PC(X)\}.$$

$$\widetilde{fs}PInt(\widetilde{\lambda}) = \cup\{\widetilde{\eta} | \widetilde{\eta} \subseteq \widetilde{\lambda}, \widetilde{\eta} \in \widetilde{FSPO}(X)\}.$$

The definitions for $\widetilde{fs}$ SCl, $\widetilde{fs}$ SInt, $\widetilde{fs}\alpha$ cl and $\widetilde{fs}\alpha$ Int are similar.

The following extension principle is used to define the mapping between the classes of fuzzy soft sets.

Definition 2.15 ([14]). *Let X and Y be any two non-empty sets. Let $g : X \to Y$ be a mapping. Let λ be a fuzzy subset of X and $\widetilde{\mu}$ be a fuzzy subset of Y. Then $g(\lambda)$ is a fuzzy subset of Y and for y in Y*

$$g(\lambda)(y) = \begin{cases} \sup\{\lambda(f(x)) : x \in g^{-1}(y)\}, & g^{-1}(y) \neq \phi \\ 0, & otherwise \end{cases}$$

$g^{-1}(\mu)$ is a fuzzy subset of X, defined by $g^{-1}(\mu)(x) = \mu(f(x))$ for all $x \in X$.

Definition 2.16 ([13]). *Let $FS(X,E)$ and $FS(Y,K)$ be classes of fuzzy soft sets over (X,E) and (Y,K) respectively $\rho : X \to Y$ and $\psi : E \to K$ be any two mappings. Then a fuzzy soft mapping $g = (\rho, \psi) : FS(X,E) \to FS(Y,K)$ is defined as follows:*
For a fuzzy soft set $\widetilde{\lambda}$ in $FS(X,E), g(\widetilde{\lambda})$ is a fuzzy soft set in $FS(Y,K)$ obtained as follows:

$$g(\widetilde{\lambda})(k) = \begin{cases} \bigcup\limits_{e \in \psi^{-1}(k)} \rho(\lambda(e)), & \psi^{-1}(k) \neq \phi \\ \overline{0}, & otherwise \end{cases}$$

for every y in Y, where

$$\rho(\lambda(e))(y) = \begin{cases} \sup\left\{\lambda(e)(x) : x \in \rho^{-1}(y)\right\}, & \rho^{-1}(y) \neq \phi \\ 0, & otherwise \end{cases}$$

That is

$$g(\widetilde{\lambda})(k)(y) = \begin{cases} \sup\limits_{e \in \psi^{-1}(k)} \left\{ \sup\limits_{x \in \rho^{-1}(y)} \lambda(e)(x) \right\}, & \rho^{-1}(y) \neq \phi, \psi^{-1}(k) \neq \phi \\ 0, & otherwise \end{cases}$$

$g(\widetilde{\lambda})$ is the image of the fuzzy soft set $\widetilde{\lambda}$ under the fuzzy mapping $g = (\rho, \psi)$. For a fuzzy soft set $\widetilde{\mu}$ in $FS(Y,K)$, $g^{-1}(\widetilde{\mu})$ is a fuzzy soft set in $FS(X,E)$ obtained as follows:
$g^{-1}(\widetilde{\mu})(e)(x) = \rho^{-1}(\widetilde{\mu}(\psi(e)))(x)$ for every x in X and $g^{-1}(\widetilde{\mu})$ is the inverse image of the fuzzy soft set $\widetilde{\mu}$.

Lemma 2.17 ([10]). *Let $(X, \widetilde{\tau}, E)$ and $(Y, \widetilde{\sigma}, K)$ be fuzzy soft topological spaces. Let $\rho : X \to Y$ and $\psi : E \to K$ be the two mappings and $g = (\rho, \psi) : FS(X,E) \to FS(Y,K)$ be a fuzzy soft mapping. Let $\widetilde{\lambda}, \widetilde{\lambda}_1, (\widetilde{\lambda})_i \in FS(X,E)$ and $\widetilde{\mu}, \widetilde{\mu}_1, (\widetilde{\mu})_i \in FS(Y,K)$, where $i \in J$ is an index set.*

(1). If $\widetilde{\lambda}_1 \subseteq \widetilde{\lambda}_2$, then $g(\widetilde{\lambda}_1) \subseteq g(\widetilde{\lambda}_2)$.

(2). If $\widetilde{\mu}_1 \subseteq \widetilde{\mu}_2$, then $g^{-1}(\widetilde{\mu}_1) \subseteq g^{-1}(\widetilde{\mu}_2)$.

(3). $\widetilde{\lambda} \subseteq g^{-1}(g(\widetilde{\lambda}))$, the equality holds if g is injective.

(4). $g(g^{-1}(\widetilde{\mu})) \subseteq \widetilde{\mu}$, the equality holds if g is surjective.

(5). $g^{-1}((\widetilde{\mu})^C) = [g^{-1}(\widetilde{\mu})]^C$.

(6). $[g(\widetilde{\lambda})]^C \subseteq g((\widetilde{\lambda})^C)$.

(7). $g^{-1}\left(\tilde{1}_K\right) = \tilde{1}_E$, $g^{-1}\left(\tilde{0}_K\right) = \tilde{0}_E$.

(8). $g\left(\tilde{1}_E\right) = \tilde{1}_K$ *if* g *is surjective.*

(9). $g\left(\tilde{0}_E\right) = \tilde{0}_K$.

Lemma 2.18 ([10])**.** *Let* $(X, \widetilde{\tau}, E)$ *and* $(Y, \widetilde{\sigma}, K)$ *be the two fuzzy soft topological spaces. Let* $\rho : X \to Y$ *and* $\psi : E \to K$ *be the two mappings and* $g = (\rho, \psi) : FS(X, E) \to FS(Y, K)$ *be a fuzzy soft mapping. Let* $\widetilde{\lambda}, \widetilde{\lambda}_1, (\widetilde{\lambda})_i \in FS(X, E)$ *and* $\widetilde{\mu}, \widetilde{\mu}_1, (\widetilde{\mu})_i \in FS(Y, K)$*, where* J *is an index set.*

(1). $g(\cup_{i\in J}\widetilde{\lambda}_i) = \cup_{i\in J} g(\widetilde{\lambda}_i)$.

(2). $g(\cap_{i\in J}\widetilde{\lambda}_i) \subseteq \cap_{i\in J} g(\widetilde{\lambda}_i)$*, the equality holds if* g *is injective.*

(3). $g^{-1}(\cup_{i\in J}\widetilde{\mu}_i) = \cup_{i\in J} g^{-1}(\widetilde{\mu}_i)$.

(4). $g^{-1}(\cap_{i\in J}\widetilde{\mu}_i) = \cap_{i\in J} g^{-1}(\widetilde{\mu}_i)$.

Definition 2.19 ([14])**.** *Fix* $x \in X$, $0 < \alpha < 1$. *Then the fuzzy subset* x^α *of* X *is called fuzzy point if* $x^\alpha(y) = \begin{cases} \alpha, & \text{if } y = x \\ 0 & \text{if } y \neq x \end{cases}$

Definition 2.20 ([14])**.** *Fix* $x \in X$, $0 < \alpha < 1$, $e \in E$. *The fuzzy soft set* x_e^α *over* (X, E) *is called fuzzy soft point if*

$$x_e^\alpha(e_1) = \begin{cases} x^\alpha, & \text{for } e_1 = e \\ \overline{0}, & \text{otherwise} \end{cases}$$

$$x_e^\alpha(e_1)(y) = \begin{cases} \alpha, & \text{for } e_1 = e,\ y = x \\ 0, & \text{otherwise} \end{cases}$$

Definition 2.21 ([3])**.** *Let* $(X, \widetilde{\tau}, E)$ *and* $(Y, \widetilde{\sigma}, K)$ *be the fuzzy soft topological spaces. Let* $\rho : X \to Y$ *and* $\psi : E \to K$ *be the two mappings and* $g = (\rho, \psi) : FS(X, E) \to FS(Y, K)$ *be a fuzzy soft mapping. Then* $g = (\rho, \psi)$ *is said to be fuzzy soft continuous if the inverse image of every fuzzy soft open set in* $(Y, \widetilde{\sigma}, K)$ *is fuzzy soft open in* $(X, \widetilde{\tau}, E)$. *That is* $g^{-1}(\widetilde{\mu}) \in \widetilde{\tau}$, *for all* $\widetilde{\mu} \in \widetilde{\sigma}$.

3. Fuzzy Soft β-continuity

Definition 3.1. *Let* $(X, \widetilde{\tau}, E)$ *and* $(Y, \widetilde{\sigma}, K)$ *be two fuzzy soft topological spaces.A fuzzy soft mapping* $g : (X, \widetilde{\tau}, E) \to (Y, \widetilde{\sigma}, K)$ *is said to be fuzzy soft* β*-continuous if for each fuzzy soft open set* $\widetilde{\mu}$ *in* $(Y, \widetilde{\sigma}, K)$*, the inverse image* $g^{-1}(\widetilde{\mu})$ *is fuzzy soft semi-pre open set in* $(X, \widetilde{\tau}, E)$.

Definition 3.2. *Let* $(X, \widetilde{\tau}, E)$ *and* $(Y, \widetilde{\sigma}, K)$ *be two fuzzy soft topological spaces. A fuzzy soft mapping* $g : (X, \widetilde{\tau}, E) \to (Y, \widetilde{\sigma}, K)$ *is said to be fuzzy soft* β*-irresolute if for each fuzzy soft* β*-open set* $\widetilde{\mu}$ *in* $(Y, \widetilde{\sigma}, K)$*, the inverse image* $g^{-1}(\widetilde{\mu})$ *is fuzzy soft* β*-open set in* $(X, \widetilde{\tau}, E)$.

Definition 3.3. *Let* $(X, \widetilde{\tau}, E)$ *and* $(Y, \widetilde{\sigma}, K)$ *be two fuzzy soft topological spaces. A fuzzy soft mapping* $g : (X, \widetilde{\tau}, E) \to (Y, \widetilde{\sigma}, K)$ *is said to be fuzzy soft* β*-open if for each fuzzy soft open set* $\widetilde{\lambda}$ *in* $(X, \widetilde{\tau}, E)$*, the image* $g(\widetilde{\lambda})$ *is fuzzy soft* β*-open set in* $(Y, \widetilde{\sigma}, K)$.

Definition 3.4. *Let* $(X, \widetilde{\tau}, E)$ *and* $(Y, \widetilde{\sigma}, K)$ *be two fuzzy soft topological spaces. A fuzzy soft mapping* $g : (X, \widetilde{\tau}, E) \to (Y, \widetilde{\sigma}, K)$ *is said to be fuzzy soft* β*-closed if for each fuzzy soft closed set* $\widetilde{\lambda}$ *in* $(X, \widetilde{\tau}, E)$*, the image* $g(\widetilde{\lambda})$ *is fuzzy soft* β*-closed set in* $(Y, \widetilde{\sigma}, K)$.

Proposition 3.5. *For a fuzzy soft mapping $g = (\rho, \psi) : (X, \widetilde{\tau}, E) \to (Y, \widetilde{\sigma}, K)$, the following are equivalent*

(i). g is fuzzy soft β-continuous.

(ii). The inverse image of every fuzzy soft closed set in $(Y, \widetilde{\sigma}, K)$ is fuzzy soft β-closed in $(X, \widetilde{\tau}, E)$.

Proof. Suppose (i) holds. Let $\widetilde{\mu}$ be a fuzzy soft closed in $(Y, \widetilde{\sigma}, K)$. Then $(\widetilde{\mu})^C$ is fuzzy soft open in $(Y, \widetilde{\sigma}, K)$. Using Definition 3.1, $g^{-1}((\widetilde{\mu})^C)$ is fuzzy soft β-open. Since $g^{-1}((\widetilde{\mu})^C) = [g^{-1}(\widetilde{\mu})]^C$, $g^{-1}(\widetilde{\mu})$ is fuzzy soft β-closed. This proves $(i) \Longrightarrow (ii)$.

Conversely we assume that (ii) holds. Let $\widetilde{\mu}$ be fuzzy soft open in $(Y, \widetilde{\sigma}, K)$. Therefore $(\widetilde{\mu})^C$ is fuzzy soft closed set in $(Y, \widetilde{\sigma}, K)$. Then by applying (ii), $[g^{-1}(\widetilde{\mu})]^C$ is fuzzy soft β-closed in $(X, \widetilde{\tau}, E)$. That implies $g^{-1}(\widetilde{\mu})$ is fuzzy soft β-open in $(X, \widetilde{\tau}, E)$. This proves $(ii) \Longrightarrow (i)$. □

Proposition 3.6. *For a fuzzy soft mapping $g = (\rho, \psi) : (X, \widetilde{\tau}, E) \to (Y, \widetilde{\sigma}, K)$. If g is fuzzy soft β-irresolute then it is fuzzy soft β-continuous.*

Proof. Suppose g is fuzzy soft β-irresolute. Let $\widetilde{\mu}$ be a fuzzy soft open set in $(Y, \widetilde{\sigma}, K)$. Since every fuzzy soft open set is fuzzy soft β-open and since g is fuzzy soft irresolute, by using Definition 3.2, $g^{-1}(\widetilde{\mu})$ is fuzzy soft β-open. That implies g is fuzzy soft β-continuous. □

Proposition 3.7. *A fuzzy soft mapping $g = (\rho, \psi) : (X, \widetilde{\tau}, E) \to (Y, \widetilde{\sigma}, K)$ is fuzzy soft β-continuous if and only if $g^{-1}\left(\widetilde{fs}\ int\widetilde{\mu}\right) \widetilde{\subseteq} \widetilde{fs}\ SPint(g^{-1}(\widetilde{\mu}))$ for every fuzzy soft set $\widetilde{\mu}$ in $(Y, \widetilde{\sigma}, K)$.*

Proof. Let $g : (X, \widetilde{\tau}, E) \to (Y, \widetilde{\sigma}, K)$ be fuzzy soft β-continuous. Let $\widetilde{\mu}$ be a fuzzy soft set in $(Y, \widetilde{\sigma}, K)$. Then $\widetilde{fs}\ int(\widetilde{\mu})$ is fuzzy soft open in Y. Since g is fuzzy soft β-continuous, by using Definition 3.1, $g^{-1}(\widetilde{fs}\ int\,(\widetilde{\mu}))$ is fuzzy soft β-open in $(X, \widetilde{\tau}, E)$. Then by using Lemma 2.18, $g^{-1}\left(\widetilde{fs}\ int\,(\widetilde{\mu})\right) \subseteq\ g^{-1}(\widetilde{\mu})$. This implies that $\widetilde{fs}\ SPintg^{-1}\left(\widetilde{fs}\ int\,(\widetilde{\mu})\right) \subseteq \widetilde{fs}\ SPint(g^{-1}(\widetilde{\mu}))$. Therefore $g^{-1}\left(\widetilde{fs}\ int\widetilde{\mu}\right) \widetilde{\subseteq} \widetilde{fs}\ SPint(g^{-1}(\widetilde{\mu}))$.

Conversely we assume that, $g^{-1}\left(\widetilde{fs}\ int\widetilde{\mu}\right) \widetilde{\subseteq} \widetilde{fs}\ SPint(g^{-1}(\widetilde{\mu}))$ for every fuzzy soft set $\widetilde{\mu}$ in $(Y, \widetilde{\sigma}, K)$. In particular the above statement is true for fuzzy soft open sets in $\widetilde{\mu}$. If $\widetilde{\mu}$ is fuzzy soft open sets in Y, $g^{-1}(\widetilde{\mu}) \widetilde{\subseteq} \widetilde{fs}\ SPint(g^{-1}(\widetilde{\mu})) \widetilde{\subseteq} g^{-1}(\widetilde{\mu})$. That implies $g^{-1}(\widetilde{\mu}) = \widetilde{fs}\ SPint(g^{-1}(\widetilde{\mu}))$ is fuzzy soft β-open. Therefore g is fuzzy soft β-continuous. □

Proposition 3.8. *A fuzzy soft mapping $g = (\rho, \psi) : (X, \widetilde{\tau}, E) \to (Y, \widetilde{\sigma}, K)$ is fuzzy soft β-continuous if and only if $g\left(\widetilde{fs}\ SPcl\widetilde{\lambda}\right) \subseteq \widetilde{fs}\ cl(g\left(\widetilde{\lambda}\right))$ for every fuzzy soft set $\widetilde{\lambda}$ in $(X, \widetilde{\tau}, E)$.*

Proof. Let $g : (X, \widetilde{\tau}, E) \to (Y, \widetilde{\sigma}, K)$ be fuzzy soft β-continuous. Let $\widetilde{\lambda}$ be fuzzy soft set in $(X, \widetilde{\tau}, E)$. Then $g(\widetilde{\lambda})$ is fuzzy soft set in $(Y, \widetilde{\sigma}, K)$. Since g is fuzzy soft β-continuous, by using Definition 3.1, $g^{-1}(\widetilde{fs}\ clg\left(\widetilde{\lambda}\right))$ is fuzzy soft β-closed in $(X, \widetilde{\tau}, E)$. Since $g\left(\widetilde{\lambda}\right) \subseteq (\widetilde{fs}\ clg\left(\widetilde{\lambda}\right))$, $g^{-1}\left(g\left(\widetilde{\lambda}\right)\right) \subseteq g^{-1}\left(\widetilde{fs}\ clg\left(\widetilde{\lambda}\right)\right)$, $\widetilde{\lambda} \subseteq g^{-1}\left(g\left(\widetilde{\lambda}\right)\right) \subseteq g^{-1}\left(\widetilde{fs}\ clg\left(\widetilde{\lambda}\right)\right)$. This implies that $(\widetilde{fs}\ SPcl\widetilde{\lambda}) \subseteq g^{-1}(\widetilde{fs}\ clg\left(\widetilde{\lambda}\right))$. Therefore $g(\widetilde{fs}\ SPcl\widetilde{\lambda}) \subseteq g(g^{-1}(\widetilde{fs}\ clg\left(\widetilde{\lambda}\right))) \subseteq \widetilde{fs}\ clg(\widetilde{\lambda})$.

Conversely we assume that, $g\left(\widetilde{fs}\ SPcl\widetilde{\lambda}\right) \subseteq \widetilde{fs}\ cl(g\left(\widetilde{\lambda}\right))$ for every fuzzy soft set $\widetilde{\lambda}$ in $(X, \widetilde{\tau}, E)$. Let $\widetilde{\mu}$ be a fuzzy soft closed in $(Y, \widetilde{\sigma}, K)$. Let $\widetilde{\lambda} = g^{-1}(\widetilde{\mu})$. Since by our assumption,

$$g(\widetilde{fs}\ SPcl\widetilde{\lambda}) \subseteq \widetilde{fs}\ cl(g\left(\widetilde{\lambda}\right)), g(\widetilde{fs}\ SPclg^{-1}(\widetilde{\mu})) \subseteq \widetilde{fs}\ clg\left(g^{-1}(\widetilde{\mu})\right) \subseteq \widetilde{fs}cl\widetilde{\mu}.$$

$$g\left(\widetilde{fs}\ SPclg^{-1}(\widetilde{\mu})\right) \subseteq \widetilde{fs}\ cl\widetilde{\mu} = \widetilde{\mu}.$$

$$g^{-1}(g\left(\widetilde{fs}\ SPclg^{-1}(\widetilde{\mu})\right) \subseteq g^{-1}(\widetilde{\mu}).$$

$$\widetilde{fs}\ SPclg^{-1}(\widetilde{\mu}) \subseteq g^{-1}(\widetilde{\mu}).$$

This implies that $g^{-1}(\widetilde{\mu}) = \widetilde{fs}\ SPclg^{-1}(\widetilde{\mu})$. Therefore $g^{-1}(\widetilde{\mu})$ is fuzzy soft β-closed. Hence g is fuzzy soft β-continuous. □

Proposition 3.9. *For a fuzzy soft mapping $g = (\rho, \psi) : (X, \widetilde{\tau}, E) \to (Y, \widetilde{\sigma}, K)$. The following are equivalent.*

(i). g is fuzzy soft β-continuous.

(ii). $g(\widetilde{fs}\ SPcl\left(\widetilde{\lambda}\right))\widetilde{\subseteq}\widetilde{fs}\ Pclg(\widetilde{\lambda})$, *for every fuzzy soft semi open set* $\widetilde{\lambda}$.

(iii). $g(\widetilde{fs}\ SPcl\left(\widetilde{\lambda}\right))\widetilde{\subseteq}\widetilde{fs}\ \alpha clg(\widetilde{\lambda})$, *for every fuzzy soft β-open set* $\widetilde{\lambda}$.

Proof. Assume (i) holds. By Proposition 3.8, $g\left(\widetilde{fs}\ SPcl\widetilde{\lambda}\right) \subseteq \widetilde{fs}\ cl(g\left(\widetilde{\lambda}\right))$ for every fuzzy soft set $\widetilde{\lambda}$ in $(X, \widetilde{\tau}, E)$. Since $\widetilde{fs}\ cl(g\left(\widetilde{\lambda}\right)) = \widetilde{fs}\ Pclg(\widetilde{\lambda})$, for every fuzzy soft semi open set $\widetilde{\lambda}$. This proves $(i) \Longrightarrow (ii)$.
Assume (ii) holds, $g(\widetilde{fs\ SPcl}\left(\widetilde{\lambda}\right))\widetilde{\subseteq}\widetilde{fs}\ Pclg(\widetilde{\lambda})$, for every fuzzy soft semi open set $\widetilde{\lambda}$. Let $\widetilde{\mu}$ be fuzzy soft closed set in $(Y, \widetilde{\sigma}, K)$ and let $\widetilde{\lambda} = g^{-1}(\widetilde{\mu})$.

$$g(\widetilde{fs}\ SPcl\left(g^{-1}(\widetilde{\mu})\right))\widetilde{\subseteq}\widetilde{fs}\ Pclg(g^{-1}(\widetilde{\mu}))\widetilde{\subseteq}\widetilde{fs}\ Pcl(\widetilde{\mu}),$$
$$g\left(\widetilde{fs}\ SPcl\left(g^{-1}(\widetilde{\mu})\right)\right)\widetilde{\subseteq}\widetilde{fs}\ Pcl(\widetilde{\mu}) = \widetilde{\mu},$$
$$g^{-1}(g\left(\widetilde{fs}\ SPcl\left(g^{-1}(\widetilde{\mu})\right)\right)\widetilde{\subseteq}g^{-1}(\widetilde{\mu}),$$
$$g^{-1}\left(\widetilde{\mu}\right) = \widetilde{fs}\ SPcl\left(g^{-1}(\widetilde{\mu}\right).$$

That implies $g^{-1}(\widetilde{\mu})$ is fuzzy soft β-closed. Therefore g is fuzzy soft β-continuous. This proves $(ii) \Longrightarrow (i)$.
Assume (i) holds. By Proposition 3.8, $g(\widetilde{fs}\ SPcl\widetilde{\lambda}) \subseteq \widetilde{fs}\ cl(g\left(\widetilde{\lambda}\right))$ for every fuzzy soft set $\widetilde{\lambda}$ in $(X, \widetilde{\tau}, E)$. Since $\widetilde{fs}\ cl(g\left(\widetilde{\lambda}\right)) = \widetilde{fs}\alpha clg(\widetilde{\lambda})$, for every fuzzy soft β-open set $\widetilde{\lambda}$. This proves $(i) \Longrightarrow (iii)$.
Assume (iii) holds, $g(\widetilde{fs}\ SPcl\left(\widetilde{\lambda}\right))\widetilde{\subseteq}\widetilde{fs}\ \alpha clg(\widetilde{\lambda})$, for every fuzzy soft β-open set $\widetilde{\lambda}$. Let $\widetilde{\mu}$ be fuzzy soft closed set in $(Y, \widetilde{\sigma}, K)$ and let $\widetilde{\lambda} = g^{-1}(\widetilde{\mu})$.

$$g(\widetilde{fs}\ SPcl\left(g^{-1}(\widetilde{\mu})\right))\widetilde{\subseteq}\widetilde{fs}\ \alpha clg(g^{-1}(\widetilde{\mu}))\widetilde{\subseteq}\widetilde{fs}\ \alpha cl(\widetilde{\mu}),$$
$$g\left(\widetilde{fs}\ SPcl\left(g^{-1}(\widetilde{\mu})\right)\right)\widetilde{\subseteq}\widetilde{fs}\ \alpha cl(\widetilde{\mu}) = \widetilde{\mu},$$
$$g^{-1}(g\left(\widetilde{fs}\ SPcl\left(g^{-1}(\widetilde{\mu})\right)\right)\widetilde{\subseteq}g^{-1}(\widetilde{\mu}),$$
$$g^{-1}\left(\widetilde{\mu}\right) = \widetilde{fs}\ SPcl\left(g^{-1}(\widetilde{\mu}\right).$$

That implies $g^{-1}(\widetilde{\mu})$ is fuzzy soft β-closed. Therefore g is fuzzy soft pre continuous. This proves $(iii) \Longrightarrow (i)$. □

Proposition 3.10. *A fuzzy soft mapping $g = (\rho, \psi) : (X, \widetilde{\tau}, E) \to (Y, \widetilde{\sigma}, K)$ is fuzzy soft β-open if and only if* $g\left(\widetilde{fs}\ int\widetilde{\lambda}\right) \subseteq \widetilde{fs}\ SPintg(\widetilde{\lambda})$ *for every fuzzy soft set* $\widetilde{\lambda}$ *in* $(X, \widetilde{\tau}, E)$.

Proof. Let $g = (\rho, \psi) : (X, \widetilde{\tau}, E) \to (Y, \widetilde{\sigma}, K)$ be fuzzy soft β-open. Let $\widetilde{\lambda}$ be fuzzy soft open set in $(X, \widetilde{\tau}, E)$. Then $\widetilde{fs}\ int\left(\widetilde{\lambda}\right)$ is fuzzy soft set in $(X, \widetilde{\tau}, E)$. Since g is fuzzy soft β-open, by Definition 3.4, $g(\widetilde{fs}\ int\left(\widetilde{\lambda}\right))$ is fuzzy soft β-open in $(Y, \widetilde{\sigma}, K)$. Then by using Lemma 2.18 $(\widetilde{fs}\ int\left(\widetilde{\lambda}\right)) \subseteq g(\widetilde{\lambda})$, $\widetilde{fs}\ SPintg(\widetilde{fs}Int\left(\widetilde{\lambda}\right)) \subseteq \widetilde{fs}\ SPintg(\widetilde{\lambda})$. Therefore $g\left(\widetilde{fs}\ int\widetilde{\lambda}\right) \subseteq \widetilde{fs}\ SPintg(\widetilde{\lambda})$.
Conversely we assume that $g\left(\widetilde{fs}\ int\widetilde{\lambda}\right) \subseteq \widetilde{fs}\ SPintg(\widetilde{\lambda})$ for every fuzzy soft set $\widetilde{\lambda}$ in $(X, \widetilde{\tau}, E)$. In particular the above statement is true for fuzzy soft open sets in $\widetilde{\lambda}$. If $\widetilde{\lambda}$ is fuzzy soft open in $\widetilde{\lambda}$, $g\left(\widetilde{\lambda}\right) \subseteq \widetilde{fs}\ SPintg\left(\widetilde{\lambda}\right) \subseteq g\left(\widetilde{\lambda}\right)$. That implies $g\left(\widetilde{\lambda}\right) = \widetilde{fs}\ SPintg\left(\widetilde{\lambda}\right)$ is fuzzy soft β-open. Therefore g is fuzzy soft β-continuous. □

Proposition 3.11. *A fuzzy soft mapping $g = (\rho, \psi) : (X, \widetilde{\tau}, E) \to (Y, \widetilde{\sigma}, K)$ is fuzzy soft β-closed if and only if* $\widetilde{fs}\ SPclg(\widetilde{\lambda}) \subseteq g(\widetilde{fs}\ cl\widetilde{\lambda})$ *for every fuzzy soft set* $\widetilde{\lambda}$ *in* $(X, \widetilde{\tau}, E)$.

Proof. Let $g = (\rho, \psi) : (X, \widetilde{\tau}, E) \to (Y, \widetilde{\sigma}, K)$ be fuzzy soft semi closed. Let $\widetilde{\lambda}$ be fuzzy soft set in $(X, \widetilde{\tau}, E)$. Then $\widetilde{fs}\ cl(\widetilde{\lambda})$ is fuzzy soft closed set in $(X, \widetilde{\tau}, E)$. Since g is fuzzy soft β-closed, by Definition 3.5, $g(\widetilde{fs}\ cl\left(\widetilde{\lambda}\right))$ is fuzzy soft β-closed in $(Y, \widetilde{\sigma}, K)$. Since $g(\widetilde{\lambda}) \subseteq g(\widetilde{fs}\ cl\left(\widetilde{\lambda}\right))$, $\widetilde{fs}\ SPclg\left(\widetilde{\lambda}\right) \subseteq \widetilde{fs}\ SPcl\ g\left(\widetilde{fs}\ cl\left(\widetilde{\lambda}\right)\right) = g\left(\widetilde{fs}\ cl\left(\widetilde{\lambda}\right)\right)$. Therefore $\widetilde{fs}\ SPclg\left(\widetilde{\lambda}\right) \subseteq g\left(\widetilde{fs}\ cl\left(\widetilde{\lambda}\right)\right)$.

Conversely we assume that, $\widetilde{fs}\ SPclg(\widetilde{\lambda}) \subseteq g(\widetilde{fs}\ cl\widetilde{\lambda})$ for every fuzzy soft set $\widetilde{\lambda}$ in $(X, \widetilde{\tau}, E)$. Let $\widetilde{\lambda}$ be fuzzy soft closed in $(X, \widetilde{\tau}, E)$. By our assumption, $\widetilde{fs}\ SPclg\left(\widetilde{\lambda}\right) \subseteq g\left(\widetilde{fs}\ cl\widetilde{\lambda}\right) = g\left(\widetilde{\lambda}\right) \subseteq \widetilde{fs}\ SPclg\left(\widetilde{\lambda}\right)$. Therefore $g\left(\widetilde{\lambda}\right) = \widetilde{fs}\ SPclg\left(\widetilde{\lambda}\right)$. Therefore $g\left(\widetilde{\lambda}\right)$ is fuzzy soft β-closed. □

Theorem 3.12. *Let $g = (\rho, \psi) : (X, \widetilde{\tau}, E) \to (Y, \widetilde{\sigma}, K)$ be fuzzy soft mapping. Then the following are equivalent.*

(1). g is fuzzy soft β-continuous.

(2). The inverse image of every fuzzy soft closed set in $(Y, \widetilde{\sigma}, K)$ is fuzzy soft β-closed in $(X, \widetilde{\tau}, E)$.

(3). $g^{-1}\left(\widetilde{fs}\ int\widetilde{\mu}\right) \widetilde{\subseteq} \widetilde{fs}\ SPint(g^{-1}(\widetilde{\mu}))$ for every fuzzy soft set $\widetilde{\mu}$ in $(Y, \widetilde{\sigma}, K)$.

(4). $g\left(\widetilde{fs}\ SPcl\widetilde{\lambda}\right) \subseteq \widetilde{fs}\ cl(g\left(\widetilde{\lambda}\right))$ for every fuzzy soft set $\widetilde{\lambda}$ in $(X, \widetilde{\tau}, E)$.

(5). $g(\widetilde{fs}\ SPcl\left(\widetilde{\lambda}\right)) \widetilde{\subseteq} \widetilde{fs}\ Pclg(\widetilde{\lambda})$, for every fuzzy soft semi open set $\widetilde{\lambda}$.

(6). $g(\widetilde{fs}\ SPcl\left(\widetilde{\lambda}\right)) \widetilde{\subseteq} \widetilde{fs}\ \alpha clg(\widetilde{\lambda})$, for every fuzzy soft β-open set $\widetilde{\lambda}$.

Proof. Follows from Proposition 3.5, Proposition 3.7, Proposition 3.8, Proposition 3.9. □

Remark 3.13. *The above discussions give the following implication diagram. Fuzzy soft continuous mapping Fuzzy soft β-continuous mapping.*

4. Conclusion

Fuzzy soft β-continuous mappings have been characterized using recent concepts in the literature of fuzzy soft topology.

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