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Discrete Heat Flow of Rod Controlled by Fibonacci Partial Delay Difference Operator

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Abstract: We introduce a new type of Fibonacci discrete heat equation model for rod using Fibonacci difference operator with delay factor σ. The dispersion of heat is analyzed by Fourier's cooling law and the findings are validated by MATLAB.

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1. Introduction

Difference operator $\underset{\alpha}{\Delta}$ was introduced in 1984 by Jerzy Popenda [1, 4]. It was further extended by M. M. S Manuel [5] and G. B. A. Xavier [3]. In this research work, we extend the above mentioned theory of α-difference operator Δ_α[2] to Fibonacci difference operator $\underset{x(\ell)}{\Delta}$, where $x = (x_1, x_2, ..., x_n)$ and $\ell = (\ell_1, \ell_2, ..., \ell_n)$ and obtain the heat equation model.

2. Fibonacci Difference Operator on r-variables with Shift Values

For $x = (x_1, x_2, x_3, ..., x_r)$, the Fibonacci difference operator on r-variables on the real valued function with shift values $k = (k_1, k_2)$ and $\ell = (\ell_1, \ell_2)$ is defined as,

$$\underset{x(\ell)}{\Delta} v(k) = v(k) - x_1 v(k - \ell_{1,2}) - x_2 v(k - 2\ell_{1,2}) - x_3 v(k - 3\ell_{1,2}) - .... - x_r v(k - r\ell_{1,2}) \tag{1}$$

The operator in (1) becomes partial Fibonacci difference operator if either ℓ_1 or ℓ_2 is zero but not both. Consider a first order partial Fibonacci difference equation,

$$\underset{x(\ell)}{\Delta} v(k) = u(k), \ell = (0, \ell_2) \quad \text{or} \quad (\ell_1, 0); \quad x = (x_1, x_2, ..., x_r). \tag{2}$$

Then the numerical solution of (2) is given as

$$v(k_1, k_2) - \sum_{i=0}^{m} \sum_{j=i}^{m} x_j F_{n+i-j} v(k_1, k_2 - (n+i)\ell_2) = \sum_{i=0}^{n} F_i u(k_1, k_2 - i\ell_2) \tag{3}$$

where $F_0 = 1$, $F_1 = x_1 F_0$, $F_2 = x_2 F_0 + x_1 F_1$, $F_r = x_r F_0 + x_{r-1} F_1 + ... + x_1 F_{r-1}$ are Fibonacci numbers and $F_n = 0$ when $n < 0$.

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3. Discrete Delay Heat Equation of a Long Rod

Consider a long rod with $v(k_1, k_2)$ as temperature, in which k_1 and k_2 denote position and time respectively[3]. By Fourier's cooling law and using (1), the discrete delay heat equation is obtained as

$$\underset{x(0,\ell_2)}{\Delta} v(k_1, k_2) = \gamma \underset{x(\pm\ell_1,0)}{\Delta} v(k_1, k_2 - \sigma); \quad x = (x_1, x_2, ..., x_r) \tag{4}$$

where σ is a delay factor and $\underset{x(\pm\ell_1,0)}{\Delta} = \underset{x(\ell_1,0)}{\Delta} + \underset{x(-\ell_1,0)}{\Delta}$. The objective of this paper is to study and discuss the solution of the heat equation (4) with Fibonacci operator of r^{th} order.

Theorem 3.1. *If* $\underset{x(\pm\ell_1)}{\Delta} v(k_1, k_2 - \sigma) = \underset{x(\pm\ell_1)}{u}(k_1, k_2 - \sigma)$ *is given then the delay heat equation has a solution*

$$v(k_1, k_2) = \sum_{i=0}^{m}\sum_{j=i}^{m} x_j F_{n+i-j} v(k_1, k_2 - (n+i)\ell_2) + \gamma \sum_{i=0}^{n} F_i \underset{x(\pm\ell_1)}{u}(k_1, k_2 - i\ell_2 - \sigma) \tag{5}$$

Proof. By representing $\underset{x(\pm\ell_1)}{\Delta} v(k_1, k_2 - \sigma) = \underset{x(\pm\ell_1)}{u}(k_1, k_2 - \sigma)$, (4) becomes

$$v(k_1, k_2) = \sum_{i=0}^{m}\sum_{j=i}^{m} x_j F_{n+i-j} v(k_1, k_2 - (n+i)\ell_2) + \gamma \underset{x(0,\ell_2)}{\overset{-1}{\Delta}} \underset{x(\pm\ell_1)}{u}(k_1, k_2 - \sigma) \tag{6}$$

The proof of (5) follows from the relation, $\underset{x(0,\ell_2)}{\overset{-1}{\Delta}} \underset{x(\pm\ell_1)}{u}(k_1, k_2) = \sum\limits_{i=1}^{n} F_i \underset{x(\pm\ell_1)}{u}(k_1 - r(0), k_2 - i\ell_2)$ and using (6). □

Theorem 3.2. *Consider* (4)*, and after denoting* $v(k_1 \pm \ell_1, *) = v(k_1 + \ell_1, *) + v(k_1 - \ell_1, *)$ *and* $v(k_1 \pm 2\ell_1, *) = v(k_1 + 2\ell_1, *) + v(k_1 - 2\ell_1, *)$ *then we obtain the four types solutions for* (4).

(a).
$$v(k_1, k_2) = {x_1}^n v(k_1, k_2 - n\ell_2) - \sum_{i=1}^{n} \gamma {x_1}^i v(k_1 \pm \ell_1, k_2 - \sigma - (i-1)\ell_2) + \sum_{i=1}^{n} \gamma {x_1}^{i-1} v(k_1, k_2 - \sigma - (i-1)\ell_2)$$
$$+ \sum_{r=2}^{p} \left\{ \sum_{i=1}^{n} x_r {x_1}^{i-1} \Big[v(k_1, k_2 - r\ell_2) - \gamma v(k_1 \pm r\ell_1, k_2 - \sigma - (i-1)\ell_2) \Big] \right\}, \tag{7}$$

(b).
$$v(k_1, k_2) = \frac{1}{{x_1}^n} v(k_1, k_2 + n\ell_2) - \sum_{i=1}^{n} \frac{\gamma}{x_1^i} \{k_1, k_2 - \sigma + i\ell_2\} + \sum_{i=1}^{n} \frac{\gamma}{x_1^{i-1}} \{k_1 \pm \ell_1, k_2 - (i-1)\sigma + i\ell_2\}$$
$$- \sum_{r=2}^{p} \left\{ \sum_{i=1}^{n} \frac{x_r}{{x_1}^i} \Big[v(k_1, k_2 + (i-r)\ell_2) - \gamma v(k_1 \pm r\ell_1, k_2 - \sigma + i\ell_2) \Big] \right\}, \tag{8}$$

(c).
$$v(k_1, k_2) = \frac{1}{\gamma^n} v(k_1 - n\ell_1, k_2 + n\sigma - n\ell_2) - \sum_{i=1}^{n} \frac{1}{\gamma^{i-1}} v(k_1 - (i+1)\ell_1, k_2 + (i-1)\sigma - (i-1)\ell_2)$$
$$- \sum_{i=1}^{n} \frac{1}{x_1\gamma^i} v(k_1 - i\ell_1, k_2 + i\sigma - (i-1)\ell_2) + \sum_{i=1}^{n} \frac{1}{x_1\gamma^{i-1}} v(k_1 - i\ell_1, k_2 + (i-1)\sigma - (i-1)\ell_2)$$
$$- \sum_{r=2}^{p} \left\{ \sum_{i=1}^{n} \frac{x_r}{x_1\gamma^i} \Big[v(k_1 - (i-r)\ell_1, k_2 + (i-1)\sigma - (i-1)\ell_2) + v(k_1 - (r+i)\ell_1, k_2 + (i-1)\sigma - (i-1)\ell_2) \Big] \right\}$$
$$+ \sum_{r=2}^{p} \left\{ \sum_{i=1}^{n} \frac{x_r}{x_1\gamma^i} v(k_1 - i\ell_1, k_2 + i\sigma - (i + (r-1))\ell_2) \right\}, \tag{9}$$

(d).
$$v(k_1, k_2) = \frac{1}{\gamma^n} v(k_1 + n\ell_1, k_2 - n\ell_2) - \sum_{i=1}^{n} \frac{1}{\gamma^{i-1}} v(k_1 + (i+1)\ell_1, k_2 + (i-1)\sigma - (i-1)\ell_2)$$
$$- \sum_{i=1}^{n} \frac{1}{x_1\gamma^i} v(k_1 + i\ell_1, k_2 + i\sigma - (i-1)\ell_2) + \sum_{i=1}^{n} \frac{1}{x_1\gamma^{i-1}} v(k_1 + i\ell_1, k_2 + (i-1)\sigma - (i-1)\ell_2)$$
$$- \sum_{r=2}^{p} \left\{ \sum_{i=1}^{n} \frac{x_r}{x_1\gamma^{i-1}} \Big[v(k_1 + (i+r)\ell_1, k_2 + (i-1)\sigma - (i-1)\ell_2) + v(k_1 + (r+i)\ell_1, k_2 + (i-1)\sigma - (i-1)\ell_2) \Big] \right\}$$
$$+ \sum_{r=2}^{p} \left\{ \sum_{i=1}^{n} \frac{x_r}{x_1\gamma^i} v(k_1 + i\ell_1, k_2 + i\sigma - (i + (r-1))\ell_2) \right\}. \tag{10}$$

Proof. (a). From (4),

$$v(k_1,k_2)=x_1v(k_1,k_2-\ell_2)+...+x_rv(k_1,k_2-r\ell_2)-x_1\gamma v(k_1\pm\ell_1,k_2)-...-x_r\gamma v(k_1\pm r\ell_1,k_2). \quad (11)$$

Replacing k_2 by $k_2-\ell_2,k_2-2\ell_2,...,k_2-n\ell_2$, we get the proof.

(b). $v(k_1,k_2)=\frac{1}{x_1}v(k_1,k_2+\ell_2)+\gamma v(k_1\pm\ell_1,k_2-\sigma+\ell_2)-\frac{\gamma}{x_1}v(k_1,k_2-\sigma+\ell_2)$

$$-\frac{x_2}{x_1}\Big[v(k_1,k_2-\ell_2)-\gamma v(k_1\pm 2\ell_1,k_2-\sigma+\ell_2)\Big]$$
$$-...-\frac{x_{(r-1)}}{x_1}\Big[v(k_1,k_2-\ell_2)-\gamma v(k_1\pm(r-1)\ell_1,k_2+\ell_2)\Big]$$
$$-\frac{x_r}{x_1}\Big[v(k_1,k_2-\ell_2)-\gamma v(k_1\pm r\ell_1,k_2-\sigma+\ell_2)\Big]. \quad (12)$$

When changing k_2 by $k_2+\ell_2,k_2+2\ell_2,...,k_2+n\ell_2$ repeatedly we get the result.

(c). The expression (4) derives as

$$v(k_1,k_2)=\frac{1}{\gamma}v(k_1-\ell_1,k_2+\sigma-\ell_2)-v(k_1-2\ell_1,k_2)+\frac{1}{x_1}v(k_1-\ell_1,k_2)$$
$$+\frac{x_2}{x_1\gamma}v(k_1-\ell_1,k_2+\sigma-2\ell_2)+\frac{x_3}{x_1\gamma}v(k_1-\ell_1,k_2+\sigma-3\ell_2)$$
$$+...+\frac{x_r}{x_1\gamma}v(k_1-\ell_1,k_2+\sigma-r\ell_2)$$
$$-\frac{x_2}{x_1}\Big[v(k_1+\ell_1,k_2)+v(k_1-3\ell_1,k_2)\Big]-\frac{x_3}{x_1}\Big[v(k_1+2\ell_1,k_2)+v(k_1-4\ell_1,k_2)\Big]$$
$$-...-\frac{x_r}{x_1}\Big[v(k_1+(r-1)\ell_1,k_2)+v(k_1-(r+1)\ell_1,k_2)\Big]-\frac{1}{x_1\gamma}v(k_1-\ell_1,k_2+\sigma).$$

Replacing k_1 by $k_1-\ell_1,k_1-2\ell_1,...,k_1-n\ell_1$ and k_2 by $k_2-\ell_2,k_2-2\ell_2,...,k_2-n\ell_2$ repeatedly.

(d). $v(k_1,k_2)=\frac{1}{\gamma}v(k_1+\ell_1,k_2+\sigma-\ell_2)-v(k_1+2\ell_1,k_2)+\frac{1}{x_1}v(k_1+\ell_1,k_2)$

$$+\frac{x_2}{x_1\gamma}v(k_1+\ell_1,k_2+\sigma-2\ell_2)+\frac{x_3}{x_1\gamma}v(k_1+\ell_1,k_2+\sigma-3\ell_2)+...+\frac{x_r}{x_1\gamma}v(k_1+\ell_1,k_2-r\ell_2)$$
$$-\frac{x_2}{x_1}\Big[v(k_1+3\ell_1,k_2)+v(k_1-\ell_1,k_2)\Big]-\frac{x_3}{x_1}\Big[v(k_1+4\ell_1,k_2)+v(k_1-2\ell_1,k_2)\Big]$$
$$-...-\frac{x_r}{x_1}\Big[v(k_1+(r+1)\ell_1,k_2)+v(k_1-(r_1)\ell_1,k_2)\Big]-\frac{1}{x_1\gamma}v(k_1+\ell_1,k_2+\sigma).$$

While changing k_1 by $k_1+\ell_1,k_1+2\ell_1,...,k_1+n\ell_1$ and k_2 by $k_2-\ell_2,k_2-2\ell_2,...,k_2-n\ell_2$ repeatedly we get the proof. □

Example 3.3. *Suppose $v(k_1,k_2)=e^{k_1+k_2}$ is an exact solution of (4), then we have the relation $\underset{x(0,\ell_2)}{\Delta}e^{k_1+k_2}=\gamma[\underset{x(\ell_1)}{\Delta}e^{k_1+k_2-\sigma}+\underset{x(-\ell_1)}{\Delta}e^{k_1+k_2-\sigma}]$, which yields*

$$e^{k_1+k_2}-x_1e^{k_1+k_2-\ell_2}-...-x_re^{k_1+k_2-r\ell_2}=\gamma\big[e^{k_1+k_2-\sigma}-x_1e^{k_1\pm\ell_1+k_2-\sigma}-...-x_re^{k_1\pm r\ell_1+k_2-\sigma}\big].$$

Canceling $e^{k_1+k_2}$ on both sides we derive

$$\gamma=\frac{1-x_1e^{-\ell_2}-...-x_re^{-r\ell_2}}{e^{-\sigma}-x_1(e^{\ell_1-\sigma}+e^{-\ell_1-\sigma})-...-x_r(e^{r\ell_1-\sigma}+e^{-r\ell_1-\sigma})}. \quad (13)$$

When $m=1$, $p=2$, $k_1=1$, $\ell_1=1$, $k_2=2$, $\ell_2=2$, $x_1=1$, $x_2=2$, $v(k_1,k_2)=e^{(k_1+k_2)}$ and γ is as given in (13), the MATLAB coding for (a) of Theorem (3.2) is as follows:

$exp(1+2) = exp(1)+symsum((0.1313589496.*(exp(3-((i-1).*2))+exp(1-((i-1).*2)))),i,1,1)+symsum((-0.1313589496.*exp(2-((i-1).*2))),i,1,1)+symsum((2.*(exp(-1)+0.131359496.*(exp(4-((i-1).*2))+exp(0-((i-1).*2))))),i,1,1)$

Verification for (b), (c), (d) is same as above.

4. Conclusion

The newly developed Fibonacci partial difference operator generates many applications in the field of numerical methods and heat Flow. The nature of propagation of heat through materials are derived using Fibonacci partial difference operator. The core Theorem 3.2 provide's the possibility of predicting the temperature with some delay either for the past or the future after getting the temperature at few finite points.

References

[1] M.J.Ablowitz and J.F.Ladik, *On the solution of a class of nonlinear partial difference equations*, Studies Appl. Math., 57(1977), 1-12.

[2] G.Britto Antony Xavier, B.Govindan, S.U.Vasantha Kumar and S.John Borg, *$m(k)$-Series solution of generalized k-difference equation with variable coefficients*, Far East Journal of Mathematical Sciences, 99(5)(2016), 699-717.

[3] G.Britto Antony Xavier, S.John Borg and M.Meganathan, *Discrete heat equation model with shift values*, Applied Mathematics, 8(2017), 1343-1350.

[4] Jerzy Popenda and Blazej Szmanda, *On the Oscillation of Solutions of Certain Difference Equations*, Demonstratio Mathematica, XVII(1)(1984), 153-164.

[5] M.Maria Susai Manuel, V.Chandrasekar and G.Britto Antony Xavier, *Solutions and Applications of Certain Class of α-Difference Equations*, International Journal of Applied Mathematics, 24(6)(2011), 943-954.