Int. J. Math. And Appl., 6(1)(2018), 41–46. (Special Issue)
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of *Mathematics And its Applications*

Discrete Heat Flow Controlled by Alpha-Beta Delay Q-Difference Equations

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Abstract: Partial q-difference equation have extended applications in q-heat equations. Having introduced the Fibonacci difference equation in the partial section by using Fibonacci difference operator with shift values, a model for heat tranfer in the rod is found having recourse to Fourier law of cooling, an involved study is carried out to evaluate that movement of heat and thus numerous are postulated. The result obtained are validated by MATLAB.

MSC: 39A70, 39A10, 47B39, 80A20.

Keywords: Partial Difference Equation, Partial Difference Operator and Delay Heat Equation.

 Accepted on: 13th April 2018

1. Introduction

The theory of $\underset{\alpha}{\Delta}$, using the definition $\underset{\alpha}{\Delta} u(k) = u(k+1) - \alpha u(k)$ was introduced by Jerzy Popenda [1, 6] in 1984. Fractional difference operators are established by Miller and Rose in 1989 ([3, 4]). The extension of Δ_α was introduced by M.M.S.Manuel [7, 8] in 2011. The q-difference operator obtained by the authors [5] in 2011 and its extended with the variable coefficients is founded in [5]. Here, we extend the operator Δ_q to a partial q-difference operator. Consider the variable $k = (k_1, k_2, ..., k_n)$, n-tuple shift factors $q = (q_1, q_2, ..., q_n) \neq 0$, not all $q_i \neq 1$, $kq = (k_1q_1, k_2q_2, ..., k_nq_n)$ and a real valued function $v(k)$ defined on R^n. The beta q-difference operator on $v(k)$ is defined as

$$\underset{\beta(q)}{\Delta} v(k) = v(kq) - \beta v(k). \tag{1}$$

When $q = (1, q_2)$, the beta q-inverse principle with respect to $\underset{\beta(1,q_2)}{\Delta}$ is given by

$$v(k_1, k_2) - \beta^m v(k_1, \frac{k_2}{q_2^m}) = \sum_{r=1}^{m} \beta^{r-1} u(k_1, \frac{k_2}{q_2^{(r+1)}}), \tag{2}$$

where $v(k_1, k_2) = \underset{\beta(q_1,q_2)}{\Delta^{-1}} u(k_1, k_2)$ [2]. Using (2), we arrive several type solutions of heat equation for a long rod with delay factor.

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2. Delay q-heat Equation of a Long Rod when γ is Constant

Let us take a long rod and assuming $v(k_1,k_2)$ be the temperature at the position k_1 and its time k_2 of the rod. Let γ be the positive dissemination rate constant of rod. By Fourier's cooling law, the discrete alpha-beta delay q-heat equation of rod is

$$v(k_1,k_2q_2)-\beta v(k_1,k_2)=\gamma\Big[v\big(\frac{k_1}{q_1},\frac{k_2}{\sigma}\big)-\alpha v\big(k_1,\frac{k_2}{\sigma}\big)+v\big(k_1q_1,\frac{k_2}{\sigma}\big)-\alpha v\big(k_1,\frac{k_2}{\sigma}\big)\Big]. \tag{3}$$

Then the discrete heat equation (3) with delay can be expressed as

$$\underset{\beta(1,q_2{}^{\pm})}{\Delta} v(k_1,k_2)=\gamma \underset{\alpha(q_1{}^{\pm},1)}{\Delta} v\big(k_1,\frac{k_2}{\sigma}\big). \tag{4}$$

Our main aim is to analyze the possible solutions of the q-heat equation (4).

Theorem 2.1. *Let us take an integer $m>0$, and a number $q_2>0$ which is real such that $v(k_1,\frac{k_2}{q_2^m})$ and $\underset{\alpha(q_1^{\pm},1)}{\Delta} v(k_1,\frac{k_2}{\sigma})=\underset{\alpha(q_1^{\pm},1)}{u}(k_1,\frac{k_2}{\sigma})$ are given then (4) has a summation solution as*

$$v(k_1,k_2)=\alpha^m v\big(k_1,\frac{k_2}{q_2^m}\big)+\gamma\sum_{r=1}^{m}\alpha^r \underset{\alpha(q_1^{\pm},1)}{u}\big(k_1,\frac{k_2}{q_2^r\sigma}\big). \tag{5}$$

Proof. Taking $\underset{\alpha(q_1^{\pm},1)}{\Delta} v(k_1,\frac{k_2}{\sigma})=\underset{\alpha(q_1^{\pm},1)}{u}(k_1,\frac{k_2}{\sigma})$ in (4) gives

$$v(k_1,k_2)=\gamma \underset{\beta(1,q_2)}{\Delta^{-1}} \underset{\alpha(q_1^{\pm},1)}{u}\big(k_1,\frac{k_2}{\sigma}\big). \tag{6}$$

The proof follows by applying inverse principle in (6). □

Theorem 2.2. *When $\beta>0$, m is a positive integer and by denoting $v(k_1q_1^{\pm},*)=v(k_1q_1,*)+v(\frac{k_1}{q_1},*)$ and $v(*,k_2q_2{}^{\pm})=v(*,k_2q_2)+v(*,\frac{k_2}{q_1})$, we get the following results.*

$$(a).\quad v(k_1,k_2)=\frac{1}{\beta^m}v(k_1,k_2q_2{}^m)-\sum_{i=1}^{m}\frac{\gamma}{\beta^i}\Big[v\big(k_1q_1^{\pm},\frac{k_2q_2^{i-1}}{\sigma}\big)-2\alpha v\big(k_1,\frac{k_2q_2^{i-1}}{\sigma}\big)\Big], \tag{7}$$

$$(b).\quad v(k_1,k_2)=\beta^m v\big(k_1,\frac{k_2}{q_2^m}\big)+\sum_{i=1}^{m}\beta^{i-1}\gamma\Big[v\big(k_1q_1^{\pm},\frac{k_2}{q_2^i\sigma}\big)-2\alpha v\big(k_1,\frac{k_2}{q_2^i\sigma}\big)\Big], \tag{8}$$

$$\text{(c).}\quad v(k_1,k_2)=\frac{1}{\gamma^m}v\big(\frac{k_1}{q_1^m},k_2q_2^m\sigma^m\big)-\sum_{i=1}^{m}\frac{\beta}{\gamma^i}v\big(\frac{k_1}{q_1^i},k_2q_2^{i-1}\sigma^i\big)$$
$$-\sum_{i=1}^{m}\frac{1}{\gamma^{i-1}}v\big(\frac{k_1}{q_1^{i+1}},k_2q_2^{i-1}\sigma^{i-1}\big)-\sum_{i=1}^{m}\frac{2\alpha}{\gamma^{i-1}}v\big(\frac{k_1}{q_1^i},k_2q_2^{i-1}\sigma^{i-1}\big), \tag{9}$$

$$\text{(d).}\quad v(k_1,k_2)=\frac{1}{\gamma^m}v\big(k_1q_1^m,k_2q_2^m\sigma^m\big)-\sum_{i=1}^{m}\frac{\beta}{\gamma^i}v\big(k_1q_1^i,k_2q_2^{i-1}\sigma^i\big)$$
$$-\sum_{i=1}^{m}\frac{1}{\gamma^{i-1}}v\big(k_1q_1^{i+1},k_2q_2^{i-1}\sigma^{i-1}\big)-\sum_{i=1}^{m}\frac{2\alpha}{\gamma^{i-1}}v\big(k_1q_1^i,k_2q_2^{i-1}\sigma^{i-1}\big). \tag{10}$$

Proof. (a). We have

$$v(k_1,k_2)=\frac{1}{\beta}v(k_1,k_2q_2)-\frac{\gamma}{\beta}\Big[v\big(k_1q_1^{\pm},\frac{k_2}{\sigma}\big)-2\alpha v\big(k_1,\frac{k_2}{\sigma}\big)\Big] \tag{11}$$

Replacing k_2 by $k_2q_2, k_2q_2^2, ..., k_2q_2^m$ in (11).

(b). Directly derives the equation

$$v(k_1,k_2)=\beta v\big(k_1,\frac{k_2}{q_2}\big)-\gamma\Big[v\big(k_1q_1^{\pm},\frac{k_2}{q_2\sigma}\big)-2\alpha v\big(k_1,\frac{k_2}{q_2\sigma}\big)\Big]. \tag{12}$$

By changing k_2 by $\frac{k_2}{q_2},\frac{k_2}{q_2^2},....,\frac{k_2}{q_2^m}$ repeatedly, we get the result (8).

(c). (4) yeilds $v(k_1,k_2)=\frac{1}{\gamma}v\big(\frac{k_1}{q_1},k_2q_2\sigma\big)-\frac{\beta}{\gamma}v\big(\frac{k_1}{q_1},\sigma k_2\big)-v\big(\frac{k_1}{{q_1}^2},k_2\big)+2\alpha v\big(\frac{k_1}{q_1},k_2\big)$. When replacing k_1 by $\frac{k_1}{q_1},\frac{k_1}{q_1^2},...,\frac{k_1}{q_1^m}$ and k_2 by $k_2q_2\sigma, k_2q_2^2\sigma^2, ..., k_2q_2^m\sigma^m$ repeatedly in the above relation we obtained (9).

(d). The proof of (10) follows by replacing k_1 by $k_1q_1, k_1q_1^2, ..., k_1q_1^m k_2$ by $k_2q_2\sigma, k_2q_2^2\sigma^2, ..., k_2q_2^m\sigma^m$ repeatedly in $v(k_1,k_2)=\frac{1}{\gamma}v(k_1q_1,k_2q_2\sigma)-\frac{\beta}{\gamma}v(k_1q_1,\sigma k_2)-v\big(k_1{q_1}^2,k_2\big)+2\alpha v\big(k_1q_1,k_2\big)$. □

Example 2.3. *Suppose that* $v(k_1,k_2)=k_1k_2$ *is a exact solution of* (4), $v(k_1,k_2)=\gamma[\underset{(q_1,1)}{\Delta}k_1k_2+\underset{(\frac{1}{q_1},1)}{\Delta}k_1k_2]$ *yields* $k_1k_2q_2-\beta k_1k_2=\gamma[k_1q_1k_2+\frac{k_1}{q_1}k_2-2k_1k_2]$. *Canceling* k_1k_2 *on the both sides derives* $\gamma=\dfrac{q_2-\beta}{\frac{q_1}{\sigma}+\frac{1}{q_1\sigma}-\frac{2\alpha}{\sigma}}$. *For numerical verification, we give the MATLAB coding for (a) by taking* $k_1=q_1=4,\ k_2=q_2=5,\ \beta=\sigma=3,\ \alpha=2,\ m=20,\ 4.*5=((1./(3).\wedge 20).*(20.*(5.\wedge 20)))-symsum((24./(3.\wedge i)).*((16.*(5.*(5.\wedge(i-1)))./3)+(1.*(5.*(5.\wedge(i-1)))./3)-(4.*(20.*((5.\wedge(i-1))./3)))),i,1,20)$.

3. Delay q-heat Equation for Thin Plate when γ is Constant

Let $v(k_1,k_2,k_3)$ be the temperature of a thin plate at position (k_1,k_2) and time k_3. The proportional amount of heat flows from left to right at $k=(k_1,k_2,k_3)$ is $\underset{(\frac{1}{q_1},1,1)}{\Delta}v(k)$, right to left $\underset{(q_1,1,1)}{\Delta}v(k)$, top to bottom $\underset{(1,q_2,1)}{\Delta}v(k)$ and bottom to top $\underset{(1,\frac{1}{q_2},1)}{\Delta}v(k)$. By Fourier law of cooling and denoting $\underset{(q_1q_2)^{\pm}}{\Delta}=\underset{(q_1,1,1)}{\Delta}+\underset{(\frac{1}{q_1},1,1)}{\Delta}+\underset{(1,q_2,1)}{\Delta}+\underset{(1,\frac{1}{q_2},1)}{\Delta}$ the heat equation for the plate is

$$\underset{\beta(1,1,q_3)}{\Delta}v(k_1,k_2,k_3)=\gamma\underset{\alpha(q_1,q_2)^{\pm}}{\Delta}v(k_1,k_2,\frac{k_3}{\sigma}) \tag{13}$$

Theorem 3.1. *Let* $m>0$ *and* $q_3>0$ *such that* $v(k_1,k_2,\frac{k_3}{q_3^m})$ *and the partial differences* $\underset{\alpha(q_1^{\pm},1)}{\Delta}v(k_1,k_2,\frac{k_3}{\sigma})=\underset{(q_1^{\pm},1)}{u}(k_1,k_2,\frac{k_3}{\sigma})$ *are known then we have*

$$v(k)=\alpha^m v(k_1,k_2,\frac{k_3}{q_3^m})+\gamma\sum_{r=1}^{m}\alpha^r\underset{\alpha(q_1^{\pm},1)}{u}(k_1,k_2,\frac{k_3}{\sigma q_3^r}). \tag{14}$$

Proof. Taking $\underset{\alpha(q_1^{\pm},1)}{\Delta}v(\frac{k}{\sigma})=\underset{\alpha(q_1^{\pm},1)}{u}(\frac{k}{\sigma})$ in (13), we arrive

$$v(k)=\gamma\underset{\beta(1,1,q_3)}{\Delta^{-1}}\underset{\alpha(q_1^{\pm},1)}{u}\Big(\frac{k}{\sigma}\Big). \tag{15}$$

By using the inverse principle of $\underset{\beta(q_3)}{\overset{-1}{\Delta}}$ in (15) we obtain (14). □

Consider the following notation which will be used in Theorem 3.2, $v(k_{(1,2)}q_{(1,2)},*)^{\pm}=v(k_1{q_1}^{\pm},k_2,*)+v(k_1,k_2{q_2}^{\pm},*)$ and also $v(k_{(2,3)}q_{(2,3)},*)^{\pm}=v(*,k_2{q_2}^{\pm},k_3)+v(*,k_2,k_3{q_3}^{\pm})$.

Theorem 3.2. *Assuming* (13)*, then we have the following results*

(a). $$v(k_1,k_2,k_3) = \frac{1}{\beta^m} v(k_1,k_2,k_3 q_3{}^m) - \sum_{i=1}^{m} \frac{\gamma}{\beta^i}\Big[v\big(k_1 q_1^{\pm}, k_2, \frac{k_3 q_3^{i-1}}{\sigma}\big) + v\big(k_1, k_2 q_2^{\pm}, \frac{k_3 q_3^{i-1}}{\sigma}\big) - 4\alpha v\big(k_1,k_2,\frac{k_3 q_3^{i-1}}{\sigma}\big)\Big], \tag{16}$$

(b). $$v(k_1,k_2,k_3) = \beta^m v\big(k_1,k_2,\frac{k_3}{q_3^m}\big) + \sum_{i=1}^{m} \beta^{i-1}\gamma\Big[v\big(k_1 q_1^{\pm}, k_2, \frac{k_3}{q_3^i \sigma}\big) + v\big(k_1, k_2 q_2^{\pm}, \frac{k_3}{q_3^i\sigma}\big) - 4\alpha v\big(k_1,k_2,\frac{k_3}{q_3^i\sigma}\big)\Big], \tag{17}$$

(c). $$v(k_1,k_2,k_3) = \frac{1}{\gamma^m} v\big(\frac{k_1}{q_1^m}, k_2, k_3 q_3^m \sigma^m\big) - \sum_{i=1}^{m}\frac{\beta}{\gamma^i} v\big(\frac{k_1}{q_1^i}, k_2, k_3 q_3^{i-1}\sigma^i\big) - \sum_{i=1}^{m}\frac{1}{\gamma^{i-1}}\Big[v\big(\frac{k_1}{q_1^{i+1}}, k_2, k_3 q_3^{i-1}\sigma^{i-1}\big) + v\big(\frac{k_1}{q_1^i}, k_2 q_2^{\pm}, k_3 q_3^{i-1}\sigma^{i-1}\big)\Big] - \sum_{i=1}^{m}\frac{4\alpha}{\gamma^{i-1}} v\big(\frac{k_1}{q_1^i}, k_2, k_3 q_3^{i-1}\sigma^{i-1}\big), \tag{18}$$

(d). $$v(k_1,k_2,k_3) = \frac{1}{\gamma^m} v\big(k_1 q_1^m, k_2, k_3 q_3^m \sigma^m\big) - \sum_{i=1}^{m}\frac{\beta}{\gamma^i} v\big(k_1 q_1^i, k_2, k_3 q_3^{i-1}\sigma^i\big) - \sum_{i=1}^{m}\frac{1}{\gamma^{i-1}}\Big[v\big(k_1 q_1^{i+1}, k_2, k_3 q_3^{i-1}\sigma^{i-1}\big) + v\big(k_1 q_1^i, k_2 q_2^{\pm}, k_3 q_3^{i-1}\sigma^{i-1}\big)\Big] - \sum_{i=1}^{m}\frac{4\alpha}{\gamma^{i-1}} v\big(k_1 q_1^i, k_2, k_3 q_3^{i-1}\sigma^{i-1}\big). \tag{19}$$

Proof. The proof and verification are as similar as in the case of long rod. □

4. Delay q-heat Equation of Medium when γ is Constant

By the Fourier law of cooling, the heat equation for medium in $\Re^3$ is

$$\underset{\beta(q_4,q_5)}{\Delta} v(k) = \gamma \underset{\alpha(q_1,q_2,q_3)^{\pm}}{\Delta} v\Big(\frac{k}{\sigma}\Big), \tag{20}$$

where $\underset{(q_1,q_2,q_3)^{\pm}}{\Delta} = \underset{(q_1)}{\Delta} + \underset{(\frac{1}{q_1})}{\Delta} + \underset{(q_2)}{\Delta} + \underset{(\frac{1}{q_2})}{\Delta} + \underset{(q_3)}{\Delta} + \underset{(\frac{1}{q_3})}{\Delta}$ and $k = (k_1,k_2,k_3,k_4,k_5)$.

Theorem 4.1. *Assume that the function* $\underset{\alpha(q_1\,q_2,q_3)^{\pm}}{\Delta} v(\frac{k}{\sigma}) = \underset{\alpha(q_1,q_2,q_3)^{\pm}}{u}(\frac{k}{\sigma})$ *is known. Then* (20) *satisfies the relation*

$$v(k) = v(k_1,k_2,k_3,\frac{k_4}{q_4{}^m},\frac{k_5}{q_5{}^m}) + \gamma \sum_{r=1}^{m} \underset{\alpha(q_1,q_2,q_3)^{\pm}}{u}\big(k_1,k_2,k_3,\frac{k_4}{q_4{}^r\sigma},\frac{k_5}{q_5{}^r\sigma}\big). \tag{21}$$

Proof. Taking $\underset{\alpha(q_1,q_2,q_3)^{\pm}}{\Delta} v(\frac{k}{\sigma}) = \underset{\alpha(q_1,q_2,q_3)^{\pm}}{u}(\frac{k}{\sigma})$ in (20), we get

$$v(k) = \gamma \underset{\beta(q_4,q_5)}{\Delta^{-1}} \underset{\alpha(q_1,q_2,q_3)^{\pm}}{u}(\frac{k}{\sigma}). \tag{22}$$

Then using the inverse principle in (22) we get (21). □

In the below theorem, we use the following notations:

$$v(k_{(1,2,3)}q^{\pm}_{(1,2,3)},*,*) = v(k_1q_1,k_2,k_3,*,*) + v(\frac{k_1}{q_1},k_2,k_3,*,*) + v(k_1,k_2q_2,k_3,*,*) + v(k_1,\frac{k_2}{q_2},k_3,*,*) + v(k_1,k_2,k_3q_3,*,*) + v(k_1,k_2,\frac{k_3}{q_3},*,*).$$

$$v(*,k_{(2,3)}q^{\pm}_{(2,3)},*,*) = v(*,k_2q_2,k_3,*,*) + v(*,\frac{k_2}{q_2},k_3,*,*) + v(*,k_2,k_3q_3,*,*) + v(*,k_2,\frac{k_3}{q_3},*,*).$$

Theorem 4.2. *If $v(k)$ is a solution of the equation (20) and $m>0$ and $k=(k_1,k_2,k_3,k_4,k_5)$ then the following relations are equivalent:*

(a). $$v(k,k_4,k_5)=\tfrac{1}{\beta^m}v(k,k_4{q_4}^m,k_5{q_5}^m)-\sum_{i=1}^{m}\frac{\gamma}{\beta^i}\Big[v\big(kq_1^{\pm},\tfrac{k_4q_4^{i-1}}{\sigma},\frac{k_5q_5^{i-1}}{\sigma}\big)$$
$$+v\big(kq_2^{\pm},\frac{k_4q_4^{i-1}}{\sigma},\frac{k_5q_5^{i-1}}{\sigma}\big)+v\big(kq_3^{\pm},\frac{k_4q_4^{i-1}}{\sigma},\frac{k_5q_5^{i-1}}{\sigma}\big)-6\alpha v\big(k,\frac{k_4q_4^{i-1}}{\sigma},\frac{k_5q_5^{i-1}}{\sigma}\big)\Big], \tag{23}$$

(b). $$v(k,k_4,k_5)=\beta^m v\big(k,\tfrac{k_4}{q_4^m},\tfrac{k_5}{q_5^m}\big)+\sum_{i=1}^{m}\beta^{i-1}\gamma\Big[v\big(kq_1^{\pm},\tfrac{k_4}{q_4^i\sigma},\frac{k_5}{q_5^i\sigma}\big)$$
$$+v\big(kq_2^{\pm},\frac{k_4}{q_4^i\sigma},\frac{k_5}{q_5^i\sigma}\big)+v\big(kq_3^{\pm},\frac{k_4}{q_4^i\sigma},\frac{k_5}{q_5^i\sigma}\big)-6\alpha v\big(k,\frac{k_4}{q_4^i\sigma},\frac{k_5}{q_5^i\sigma}\big)\Big], \tag{24}$$

(c). $$v(k,k_4,k_5)=\tfrac{1}{\gamma^m}v\big(\tfrac{k}{q_1^m},k_4q_4^m\sigma^m,k_5q_5^m\sigma^m\big)-\sum_{i=1}^{m}\tfrac{\beta}{\gamma^i}v\big(\tfrac{k}{q_1^i},k_4q_4^{i-1}\sigma^i,k_5q_5^{i-1}\sigma^i\big)$$
$$-\sum_{i=1}^{m}\tfrac{1}{\gamma^{i-1}}\Big[v\big(\tfrac{k}{q_1^{i+1}},k_4q_4^{i-1}\sigma^{i-1},k_5q_5^{i-1}\sigma^{i-1}\big)+v\big(\tfrac{kq_2^{\pm}}{q_1^i},k_4q_4^{i-1}\sigma^{i-1},k_5q_5^{i-1}\sigma^{i-1}\big)\Big]$$
$$-\sum_{i=1}^{m}\frac{6\alpha}{\gamma^{i-1}}v\big(\frac{k}{q_1^i},k_4q_4^{i-1}\sigma^{i-1},k_5q_5^{i-1}\sigma^{i-1}\big), \tag{25}$$

(d). $$v(k,k_4,k_5)=\tfrac{1}{\gamma^m}v\big(kq_1^m,k_4q_4^m\sigma^m,k_5q_5^m\sigma^m\big)-\sum_{i=1}^{m}\tfrac{\beta}{\gamma^i}v\big(kq_1^i,k_4q_4^{i-1}\sigma^i,k_5q_5^{i-1}\sigma^i\big)$$
$$-\sum_{i=1}^{m}\tfrac{1}{\gamma^{i-1}}\Big[v\big(kq_1^{i+1},k_4q_4^{i-1}\sigma^{i-1},k_5q_5^{i-1}\sigma^{i-1}\big)+v\big(kq_2^{\pm}q_1^i,k_4q_4^{i-1}\sigma^{i-1},k_5q_5^{i-1}\sigma^{i-1}\big)\Big]$$
$$-\sum_{i=1}^{m}\frac{6\alpha}{\gamma^{i-1}}v\big(kq_1^i,k_4q_4^{i-1}\sigma^{i-1},k_5q_5^{i-1}\sigma^{i-1}\big), \tag{26}$$

Proof. The proof and verification are as similar as Theorem 3.2. □

5. Conclusion

The newly introduced partial q-difference operator with its corresponding equations has many applications in the field of finite difference methods and q-heat equations. The nature of propagation of heat through the rod is studied using partial Fibonacci difference operator. The results obtained above gives us the tool to predict the temperature and also gives us the possibility to determine the nature of the rod under study for better transmission.

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