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Stability of a Quadratic Functional Equation in IFNS

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Abstract: In this paper, we study the generalized Hyers-Ulam-Rassias stability for the Quadratic type functional equation in IFNS. $f(x+y) - f(x-y) = f(2x+y) - 4f(x) - f(y)$.

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1. Introduction

The stability problems of functional equations originated from a question by S. M.Ulam [19] in the year 1940. The solution for that question given by D. H. Hyers [5] in the year 1941, under certain statement. In 2008, on the stability of quadratic mappings in random normed spaces was given some authors E.Baktash, Y. J. Cho, M. Jalili, R.Saadati, S.M.Vaezpour [4]. Fuzzy theory has become very interesting area of research and a lot of developments have been made in theory of fuzzy sets to find the fuzzy classical set theory. There are many situations where the norm of a vector is not possible to find and the concept of intuitionistic fuzzy norm to be more suitable in such cases. we can deal with such situations by modeling in exactness through the intuitionistic fuzzy norm. The paper On the stability of the linear mapping in Banach spaces given by Th. M. Rassias has provided a lot of influence in the development of what we call generalized Hyers-Ulam-Rassias stability of functional equations. The stability concept that was introduced and investigated by Rassias is called the Hyers-Ulam-Rassias stability. We refer the interested authors for more information to the papers [1, 2, 8, 9, 11–16] and references therein.

In the present paper, the authors finds the stability results concerning the following Quadratic functional equation

$$f(x+y) - f(x-y) = f(2x+y) - 4f(x) - f(y)$$

in intuitionistic fuzzy normed spaces (IFNS). We also study the intuitionistic fuzzy continuity through the existence of a certain solution of a fuzzy stability problem for approximately Quadratic functional equation. Before going to find, first we recall some notations and basic definitions here.

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1.1. Preliminaries

Definition 1.1. *A binary operation* $* : [0,1] \times [0,1] \to [0,1]$ *is said to be a continuous t-norm if it satisfies the following conditions:*

(a). $*$ *is associative and commutative*

(b). $*$ *is continuous*

(c). $a * 1 = a$ *for all* $a \in [0,1]$

(d). $a * b \leq c * d$ *whenever* $a \leq c$ *and* $b \leq d$ *for each* $a, b, c, d \in [0,1]$.

Definition 1.2. *A binary operation* $\diamond : [0,1] \times [0,1] \to [0,1]$ *is said to be a continuous t-conorm if it satisfies the following conditions:*

(a). $\diamond$ *is associative and commutative*

(b). $\diamond$ *is continuous*

(c). $a \diamond 0 = a$ *for all* $a \in [0,1]$

(d). $a \diamond b \leq c \diamond d$ *whenever* $a \leq c$ *and* $b \leq d$ *for each* $a, b, c, d \in [0,1]$.

Using the above two definitions, Saadati and Park [17] introduced the concept of intuitionistic fuzzy normed spaces as follows:

Definition 1.3. *The five-tuple*$(X, \mu, \nu, *, \diamond)$ *is said to be an intuitionistic fuzzy normed spaces(IFNS) if* X *is a vector space,* $*$ *is continuous t-norm,* $\diamond$ *is a continuous t-conorm and* μ, ν *are fuzzy sets on* $X \times (0, \infty)$ *satisfying the following conditions. For every* $x, y \in X$ *and* $s, t > 0$

(a). $\mu(x,t) + \nu(x,t) \leq 1$

(b). $\mu(x,t) > 0$

(c). $\mu(x,t) = 1$ *iff* $x = 0$

(d). $\mu(\alpha x, t) = \mu\left(x, \frac{t}{|\alpha|}\right)$ *for each* $\alpha \neq 0$

(e). $\mu(x,t) * \mu(y,s) \leq \mu(x+y, t+s)$

(f). $\mu(x,.) : (0,\infty) \to [0,1]$ *is continuous*

(g). $\lim_{t\to\infty} \mu(x,t) = 1$ *and* $\lim_{t\to 0} \mu(x,t) = 0$

(h). $\nu(x,t) < 1$

(i). $\nu(x,t) = 0$ *iff* $x = 0$

(j). $\nu(\alpha x, t) = \nu\left(x, \frac{t}{|\alpha|}\right)$ *for each* $\alpha \neq 0$

(k). $\nu(x,t) \diamond \nu(y,s) \geq \nu(x+y, t+s)$

(l). $\nu(x,.) : (0,\infty) \to [0,1]$ *is continuous*

(m). $\lim_{t\to\infty} \nu(x,t) = 0$ *and* $\lim_{t\to 0} \nu(x,t) = 1$.

In this case (μ,ν) *is called an intuitionistic fuzzy norm.*

Let $(X,\mu,\nu,*,\diamond)$ be an IFNS. Then, a sequence $x = (x_n)$ is said to be intuitinistic fuzzy convergent to $L \in X$ if lim $\mu(x_n - L,t) = 1$ and lim $\nu(x_n - L,t) = 0$ for all $t > 0$. In this case we write $x_n \overset{IF}{\to} L$ as $n \to \infty$. Let $(X,\mu,\nu,*,\diamond)$ be an IFNS. Then $x = (x_n)$ is said to be intuitinistic fuzzy Cauchy sequence if lim $\mu(x_{n+p} - x_n,t) = 1$ and lim $\nu(x_{n+p} - x_n,t) = 0$ for all $t > 0$ and $p = 1,2,\ldots$. Let $(X,\mu,\nu,*,\diamond)$ be an IFNS. Then $(X,\mu,\nu,*,\diamond)$ is said to be complete if every intuitinistic fuzzy Cauchy sequence in $(X,\mu,\nu,*,\diamond)$ in intuitinistic fuzzy convergent in $(X,\mu,\nu,*,\diamond)$.

2. Intuitionistic fuzzy stability

The functional equation

$$f(x+y) - f(x-y) = f(2x+y) - 4f(x) - f(y) \tag{1}$$

is called an Quadratic functional equation, since the function $f(x) = cx^2$ is its solution of the above functional equation (1). Thus, it is called the Quadratic functional equation and every solution of the Quadratic functional equation (1) is said to be a Quadratic function. Every solutions of the Quadratic functional equation is said to be an Quadratic mapping. We start with a generalized Hyers-Ulam-Rassias type theorem in IFNS for an Quadratic functional equation.

Theorem 2.1. *Let* X *be a linear space and let* (Z,μ',ν') *be an IFNS. Let* $\varphi : X \times X \to Z$ *be a function such that for some* $\alpha > 2^2$

$$\begin{aligned} \mu'\left(\varphi\left(\frac{x}{2},0\right),t\right) &\geq \mu'\left(\varphi(x,0),\alpha t\right) \\ \nu'\left(\varphi\left(\frac{x}{2},0\right),t\right) &\leq \nu'\left(\varphi(x,0),\alpha t\right) \end{aligned} \tag{2}$$

and

$$\begin{aligned} \lim_{n\to\infty} \mu'\left(4^n\varphi\left(\frac{x}{2^n},\frac{y}{2^n}\right),t\right) &= 1 \\ \lim_{n\to\infty} \nu'\left(4^n\varphi\left(\frac{x}{2^n},\frac{y}{2^n}\right),t\right) &= 0 \end{aligned}$$

for all $x,y \in X$ *and* $t > 0$. *Let* (Y,μ,ν) *be an intuitionistic fuzzy Banach space and let* $f : X \to Y$ *be a* φ*-approximately Quadratic mapping and that*

$$\begin{aligned} \mu\left(2f(x+2y) + f(2x-y) - 5[f(x+y) + f(x-y)] - 15f(y),t\right) &\geq \mu'\left(\varphi(x,y),t\right), \\ \nu\left(2f(x+2y) + f(2x-y) - 5[f(x+y) + f(x-y)] - 15f(y),t\right) &\leq \nu'\left(\varphi(x,y),t\right) \end{aligned} \tag{3}$$

for all $t > 0$ *and all* $x,y \in X$. *Then there exists a unique Quadratic mapping* $B : X \to Y$ *such that*

$$\begin{aligned} \mu\left(B(x) - f(x),t\right) &\geq \mu'\left(\varphi(x,0),\frac{(\alpha-4)t}{2}\right) \\ \nu\left(B(x) - f(x),t\right) &\leq \nu'\left(\varphi(x,0),\frac{(\alpha-4)t}{2}\right) \end{aligned} \tag{4}$$

for all $x \in X$ *and all* $t > o$.

Proof. Put $y = 0$ in (3). Then for all $x \in X$ and $t > 0$

$$\mu\left(f(2x) - 4f(x), t\right) \geq \mu'\left(\varphi(x,0), t\right)$$

which gives

$$\begin{aligned} &\mu\left(4f\left(\frac{x}{2}\right) - f(x), t\right) \geq \mu'\left(\varphi\left(\frac{x}{2}, 0\right), t\right) \geq \mu'\left(\varphi(x,0), \alpha t\right), \\ &\nu\left(4f\left(\frac{x}{2}\right) - f(x), t\right) \leq \nu'\left(\varphi\left(\frac{x}{2}, 0\right), t\right) \leq \nu'\left(\varphi(x,0), \alpha t\right). \end{aligned} \tag{5}$$

Changes from x by $\frac{x}{2^n}$ in (5), we get

$$\begin{aligned} &\mu\left(4^{n+1}f\left(\frac{x}{2^{n+1}}\right) - 4^n f\left(\frac{x}{2^n}\right), 4^n t\right) \geq \mu'\left(\varphi\left(\frac{x}{2^n}, 0\right), \alpha t\right) \geq \mu'\left(\varphi(x,0), \alpha^{n+1} t\right) \quad \textit{and} \\ &\nu\left(4^{n+1}f\left(\frac{x}{2^{n+1}}\right) - 4^n f\left(\frac{x}{2^n}\right), 4^n t\right) \leq \nu'\left(\varphi\left(\frac{x}{2^n}, 0\right), \alpha t\right) \leq \nu'\left(\varphi(x,0), \alpha^{n+1} t\right). \end{aligned} \tag{6}$$

Changes from t by $\frac{t}{\alpha^{n+1}}$, we get

$$\begin{aligned} &\mu\left(4^{n+1}f\left(\frac{x}{2^{n+1}}\right) - 4^n f\left(\frac{x}{2^n}\right), \frac{4^n t}{\alpha^{n+1}}\right) \geq \mu'\left(\varphi(x,0), t\right) \quad \textit{and} \\ &\nu\left(4^{n+1}f\left(\frac{x}{2^{n+1}}\right) - 4^n f\left(\frac{x}{2^n}\right), \frac{4^n t}{\alpha^{n+1}}\right) \leq \nu'\left(\varphi(x,0), t\right). \end{aligned} \tag{7}$$

It follows from $4^n f\left(\frac{x}{2^n}\right) - f(x) = \sum_{j=0}^{n-1}\left(4^{j+1}f\left(\frac{x}{2^{j+1}}\right) - 4^j f\left(\frac{x}{2^j}\right)\right)$ and (7) that

$$\begin{aligned} &\mu\left(4^n f\left(\frac{x}{2^n}\right) - f(x), \sum_{j=0}^{n-1}\frac{4^j t}{\alpha^{j+1}}\right) \geq \prod_{j=0}^{n-1}\mu\left(4^{j+1}f\left(\frac{x}{2^{j+1}}\right) - 4^j f\left(\frac{x}{2^j}\right), \frac{4^j t}{\alpha^{j+1}}\right) \geq \mu'\left(\varphi(x,0), t\right) \quad \textit{and} \\ &\nu\left(4^n f\left(\frac{x}{2^n}\right) - f(x), \sum_{j=0}^{n-1}\frac{4^j t}{\alpha^{j+1}}\right) \leq \coprod_{j=0}^{n-1}\nu\left(4^{j+1}f\left(\frac{x}{2^{j+1}}\right) - 4^j f\left(\frac{x}{2^j}\right), \frac{4^j t}{\alpha^{j+1}}\right) \leq \nu'\left(\varphi(x,0), t\right) \end{aligned} \tag{8}$$

for all $x \in X, t > 0$ and $n > 0$ where $\prod_{j=0}^{n-1} a_j = a_1 * a_2 * ... * a_n, \coprod_{j=0}^{n-1} b_j = b_1 \diamond b_2 \diamond ... \diamond b_n$. By replacing x with $\frac{x}{2^m}$ in (8), we have

$$\begin{aligned} &\mu\left(4^{n+m}f\left(\frac{x}{2^{n+m}}\right) - 4^m f\left(\frac{x}{2^m}\right), \sum_{j=0}^{n-1}\frac{4^{j+m} t}{\alpha^{j+m+1}}\right) \geq \mu'\left(\varphi\left(\frac{x}{2^m}, 0\right), t\right) \geq \mu'\left(\varphi(x,0), t\right) \quad \textit{and} \\ &\nu\left(4^{n+m}f\left(\frac{x}{2^{n+m}}\right) - 4^m f\left(\frac{x}{2^m}\right), \sum_{j=0}^{n-1}\frac{4^{j+m} t}{\alpha^{j+m+1}}\right) \leq \nu'\left(\varphi\left(\frac{x}{2^m}, 0\right), t\right) \leq \nu'\left(\varphi(x,0), t\right) \end{aligned}$$

Thus,

$$\begin{aligned} &\mu\left(4^{n+m}f\left(\frac{x}{2^{n+m}}\right) - 4^m f\left(\frac{x}{2^m}\right), \sum_{j=m}^{n+m-1}\frac{4^j t}{\alpha^{j+1}}\right) \geq \mu'\left(\varphi(x,0), t\right) \quad \textit{and} \\ &\nu\left(4^{n+m}f\left(\frac{x}{2^{n+m}}\right) - 4^m f\left(\frac{x}{2^m}\right), \sum_{j=m}^{n+m-1}\frac{4^j t}{\alpha^{j+1}}\right) \leq \nu'\left(\varphi(x,0), t\right) \end{aligned}$$

for all $x \in X, t > 0, m \geq 0$ and $n \geq 0$. Hence

$$\mu\left(4^{n+m}f\left(\frac{x}{2^{n+m}}\right) - 4^m f\left(\frac{x}{2^m}\right), t\right) \geq \mu'\left(\varphi(x,0), \frac{t}{\sum_{j=m}^{n+m-1}\frac{4^j t}{\alpha^{j+1}}}\right) \quad \textit{and}$$

$$\nu\left(4^{n+m} f\left(\frac{x}{2^{n+m}}\right)-4^m f\left(\frac{x}{2^m}\right), t\right) \leq \nu'\left(\varphi(x, 0), \frac{t}{\sum_{j=m}^{n+m-1} \frac{4^j t}{\alpha^{j+1}}}\right) \tag{9}$$

for all $x \in X, t>0, m \geq 0$ and $n \geq 0$. Since $\alpha>4$ and $\sum_{j=0}^{\infty}\left(\frac{4}{\alpha}\right)<\infty$, the Cauchy criterion for convergence in IFNS shows that $4^n f\left(\frac{x}{2^n}\right)$ is a Cauchy sequence in (Y, μ, ν). Since (Y, μ, ν) is complete, this sequence converges to some point B(x) $\in Y$. Fix $x \in X$ and $m=0$ in (9), we have

$$\mu\left(4^n f\left(\frac{x}{2^n}\right)-f(x), t\right) \geq \mu'\left(\varphi(x, 0), \frac{t}{\sum_{j=0}^{n-1} \frac{4^j}{\alpha^{j+1}}}\right) \text{ and}$$
$$\nu\left(4^n f\left(\frac{x}{2^n}\right)-f(x), t\right) \leq \nu'\left(\varphi(x, 0), \frac{t}{\sum_{j=0}^{n-1} \frac{4^j}{\alpha^{j+1}}}\right)$$

for all $t>0$ and $n>0$. Therefore, we get

$$\mu(B(x)-f(x), t) \geq \mu\left(B(x)-4^n f\left(\frac{x}{2^n}\right), \frac{t}{2}\right) * \mu\left(4^n f\left(\frac{x}{2^n}\right)-f(x), \frac{t}{2}\right) \geq \mu'\left(\varphi(x, 0), \frac{t}{2 \sum_{j=0}^{n-1} \frac{4^j}{\alpha^{j+1}}}\right),$$
$$\nu(B(x)-f(x), t) \leq \nu\left(B(x)-4^n f\left(\frac{x}{2^n}\right), \frac{t}{2}\right) \diamond \nu\left(4^n f\left(\frac{x}{2^n}\right)-f(x), \frac{t}{2}\right) \leq \nu'\left(\varphi(x, 0), \frac{t}{2 \sum_{j=0}^{n-1} \frac{4^j}{\alpha^{j+1}}}\right),$$

for large n. By taking the limit as $n \rightarrow \infty$ and using the definition of IFNS, we obtain

$$\mu(B(x)-f(x), t) \geq \mu'\left(\varphi(x, 0), \frac{(\alpha-4) t}{2}\right) \quad \text{and}$$
$$\nu(B(x)-f(x), t) \leq \nu'\left(\varphi(x, 0), \frac{(\alpha-4) t}{2}\right)$$

for all $x \in X, t>0$. Changes from x to $\frac{x}{2^n}$ and y to $\frac{y}{2^n}$, in (3), we obtain

$$\mu\left(4^n f\left(\frac{x+2 y}{2^n}\right)+4^n f\left(\frac{2 x-y}{2^n}\right)-5\left[4^n f\left(\frac{x+y}{2^n}\right)+4^n f\left(\frac{x-y}{2^n}\right)\right]+4^n 15 f\left(\frac{y}{2^n}, t\right)\right) \geq \mu'\left(\varphi\left(\frac{x}{2^n}, \frac{y}{2^n}\right), \frac{t}{4^n}\right) \text{ and}$$
$$\nu\left(4^n f\left(\frac{x+2 y}{2^n}\right)+4^n f\left(\frac{2 x-y}{2^n}\right)-5\left[4^n f\left(\frac{x+y}{2^n}\right)+4^n f\left(\frac{x-y}{2^n}\right)\right]+4^n 15 f\left(\frac{y}{2^n}, t\right)\right) \leq \nu'\left(\varphi\left(\frac{x}{2^n}, \frac{y}{2^n}\right), \frac{t}{4^n}\right)$$

for all $x, y \in X, t>0$. Since

$$\lim_{n \rightarrow \infty} \mu'\left(4^n \varphi\left(\frac{x}{2^n}, \frac{y}{2^n}\right), t\right)=1,$$
$$\lim_{n \rightarrow \infty} \nu'\left(4^n \varphi\left(\frac{x}{2^n}, \frac{y}{2^n}\right), t\right)=0,$$

for all $x, y \in X, t>0$. We notice that B satisfies (1). Therefore B is an quadratic mapping.

To prove the uniqueness of the quadratic mapping B, assume that there exists a quadratic mapping $B': X \rightarrow Y$ which satisfies (4). For fix $x \in X$, clearly $4^n B\left(\frac{x}{2^n}\right)=B(x)$ and $4^n B'\left(\frac{x}{2^n}\right)=B'(x)$ for all $n \in N$. It follows from (4) that

$$\begin{aligned}
\mu\left(B(x)-B'(x), t\right) & =\mu\left(4^n B\left(\frac{x}{2^n}\right)-4^n B'\left(\frac{x}{2^n}\right), t\right) \\
& \geq \mu\left(4^n B\left(\frac{x}{2^n}\right)-4^n f\left(\frac{x}{2^n}\right), \frac{t}{2}\right) * \mu\left(4^n f\left(\frac{x}{2^n}\right)-4^n B'\left(\frac{x}{2^n}\right), \frac{t}{2}\right) \\
& \geq \mu'\left(\varphi\left(\frac{x}{2^n}, 0\right), \frac{2(\alpha-4) t}{4^{n+1}}\right) \\
& \geq \mu'\left(\varphi(x, 0), \frac{2 \alpha^n(\alpha-4) t}{4^{n+1}}\right)
\end{aligned}$$

and similarly

$$\nu\left(B(x)-B'(x), t\right) \leq \nu'\left(\varphi(x, 0), \frac{2 \alpha^n(\alpha-4) t}{4^{n+1}}\right).$$

Since $lim_{n \rightarrow \infty} \frac{2 \alpha^n(\alpha-4)}{4^{n+1}}=\infty$ as $\alpha>4$, we get $lim_{n \rightarrow \infty} \mu'\left(\varphi(x, 0), \frac{2 \alpha^n(\alpha-4) t}{4^{n+1}}\right)=1$, and $lim_{n \rightarrow \infty} \nu'\left(\varphi(x, 0), \frac{2 \alpha^n(\alpha-4) t}{4^{n+1}}\right)=0$. Therefore $\mu\left(B(x)-B'(x), t\right)=1$ and $\nu\left(B(x)-B'(x), t\right)=0$, for all $t>0$. Hence $B(x)=B'(x)$. □

In the next theorem, let us consider $0 < \alpha < 4$.

Theorem 2.2. *Let X be a linear space and let (Z, μ', ν') be an IFNS. Let $\varphi : X \times X \to Z$ be a function such that for some $0 < \alpha < 4$*

$$\mu'(\varphi(2x, 0), t) \geq \mu'(\alpha\varphi(x, 0), t) \quad and \quad \nu'(\varphi(2x, 0), t) \leq \nu'(\alpha\varphi(x, 0), t),$$

$\lim_{n\to\infty} \mu'(\varphi(2^n x, 2^n y), 4^n t) = 1$ *and* $\lim_{n\to\infty} \nu'(\varphi(2^n x, 2^n y), 4^n t) = 0$ *for all $x, y \in X$ and $t > 0$. Let (Y, μ, ν) be an intuitionistic fuzzy Banach space and let $f : X \to Y$ be a φ-approximately quadratic mapping in the sense that*

$$\begin{aligned} &\mu \{(2f(x + 2y) + f(2x - y) - 5[f(x + y) + f(x - y))] - 15f(y, t)\} \geq \mu'(\varphi(x, y), t) \quad and \\ &\nu \{(2f(x + 2y) + f(2x - y) - 5[f(x + y) + f(x - y))] - 15f(y, t)\} \leq \nu'(\varphi(x, y), t) \end{aligned}$$

for all $x, y \in X$ and $t > 0$. Then there exists a unique quadratic mapping $B : X \to Y$ such that

$$\mu(B(x) - f(x), t) \geq \mu'\left(\varphi(x, 0), \frac{(4-\alpha)t}{2}\right) \quad and \quad \nu(B(x) - f(x), t) \leq \nu'\left(\varphi(x, 0), \frac{(4-\alpha)t}{2}\right)$$

for all $x \in X$ and $t > 0$.

Proof. The proof of this theorem is similarly as Theorem 2.1. Here we outline the proof. Put $y = 0$ in (3) we get

$$\mu\left(\frac{f(2x)}{4} - f(x), t\right) \geq \mu'(\varphi(x, 0), t) \quad and \quad \nu\left(\frac{f(2x)}{4} - f(x), t\right) \leq \nu'(\varphi(x, 0), t),$$

for all $x \in X$ and $t > 0$. So

$$\mu\left(\frac{f(2^{n+1}x)}{4} - f(2^n x), t\right) \geq \mu'\left(\varphi(x, 0), \frac{t}{\alpha^n}\right) \quad and \quad \nu\left(\frac{f(2^{n+1}x)}{4} - f(2^n x), t\right) \leq \nu'\left(\varphi(x, 0), \frac{t}{\alpha^n}\right),$$

for all $x \in X$ and $t > 0$. For each $x \in X, n \geq 0, m \geq 0$ and $t > 0$, we reduces

$$\begin{aligned} &\mu\left(\frac{f(2^{n+m}x)}{4^{n+m}} - \frac{f(2^m x)}{4^m}, t\right) \geq \mu'\left(\varphi(x, 0), \frac{t}{\sum_{j=m}^{n+m-1} \frac{\alpha^j}{4^{j+1}}}\right) \quad and \\ &\nu\left(\frac{f(2^{n+m}x)}{4^{n+m}} - \frac{f(2^m x)}{4^m}, t\right) \leq \nu'\left(\varphi(x, 0), \frac{t}{\sum_{j=m}^{n+m-1} \frac{\alpha^j}{4^{j+1}}}\right) \end{aligned} \tag{10}$$

for all $x \in X$, $t > 0$, and $m, n \geq 0$. Thus, $\left\{\frac{f(2^n x)}{4^n}\right\}$ is a Cauchy sequence in intuitionistic fuzzy Banach space. There exist a function $B : X \to Y$ defined by $B(x) = lim_{n\to\infty} \frac{f(2^n x)}{4^n}$ and put $m = 0$ in (10) we obtain

$$\mu(B(x) - f(x), t) \geq \mu'\left(\varphi(x, 0), \frac{(4-\alpha)t}{2}\right) \quad and \quad \nu(B(x) - f(x), t) \leq \nu'\left(\varphi(x, 0), \frac{(4-\alpha)t}{2}\right)$$

for all $x \in X$ and $t > 0$. Hence proved. □

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