Int. J. Math. And Appl., 6(1)(2018), 61–66. (Special Issue)
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Ulam Stability of a LCR Electric Circuit with Electromotive Force

R. Murali[1,*] and A. Ponmana Selvan[1]

1 PG and Research Department of Mathematics, Sacred Heart College (Autonomous), Tirupattur, Tamil Nadu, India.

Abstract: We enumerate the approximate solution of the second order differential equation of the LCR electric circuit. That is, we study the Hyers-Ulam stability and Hyers-Ulam-Rassias of the dirrential equations $l''(t)+\frac{R}{L}\,l'(t)+\frac{1}{LC}\,l(t)=0$ and the non-homogeneous differential equation $l''(t)+\frac{R}{L}\,l'(t)+\frac{1}{LC}\,l(t)=p(t)$, with initial conditions $H(a)=H'(a)=0$, where R, L, C are constants and $l\in C^2(I)$, $p(t)\in C(I)$, $I=[a,b]\subseteq\mathbb{R}$.

MSC: 35B35, 34K20, 26D10, 44A10, 39B82.

Keywords: Hyers-Ulam Stability, Hyers-Ulam-Rassias stability, linear differential equations, homogeneous and non-homogeneous.

Accepted on: 13th April 2018

1. Introduction

The Hyers-Ulam stability property issue for different functional equation was presented by Ulam [15] in 1940. Then in the next year, D.H. Hyers [7] was handled the issue of Ulam for Cauchy additive functional equation in Banach spaces. From that point forward, Aoki [16], Bourgin [2] and Rassias [11] are generalized the Hyers result. Starting there onwards, colossal number of authors are shown the Hyers-Ulam problem for various functional equation on different spaces [3, 4, 17]. As of late, the Hyers-Ulam stability issue was proposed by supplanting functional equation by differential equation. In 1998, Alsina [1] seems to be the primary authors who contemplated the Ulam-Hyers stability of $x'(t)=x(t)$. Then Takashi [14] are generalized the result reported in [1] for Banach space valued function. These outcomes were stretched out to the Ulam-Hyers stability and Ulam-Hyers-Rassias stability of the linear and non-linear differential equation of first order, second order and higher order in [5, 6, 8–10, 12, 13, 18–20]. Encouraged by the above discussions, our foremost goal is towards enumerate the stability of the differential equations in the sense of Hyers-Ulam and Hyers-Ulam-Rassias for simple electric circuit of the shape

$$l''(t)+\frac{R}{L}\,l'(t)+\frac{1}{LC}\,l(t)=0 \tag{1}$$

and the non-homogeneous differential equation

$$l''(t)+\frac{R}{L}\,l'(t)+\frac{1}{LC}\,l(t)=p(t), \tag{2}$$

* *E-mail: shcrmurali@yahoo.co.in*

with initial condition

$$l(a) = l'(a) = 0, \tag{3}$$

here R, L, C are constants and $l \in C^2(I)$, $p(t) \in C(I)$, $I = [a, b] \subseteq \mathbb{R}$.

2. Preliminaries

Firstly, we give the definition of Hyers-Ulam stability and Hyers-Ulam-Rassias stability of the linear differential equation (1) and (2).

Definition 2.1. *The homogeneous differential equation (1) is said to have Hyers-Ulam stability, if for every $\epsilon > 0$, there is a constant $K > 0$, $l \in C^2[a, b]$, such that*

$$\left| l''(t) + \frac{R}{L}\, l'(t) + \frac{1}{LC}\, l(t) \right| \leq \epsilon,$$

with (3) then there is some $j \in C^2[a, b]$, satisfies the differential equation $j''(t) + \frac{R}{L}\, j'(t) + \frac{1}{LC}\, j(t) = 0$ with $j(a) = j'(a) = 0$ such that $|l(t) - j(t)| \leq K\, \epsilon$. Where K is called as the Hyers-Ulam stability constant for (1).

Definition 2.2. *The non-homogeneous linear differential equation (2) is said to have the Hyers-Ulam stability, if for every $\epsilon > 0$, there is a constant $K > 0$, $l \in C^2[a, b]$, such that*

$$\left| l''(t) + \frac{R}{L}\, l'(t) + \frac{1}{LC}\, l(t) - p(t) \right| \leq \epsilon,$$

with (3) then there is a $j \in C^2[a, b]$, satisfying $j''(t) + \frac{R}{L}\, j'(t) + \frac{1}{LC}\, j(t) = p(t)$ with $j(a) = j'(a) = 0$ such that $|l(t) - j(t)| \leq K\, \epsilon$. Where K is called as a Hyers-Ulam stability constant for (2).

Definition 2.3. *The homogeneous linear differential equation (1) is said to have the Hyers-Ulam-Rassias stability, if there is a constant $K > 0$, for every $\epsilon > 0$ and $l \in C^2[a, b]$, if there exists a function $\phi : [0, \infty) \to [0, \infty)$ such that*

$$\left| l''(t) + \frac{R}{L}\, l'(t) + \frac{1}{LC}\, l(t) \right| \leq \phi(t)\, \epsilon,$$

with (3) then there is $j \in C^2[a, b]$, satisfying $j''(t) + \frac{R}{L}\, j'(t) + \frac{1}{LC}\, j(t) = 0$ with $j(a) = j'(a) = 0$ such that $|l(t) - j(t)| \leq K\, \phi(t)\, \epsilon$. Where K is called as a Hyers-Ulam-Rassias stability constant for (1).

Definition 2.4. *The non-homogeneous linear differential equation (2) has the Hyers-Ulam-Rassias stability, if there is a constant $K > 0$, for every $\epsilon > 0$ and $l \in C^2[a, b]$, if there exists $\phi : [0, \infty) \to [0, \infty)$ such that*

$$\left| l''(t) + \frac{R}{L}\, l'(t) + \frac{1}{LC}\, l(t) - p(t) \right| \leq \phi(t)\, \epsilon,$$

with (3) then there exists some $j \in C^2[a, b]$, satisfying $j''(t) + \frac{R}{L}\, j'(t) + \frac{1}{LC}\, j(t) = p(t)$ with $j(a) = j'(a) = 0$ such that $|l(t) - j(t)| \leq K\, \phi(t)\, \epsilon$. Where K is called as a Hyers-Ulam-Rassias stability constant (2).

3. Hyers-Ulam Stability

Now, we enumerate the Hyers-Ulam stability of (1) with (3).

Theorem 3.1. *Suppose that R, L, C are constants and $l \in C^2[a,b]$ such that $|l'(t)| \leq |l(t)|$ and satisfies the inequality*

$$\left| l''(t) + \frac{R}{L}\, l'(t) + \frac{1}{LC}\, l(t) \right| \leq \epsilon$$

with initial condition (3), then (1) has Hyers-Ulam stability.

Proof. For every $\epsilon > 0$, there is a twice continously differentiable function $l : [a,b] \to C$ such that $|l'(t)| \leq |l(t)|$ and satisfies the inequality

$$\left| l''(t) + \frac{R}{L}\, l'(t) + \frac{1}{LC}\, l(t) \right| \leq \epsilon, \tag{4}$$

with initial condition (3) and $M = \max_{t \in I} |l(t)|$. From the inequality (4), we have

$$-\epsilon \leq l''(t) + \frac{R}{L}\, l'(t) + \frac{1}{LC}\, l(t) \leq \epsilon. \tag{5}$$

Multiplying the above inequality (5) by $l'(t)$ and then integrating, we get

$$-\epsilon \int_a^t l'(t)\ dt \leq \int_a^t l''(t)\ l'(t)\ dt + \int_a^t \frac{R}{L}\ l'(t)^2\ dt + \int_a^t \frac{1}{LC}\ l(t)\ l'(t)\ dt \leq \epsilon \int_a^t l'(t)\ dt.$$

From which we obtain

$$\begin{aligned}
-2\epsilon \int_a^t l'(t)\ dt \leq l'(t)^2 + \frac{l(t)^2}{LC} + \frac{2R}{L} \int_a^t l'(t)^2\ dt \leq 2\ \epsilon \int_a^t l'(t)\ dt \\
\frac{l(t)^2}{LC} \leq 2\ \epsilon \int_a^t l'(t)\ dt + \frac{2R}{L} \int_a^t l'(t)^2\ dt \\
M^2 \leq 2LC\epsilon M(b-a) + 2RC(b-a)M^2 \\
M \leq \frac{2LC\epsilon(b-a)}{1-\nu}, \qquad \text{where} \quad \nu = 2RC(b-a).
\end{aligned}$$

Hence $|l(t)| \leq K\ \epsilon$, for all $t \in [a,b]$, where $K = \dfrac{2LC(b-a)}{1-\nu}$. Obviously, $j(t) = 0$ is a solution of (1) with initial condition (3) such that $|l(t) - j(t)| \leq K\ \epsilon$. Then by the virtue of Definition 2.1, the Theorem holds good. □

In the next theorem, we study the Hyers-Ulam stability of the non-homogeneous differential equation (2) with (3).

Theorem 3.2. *Suppose that R, L, C are constants and $l \in C^2[a,b]$ such that $|l'(t)| \leq |l(t)|$ and satisfies the inequality*

$$\left| l''(t) + \frac{R}{L}\, l'(t) + \frac{1}{LC}\, l(t) - p(t) \right| \leq \epsilon$$

with (3), then (2) has the Hyers-Ulam stability.

Proof. For every $\epsilon > 0$, there is a twice continously differentiable function $l : [a,b] \to C$ such that $|l'(t)| \leq |l(t)|$ and satisfies the inequality

$$\left| l''(t) + \frac{R}{L}\, l'(t) + \frac{1}{LC}\, l(t) - p(t) \right| \leq \epsilon, \tag{6}$$

with initial condition (3) and $M = \max_{t \in I} |l(t)|$. From the inequality (6), we have

$$-\epsilon \leq l''(t) + \frac{R}{L}\, l'(t) + \frac{1}{LC}\, l(t) - p(t) \leq \epsilon. \tag{7}$$

Multiplying the above inequality (7) by $l'(t)$ and then integrating, we get

$$-\epsilon\int_a^t l'(t)\ dt \leq \int_a^t l''(t)\ l'(t)\ dt + \int_a^t \frac{R}{L}\ l'(t)^2\ dt + \int_a^t \frac{1}{LC}\ l(t)\ l'(t)\ dt - \int_a^t p(t)\ l'(t)\ dt \leq \epsilon\int_a^t l'(t)\ dt.$$

From which we get that

$$\begin{aligned}
-2\ \epsilon\int_a^t l'(t)\ dt \leq l'(t)^2 + \frac{l(t)^2}{LC} + \frac{2R}{L}\int_a^t l'(t)^2\ dt - \int_a^t p(t)\ l'(t)\ dt \leq 2\ \epsilon\int_a^t l'(t)\ dt \\
\frac{l(t)^2}{LC} \leq 2\ \epsilon\int_a^t l'(t)\ dt + \frac{2R}{L}\int_a^t l'(t)^2\ dt + 2\int_a^t p(t)\ l'(t)\ dt \\
M^2 \leq 2LC\epsilon M(b-a) + 2RC(b-a)M^2 + 2LCMN(b-a) \\
M \leq \frac{2LC(N+\epsilon)(b-a)}{1-\nu}, \qquad \text{where} \ \ \nu = 2RC(b-a).
\end{aligned}$$

Hence $|l(t)| \leq K\ (\epsilon)$, for all $t \in [a,b]$. Obviously, $j(t) = 0$ is a solution of (2) with initial condition (3) such that $|l(t) - j(t)| \leq K\ (\epsilon)$. Then by the virtue of Definition 2.2, equation (2) has the Ulam-Hyers stability. □

4. Hyers-Ulam-Rassias Stability

Now, we study the Hyers-Ulam-Rassias stability of (1) with (3).

Theorem 4.1. *Suppose that R, L, C are constants and $l \in C^2[a,b]$ such that $|l'(t)| \leq |l(t)|$ and if there is a function $\phi : [0,\infty) \to [0,\infty)$ satisfies the inequality*

$$\left| l''(t) + \frac{R}{L}\ l'(t) + \frac{1}{LC}\ l(t)\right| \leq \phi(t)\ \epsilon,$$

with initial condition (3), then (1) has the Hyers-Ulam-Rassias stability with $\phi(a) = 0$.

Proof. For every $\epsilon > 0$, there is a twice continously differentiable function $l : [a,b] \to C$ such that $|l'(t)| \leq |l(t)|$ and satisfies the inequality

$$\left| l''(t) + \frac{R}{L}\ l'(t) + \frac{1}{LC}\ l(t)\right| \leq \phi(t)\ \epsilon, \tag{8}$$

with initial condition (3) and $M = \max_{t\in I} |l(t)|$. From the inequality (8), we have

$$-\phi(t)\ \epsilon \leq l''(t) + \frac{R}{L}\ l'(t) + \frac{1}{LC}\ l(t) \leq \phi(t)\ \epsilon. \tag{9}$$

Multiplying the above inequality (9) by $l'(t)$ and then integrating, we get

$$-\epsilon\int_a^t \phi(t)\ l'(t)\ dt \leq \int_a^t l''(t)\ l'(t)\ dt + \int_a^t \frac{R}{L}\ l'(t)^2\ dt + \int_a^t \frac{1}{LC}\ l(t)\ l'(t)\ dt \leq \epsilon\int_a^t \phi(t)\ l'(t)\ dt.$$

From which we obtain

$$-2\ \epsilon\int_a^t \phi(t)\ l'(t)\ dt \leq l'(t)^2 + \frac{l(t)^2}{LC} + \frac{2R}{L}\int_a^t l'(t)^2\ dt \leq 2\ \epsilon\int_a^t \phi(t)\ l'(t)\ dt$$

$$\frac{l(t)^2}{LC} \leq 2\ \epsilon \int_a^t \phi(t)\ l'(t)\ dt + \frac{2R}{L} \int_a^t l'(t)^2\ dt$$

$$M^2 \leq 2LC\phi(t)\ \epsilon M + 2RC(b-a)M^2$$

$$M \leq \frac{2LC\phi(t)\ \epsilon}{1-\nu}, \qquad \text{where} \quad \nu = 2RC(b-a).$$

Hence $|l(t)| \leq K\ \phi(t)\ \epsilon$, for all $t \in [a,b]$, where $K = \dfrac{2LC}{1-\nu}$. Obviously, $j(t) = 0$ is a solution of (1) with initial condition (3) such that $|l(t) - j(t)| \leq K\ \phi(t)\ \epsilon$. Then by the virtue of Definition 2.3, the differential equation (1) has the Hyers-Ulam-Rassias stability. □

Finally, we study the Hyers-Ulam-Rassias stability of the non-homogeneous differential equation (2) with (3).

Theorem 4.2. *Suppose that R, L, C are constants and $l \in C^2[a,b]$ such that $|l'(t)| \leq |l(t)|$ and if there exists $\phi : [0,\infty) \to [0,\infty)$ satisfies the inequality*

$$\left| l''(t) + \frac{R}{L}\ l'(t) + \frac{1}{LC}\ l(t) - p(t) \right| \leq \phi(t)\ \epsilon$$

with initial condition (3), then the differential equation (2) has the Hyers-Ulam-Rassias stability with $\phi(a) = 0$.

Proof. For every $\epsilon > 0$, there exists a $l : [a,b] \to C$ be twice continously differentiable function such that $|l'(t)| \leq |l(t)|$ and if there exists $\phi : [0,\infty) \to [0,\infty)$ satisfies the inequality

$$\left| l''(t) + \frac{R}{L}\ l'(t) + \frac{1}{LC}\ l(t) - p(t) \right| \leq \phi(t)\ \epsilon, \tag{10}$$

with initial condition (3) and $M = \max\limits_{t\in[a,b]} |l(t)|$. From the inequality (10), we have

$$-\phi(t)\ \epsilon \leq l''(t) + \frac{R}{L}\ l'(t) + \frac{1}{LC}\ l(t) - p(t) \leq \phi(t)\ \epsilon. \tag{11}$$

Multiplying the above inequality (11) by $l'(t)$ and then integrating, we get

$$-\epsilon \int_a^t \phi(t)\ l'(t)\ dt \leq \int_a^t l''(t)\ l'(t)\ dt + \int_a^t \frac{R}{L}\ l'(t)^2\ dt + \int_a^t \frac{1}{LC}\ l(t)\ l'(t)\ dt - \int_a^t p(t)\ l'(t)\ dt \leq \epsilon \int_a^t \phi(t)\ l'(t)\ dt.$$

From which we get that

$$-2\ \epsilon \int_a^t \phi(t)\ l'(t)\ dt \leq l'(t)^2 + \frac{l(t)^2}{LC} + \frac{2R}{L} \int_a^t l'(t)^2\ dt - \int_a^t p(t)\ l'(t)\ dt \leq 2\ \epsilon \int_a^t \phi(t)\ l'(t)\ dt$$

$$\frac{l(t)^2}{LC} \leq 2\ \epsilon \int_a^t \phi(t)\ l'(t)\ dt + \frac{2R}{L} \int_a^t l'(t)^2\ dt + 2 \int_a^t p(t)\ l'(t)\ dt$$

$$M^2 \leq 2LC\phi(t)\ \epsilon M + 2RC(b-a)M^2 + 2LCMN(b-a)$$

$$M \leq \frac{2LC(N(b-a) + \phi(t)\ \epsilon)}{1-\nu}, \qquad \text{where} \quad \nu = 2RC(b-a).$$

Hence $|l(t)| \leq K\ (\epsilon)\phi(t)$, for all $t \in [a,b]$. Obviously, $j(t) = 0$ is a solution of (2) with initial condition (3) such that $|l(t) - j(t)| \leq K\ (\epsilon)\phi(t)$. Then by the virtue of Definition 2.4, the differential equation (2) has the Hyers-Ulam-Rassias stability. □

References

[1] C.Alsina and R.Ger, *On Some inequalities and stability results related to the exponential function*, Journal of Inequalities Appl., 2(1998), 373-380.

[2] D.G.Bourgin, *Classes of transformations and bordering transformations*, Bull. Amer. Math. Soc., 57(1951), 223-237.

[3] M.Burger, N.Ozawa and A.Thom, *On Ulam Stability*, Israel J. Math., 193(2013), 109-129.

[4] J.Chung, *Hyers-Ulam stability theorems for Pexiders equations in the space of Schwartz distributions*, Arch. Math., 84(2005), 527-537.

[5] P.Gavruta, S.M.Jung and Y.Li, *Hyers-Ulam stability for the second order linear differential equations with boundary conditions*, Elec. Journal of Diff. Equations, 2011(80)(2011), 1-5.

[6] P.Gavruta and L.Gavruta, *A New method for the generalized Hyers-Ulam-Rassias stability*, International Journal of Non-linear Analysis, 2(2010), 11-18.

[7] D.H.Hyers, *On the Stability of a Linear functional equation*, Proc. Natl. Acad. Sci. USA, 27(1941), 222-224.

[8] S.M.Jung, *Hyers-Ulam stability of a system of first order linear differential equations with constant coefficients*, J. Math. Anal. Appl., 320(2006), 549-561.

[9] T.Li, A.Zada and S.Faisal, *Hyers-Ulam stability of nth order linear differential equations*, Journal of Nonlinear Science and Applications, 9(2016), 2070-2075.

[10] J.Huang, S.M.Jung and Y.Li, *On Hyers-Ulam stability of nonlinear differential equations*, Bull. Korean Math. Soc., 52(2015), 685-697.

[11] Th.M.Rassias, *On the stability of the linear mappings in Banach Spaces*, Proc. Amer. Math. Soc., 72(1978), 297-300.

[12] K.Ravi, R.Murali and A.Ponmana Selvan, *Ulam stability of a General nth order linear differential equation with constant coefficients*, Asian Journal of Mathematics and Computer Research, 11(1)(2016), 61-68.

[13] K.Ravi, R.Murali and A.Ponmana Selvan, *Hyers-Ulam stability of nth order linear differential equation with initial and boundary condition*, Asian Journal of Mathematics and Computer Research, 11(3)(2016), 201-207.

[14] S.E.Takahasi, T.Miura and S.Miyajima, *On the Hyers-Ulam stability of the Banach space-valued differential equation* $y' = \alpha y$, Bulletin Korean Math. Soc., 39(2002), 309-315.

[15] S.M.Ulam, *Problem in Modern Mathematics*, Chapter IV, Science Editors, Willey, New York, (1960).

[16] T.Aoki, *On the stability of the linear transformation in Banach Spaces*, J. Math. Soc. Japan, 2(1950), 64-66.

[17] T.Z.Xu, *On the stability of Multi-Jensens mappings in β-normed spaces*, Appl. Math. Lett., 25(11)(2012), 1866-1870.

[18] R.Murali and A.Ponmana Selvan, *On the Generalized Hyers-Ulam Stability of Linear Ordinary Differential Equations of Higher Order*, International Journal of Pure and Applied Mathematics, 117(12)(2017), 317-326.

[19] R.Murali and A.Ponmana Selvan, *Hyers-Ulam Stability of nth order Differential Equation*, Contemporary Studies in Discrete Mathematics, 2(1)(2018), 45-50.

[20] R.Murali and A.Ponmana Selvan, *Hyers-Ulam-Rassias Stability for the Linear Ordinary Differential Equation of Third order*, Kragujevac Journal of Mathematics, 42(4)(2018), 579-590.