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Octagonal Fuzzy Number and its Corresponding Matrices

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Abstract: In this paper we defined Octagonal Fuzzy Number(OFN) in continuation with the other defined fuzzy numbers. We also include basic arithmetic operations of octagonal fuzzy numbers. Finally we defined Octagonal Fuzzy Matrix(OFM) with some matrix properties.

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1. Introduction

Fuzzy sets have been introduced by Lotfi.A.Zadeh. Fuzzy set theory permits the gradualassessment of the membership of elements in a set which is described in the interval [0,1]. It can be used in a wide range of domains where information is incomplete and imprecise. Interval arithmetic was first suggested by Dwyer in 1951. By means ofzadeh's extension principle, the usual arithmetic operations on real numbers can be extended to the ones defined on fuzzy numbers. Dubois and prade has defined any of the real line [2–4, 7]. A fuzzy number is a quantity whose values are imprecise, rather than exact as is the case with single-valued numbers. Among the various shapes of fuzzy numbers, Triangular fuzzy numbers are the must commonly used membership function. In this paper, a new fuzzy number called 'Octagonal Fuzzy Numbers' is utilized in developing the notion of "Corresponding matrices". Proposed the above said fuzzy number without any restrictions of parameter. Recently introduced Dinagar and Rajesh kanna the "Modified definition" of the Octagonal fuzzy number by including conditions for the convexity of the number and few more results have been verified in this work. The paper is organized as follows, firstly in section 2, of this paper, Preliminaries. In section 3, we recall the definition of octagonal fuzzy number and some operations on octagonal fuzzy numbers. In section 4, we defined Octagonal fuzzy matrix definition and some operations on octagonal fuzzy matrices (OFM). In section 5, Fundamental properties of Octagonal Fuzzy Matrices. In section 6, Trace of a octagonal Fuzzy Matrices (OFM) and some property. In section 7, Determinant of a Octagonal Fuzzy matrices. In section 8, Fuzzy Comparable OFM. Finally In section 9, conclusion included.

2. Preliminaries

Definition 2.1 (Fuzzy Set)**.** *A Fuzzy Set is characterized by its membership function, taking values from the domain, space or universe of discourse mapped into the unit interval [0,1]. A fuzzy set A in the universal set X is defined as*

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$A = (x, \mu(x) : x \in X)$.*here,* $\mu_A : A \to [0,1]$ *is the grade of the membership function and* $\mu_A(x)$ *is the grade value of* $x \in X$ *in the fuzzy set* A.

Definition 2.2 (Normal Fuzzy Set). *A fuzzy set* A *is called normal if there exists an element* $x \in X$ *whose membership value is one, i.e.,* $\mu_A(X) = 1$.

Definition 2.3 (Fuzzy Number). *A fuzzy number* A *is a subset of real line* R*, with the membership function* μ_A *satisfying the following properties:*

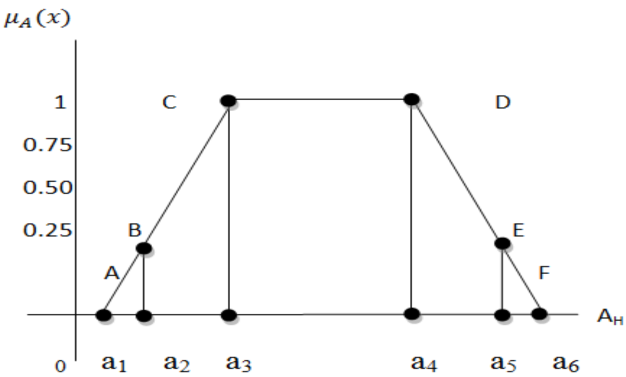
(1). $\mu_A(x)$ *is piecewise continuous in its domain.*

(2). A *is normal, i.e., there is* $ax_0 \in A$ *such that* $\mu_A(x_0) = 1$.

(3). A *is convex, i.e.,* $\mu_A(\lambda x_1 + (1-\lambda)x_2) = \min(\mu_A(x_1), \mu_A(x_2)).\vee\ x_1, x_2$ *in* X.

Definition 2.4 (Hexagonal Fuzzy Number). *A fuzzy number on* $\tilde{A}_h$ *is hexagonal fuzzy number denoted by* $\tilde{A}_h = (a_1, a_2, a_3, a_4, a_5, a_6)$*, where* $(a_1 \le a_2 \le a_3 \le a_4 \le a_5 \le a_6)$ *are real number satisfying* $a_2 - a_1 \le a_3 - a_2$ *and* $a_5 - a_4 \ge a_6 - a_5$ *and its membership function* $\mu\tilde{A}_h(x)$ *is given by*

$$\mu\tilde{A}_h(X) = \begin{cases} 0, & x \le a_1 \\ \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right), & a_1 \le x \le a_2 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x-a_3}{a_4-a_3}\right), & a_2 \le x \le a_3 \\ 1, & a_3 \le x \le a_4 \\ \frac{1}{2} - \frac{1}{2}\left(\frac{x-a_3}{a_4-a_3}\right), & a_4 \le x \le a_5 \\ \frac{1}{2}\left(\frac{x-a_6}{a_6-a_5}\right), & a_5 \le x \le a_6 \\ 0, & x \ge a_6 \end{cases}$$



3. Octagonal Fuzzy Numbers

$l_1(r)$ is a bounded left continuous non decreasing function over $[o, w_1], 0 \le w_1 \le k$ Octagonal fuzzy numbers are proposed by Malini.S.U and Kennedy Felbin .C in 2013.

Definition 3.1. *An octagonal Fuzzy Number denoted by* $\tilde{A}_w$ *is defined to be the ordered quadruple* $\tilde{A}_w = (l_1(r), s_1(t), s_2(t), l_2(r))$ *for* $r \in [0,k]$ *and* $t \in [k,w]$*, where,*

(1). $l_1(r)$ *is a bounded left continuous non decreasing function over* $[o, w_1]$, $0 \le w_1 \le k$.

(2). $s_1(t)$ is a bounded left continuous non decreasing function over $[k, w_2]$, $k \le w_2 \le w$.

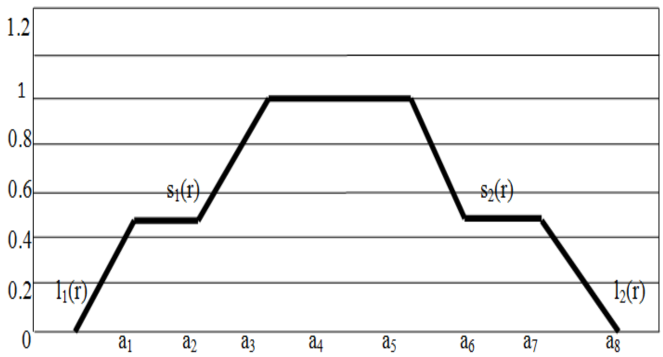
(3). $s_2(t)$ is a bounded left continuous non increasing function over $[k, w_2]$, $k \le w_2 \le w$.

(4). $l_2(r)$ is a bounded left continuous non increasing function over $[o, w_1]$, $0 \le w_1 \le k$.

Remark 3.2. *If $w = 1$ then the above defined number is called a normal octagonal fuzzy number.*

Definition 3.3. *A fuzzy number $\tilde{A}$ is a normal octagonal fuzzy number denoted by where $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8$ are real numbers and its membership function is given by,*

$$[\tilde{A}]_\alpha = \begin{cases} 0, & for\ X \le a_1 \\ k\left(\frac{x-a_1}{a_2-a_1}\right), & for\ a_1 \le X \le a_2 \\ k, & for\ a_1 \le X \le a_3 \\ k + (1-k)\left(\frac{x-a_3}{a_4-a_3}\right), & for\ a_3 \le X \le a_4 \\ 1, & for\ a_4 \le X \le a_5 \\ k + (1-k)\left(\frac{a_6-x}{a_6-a_5}\right), & for\ a_5 \le X \le a_6 \\ k, & for\ a_6 \le X \le a_7 \\ k\left(\frac{a_8-x}{a_8-a_7}\right), & for a_7 \le X \le a_8 \\ 0, & for\ X \ge a_8 \end{cases}$$



α-Cut of ANOFM: To find the α-cut of a normal octagonal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ for $\alpha \in [0, 1]$

$$\begin{cases} \left[a_1 + \left(\frac{\alpha}{k}\right)(a_2 - a_1), a_8 - \left(\frac{\alpha}{k}\right)(a_8 - a_7)\right] & for\ \alpha \in [0, k] \\ \left[a_3 + \left(\frac{\alpha-k}{1-k}\right)(a_4 - a_3), a_6 - \left(\frac{\alpha-k}{1-k}\right)(a_6 - a_5)\right] & for\ \alpha \in [k, 1] \end{cases}$$

Definition 3.4 (Arithmetic operations of Octagonal Fuzzy Number)**.** *Formation of an arithmetic operation is crucial in the study of fuzzy numbers. Note that every OFN is associated with two weights:w_1 and w_2. To avoid confusion, we use the notation w_{iA} for $i = 1, 2$ to represent w_1 and w_2 as the weights of the OFN.*

(a). Addition:

Let $A = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ and $B = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8)$ be two OFN's then, $A + B = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6, a_7 + b_7, a_8 + b_8)$ with $w_{i(A+B)} = \max(w_{iA}, w_{iB})$ for $i = 1, 2$.

(b). Substraction:

We define $A = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ and $B = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8)$ be two OFN's then, $(A - B) = (a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4, a_5 - b_5, a_6 - b_6, a_7 - b_7, a_8 - b_8)$ with, $w_{i(A-B)} = \max(w_i, , w_{iB})$ for $i = 1, 2$

(c). Scalar Multiplication:

Let, $A = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ be a OFN and $k \in R$ be any scalar. If $k \geq 0$, $kA = (ka_1, ka_2, ka_3, ka_4, ka_5, ka_6, ka_7, ka_8)$ and if $k \leq 0$, $kA = (ka_1, ka_2, ka_3, ka_4, ka_5, ka_6, ka_7, ka_8)$.

(d). Multiplication:

Let $A = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ and Let $B = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8)$ be two OFNs, then $AB = (a_1b_1, a_2b_2, a_3b_3, a_4b_4, a_5b_5, a_6b_6, a_7b_7, a_8b_8)$ with $w_{i(AB)} = \max(w_{iA}, w_{iB})$ $i = 1, 2$.

(e). Inverse:

We define the inverse of a OFN when all its components are non-zero. Suppose $A = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ is a OFN; then $A^{-1} \approx \frac{1}{A} \approx \left[\frac{1}{a_8}, \frac{1}{a_7}, \frac{1}{a_6}, \frac{1}{a_5}, \frac{1}{a_4}, \frac{1}{a_3}, \frac{1}{a_2}, \frac{1}{a_1}\right]$ if one of the components of a OFN becomes 0, then we cannot find its inverse.

(f). Division:

The division of two OFN's $A = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ and $B = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8)$ is approximated as the multiplication with inverse. $\frac{A}{B} \approx AB{-}1 \approx \left[\frac{a_1}{b_8}, \frac{a_2}{b_7}, \frac{a_3}{b_6}, \frac{a_4}{b_5}, \frac{a_5}{b_4}, \frac{a_6}{b_3}, \frac{a_7}{b_2}, \frac{a_8}{b_1}\right]$ AOFN A is divisible by B only when B is a non-null OFN having non-zero components.

(g). Exponent:

The exponent of a OFN $A = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ is defined as the power of its components. $A^n = (a_1^n, a_2^n, a_3^n, a_4^n, a_5^n, a_6^n, a_7^n, a_8^n)$ with n being a real number.

4. Octagonal Fuzzy MAtrix (OFM)

Definition 4.1. *A OFM of order $m \times n$ is defined as $A = (aoij)_{m\times n}$ where, $A_{oij} = (a_{ij1}, a_{ij2}, a_{ij3}, a_{ij4}, a_{ij5}, a_{ij6}, a_{ij7}, a_{ij8})$ is ij^{th} element of $\hat{A}$.*

Definition 4.2 (Operations on OFM). *Let $A = (a_{oij})$ and $B = (b_{oij})$ be two OFM of same order.then we have the following*

(a). $A + B = (a_{oij} + b_{oij})$.

(b). $A - B = (a_{oij} - b_{oij})$.

(c). For $A = (a_{oij})$ and $B = (b_{oij})$ then $A.B = (c_{oij})$, where $(c_{oij}) = \sum_{k=1}^{n} a_{oik}\ b_{okj}$ for $i = 1, 2, ..., m$; $j = 1, 2, ..., n$.

(d). $A^T = (a_{ij})$, the transpose of A.

(e). $kA = (ka_{ij})$, where k is any scalar.

Some special types of octagonal fuzzy matrices corresponding to classical matrices are now introduced in this section. However in fuzzy matrix algebra, we define some other types of octagonal fuzzy matrices and their algebraic properties.

5. Fundamental Properties of OFM

Here we introduce some fundamental properties of OFMs. Here we furnish the commutative and associative laws, which are well defined, for OFM under the arithmetic operations addition and multiplication.

Proposition 5.1. *For any 3 square OFMs L, M, N of the same order $s \times n$, we have the following results:*

(1). $L + M = M + L$

(2). $L + (M + N) = (L + M) + N$

(3). $L + L = 2L$

(4). $L - L = 0$, a null equivalent OFM.

(5). $L + 0 = 0 - L = L$

Proposition 5.2. *Let L and M be any two OFNs of the same order and s, t be any two scalars. Then,*

(1). $s(tL) = (st)L$

(2). $s(L + M) = sL + sM$

(3). $(s + t)L = s\ L + t\ L$

(4). $s(L - M) = sL - sM$

Proposition 5.3. *Let L and M be any two OFNs such that $L + M$ and $L.M$ are well defined. Then,*

(1). $(L^T)^T = L$

(2). $(L + M)^T = L^T + M^T$

(3). $(L.M)^T = M^T.L^T$

Proposition 5.4. *Let L and M be any two OFNs of the same order and s, t be any two scalars. Then,*

(1). $(sL)^T = sL^T$

(2). $(sL + tM)^T = sL^T + tM^T$

Proposition 5.5. *Let l be any square OFM. Then,*

(1). LL^T *and* L^T *are both symmetric.*

(2). $L + L^T$ *is a fuzzy symmetric OFM.*

(3). $L - L^T$ *is a fuzzy skew- symmetric OFM.*

6. Trace of a OFM

Definition 6.1. *The trace of a square OFM $A = (aij)$ is defined as the sum of the elements of the principle diagonal. It is denoted by $tr(A)$, i.e., $\sum_{i=1}^{n} a_{ii}$.*

Proposition 6.2. *The product of two pure lower triangular OFMs is also a pure lower triangular OFM.*

Proof. Let $L = (l_{ij})$ and $M = (m_{ij})$ be two pure lower triangular OFM of the same order k. Where, $l_{ij} = (l_{1ij}, l_{2ij}, l_{3ij}, l_{4ij}, l_{5ij}, l_{6ij}, l_{7ij}, l_{8ij})$ and $m_{ij} = (m_{1ij}, m_{2ij}, m_{3ij}, m_{4ij}, m_{5ij}, m_{6ij}, m_{7ij}, m_{8ij})$. Because L, M are both lower triangular OFM's. $l_{ij} = (0,0,0,0,0,0,0,0)$ and $m_{ij} = (0,0,0,0,0,0,0,0)$ for $i < j$; $i,j = 1,2,...,k$. Let, $Y = L.M = (y_{ij})$. Then

$$\begin{aligned} l_{ij} &= \sum_{r=1}^{k} l_{ir} m_{rj} \\ &= \sum_{r=1}^{k} (l_{1ij}, l_{2ij}, l_{3ij}, l_{4ij}, l_{5ij}, l_{6ij}, l_{7ij}, l_{8ij})(m_{1ij}, m_{2ij}, m_{3ij}, m_{4ij}, m_{5ij}, m_{6ij}, m_{7ij}, m_{8ij}) \end{aligned}$$

Now, it is enough to establish that $(y_{ij}) = (0,0,0,0,0,0,0,0)$. For $i < j$, $ij = 1,2,...,k$ for $i < r$, we have $p_{ir} = (0,0,0,0,0,0,0,0)$, $r = 1,2,...,i-1$ and $q_{ir} = (0,0,0,0,0,0,0,0)$, $r = i, i+1,..,k$. Therefore

$$(y_{ij}) = \sum_{r=1}^{k} l_{ir} m_{rj} = \sum_{r=1}^{i-1} l_{ir} m_{rj} + \sum_{r=i-1}^{k} l_{ir} m_{rj}$$

now,

$$\begin{aligned} (y_{ij}) &= \sum_{r=1}^{k} l_{ir} m_{rj} = \sum_{r=1}^{i-1} l_{ir} m_{rj} + \sum_{r=i-1}^{k} l_{ir} m_{rj} \\ &= l_{ii} m_{ii} = (0,0,0,0,0,0,0,0) \end{aligned}$$

Because, $p_{ir} = (0,0,0,0,0,0,0,0)$, $r = 1,2,...,i-1$ and $m_{ir} = (0,0,0,0,0,0,0,0)$, $r = i, i+1,...,k$. Hence the result. □

7. Determinant of a OFM

Definition 7.1. *The determinant of $n \times n$ OFM $A = (a_{oij})$ is denoted by $det(a)$ or $|A|$ and defined as*

$$|A| = \sum_{z \in sn} \left(\text{sgn} z . \prod_{i=i}^{n} a_{izi} \right)$$

Where, $a_{izi} = (a_{1izi}, a_{2izi}, a_{3izi}, a_{4izi}, a_{5izi}, a_{6izi}, a_{7izi}, a_{8izi})$ are OFN and sn denotes the symmetric group of all permutations of the indices $(1,2,...,n)$ and $sgnz = 1$ or -1 according the permutations $z = \begin{bmatrix} 1 & 2 & \dots & n \\ z(1) & z(2) & \dots & n \end{bmatrix}$ is even or odd respectively.

Proposition 7.2. *If any two columns(rows) of a square OFM A are interchanged, then only the sign of determinant $|A$ of A changes.*

Proof. Let$A = (a_i j)$ be a square OFM of order $n \times n$. if $L = l_{ij}$ is obtained from A by interchanging the $r^t h$ and $s^t h$ column $(r < s)$ of A, then it is clear that $l_{ij} = a_{ij}$, $i \neq r$, $i \neq s$ and $l_{rj} = a_{sj}$, $l_{sj} = a_{rj}$. Now,

$$\begin{aligned} |P| &= \sum_{z \in sn} sgnz(ls_{1(z1)}, l_{2z(2)}...l_{rz(r)}...l_{sz(s)}...l_{nz(n)}) \\ &= \sum_{z \in sn} s_{gnz}(a_{1(z1)}, a_{2z(2)}....a_{rz(r)}...a_{sz(s)}...a_{nz(n)}) \end{aligned}$$

Let, $\gamma = \begin{bmatrix} 1 & 2 & \dots & r & \dots & s & \dots & n \\ 1 & 2 & \dots & r & \dots & s & \dots & n \end{bmatrix}$. Then, γ is a transposition of interchanging r and s. Thus, γ is an odd permutation; thus, sgn$\lambda = -1$. Let $\gamma\sigma = \delta$. As σ runs through all permutations on $(1,2,...,n)$, δ also runs over the same permutations. Because $\sigma_{1\gamma} = \sigma_{2\gamma}$ or $\sigma_1 = \sigma_2$. Now,

$$\delta = \sigma\gamma = \begin{bmatrix} 1 & 2 & \dots & r & \dots & s & \dots & n \\ \sigma(1) & \sigma(2) & \dots & \sigma(r) & \dots & \sigma(s) & \dots & n \end{bmatrix} \begin{bmatrix} 1 & 2 & \dots & r & \dots & s & \dots & n \\ 1 & 2 & \dots & r & \dots & s & \dots & n \end{bmatrix}$$

Therefore $\sigma(i) = i$; $i \neq r, s$; $\sigma(r) = \sigma(s)$, $\sigma(s) = \sigma(r)$. Because γ is an odd permutation, δ is even or odd if σ is even or odd, i.e., sgnδ =-sgnσ. Then,

$$\begin{aligned}|L| &= \sum_{z\in sn} s_{gnz} \prod_{i=1}^{n} a_i\sigma_i \\ &= -\sum_{z\in sn} s_{\text{gnz}} \prod_{i=1}^{n} a_i\sigma_i \\ &= -|A|.\end{aligned}$$

Hence the result. □

8. Fuzzy Comparable OFM

Definition 8.1. *Let L and M be two OFMs of order $n \times n$. We say that L is comparable to M if either $L = M$ or $M = L$, i.e., when $l_{ij} \leq m_{ij} \rightarrow L \leq M$ or $l_{ij} \geq m_{ij} \rightarrow M \leq L$. when both are equal, we called them equivalent OFM.*

Proposition 8.2. *Let L and M be two OFMs of order $a \times n$. Then, we have the, for any OFM of order $n \times l$, we have $L \leq M$ which implies $L.N \leq M.N$.*

Proof. Let, $L = M$, then, we have $l_{ij} = m_{ij} \Rightarrow (l_{1ij}, l_{2ij}, l_{3ij}, l_{4ij}, l_{5ij}, l_{6ij}, l_{7ij}, l_{8ij}) \leq (m_{1ij}, m_{2ij}, m_{3ij}, m_{4ij}, m_{5ij}, m_{6ij}, m_{7ij}, m_{8ij})$. Now

$$\begin{aligned}L.M &= L_{ij}.M_{ij} \\ &= (l_{1ij}, l_{2ij}, l_{3ij}, l_{4ij}, l_{5ij}, l_{6ij}, l_{7ij}, l_{8ij}).(m_{1ij}, m_{2ij}, m_{3ij}, m_{4ij}, m_{5ij}, m_{6ij}, m_{7ij}, m_{8ij})\end{aligned}$$

(because $L \leq M$). This implies $L.N \leq M.N$, then, $l_{ij}.n_{ij} \leq m_{ij}.n_{ij} \Rightarrow (l_{1ij}, l_{2ij}, l_{3ij}, l_{4ij}, l_{5ij}, l_{6ij}, l_{7ij}, l_{8ij})$ $(n_{1ij}, n_{2ij}, n_{3ij}, n_{4ij}, n_{5ij}, n_{6ij}, n_{7ij}, n_{8ij}) \leq (m_{1ij}, m_{2ij}, m_{3ij}, m_{4ij}, m_{5ij}, m_{6ij}, m_{7ij}, m_{8ij})$ $(n_{1ij}, n_{2ij}, n_{3ij}, n_{4ij}, n_{5ij}, n_{6ij}, n_{7ij}, n_{8ij})$. This implies $L \leq M$. Hence the proof. □

9. Conclusion

In this article, special attention is paid to the OFM and corresponding OFM, along with the related mathematical expressions. Apply for elementary algebraic operations for OFM, then many types of OFM and their properties (trace, determinant, etc). Second, this paper addresses the nature of nilpotent OFM, with some interesting properties. There are several choices to develop the applications of such OFN. We are trying to investigate such applications.

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