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Mathematical Modeling on Unsteady MHD Oscillatory Blood Flow in an Inclined Tapered Stenosed Artery with Permeable Wall: Effects of Slip Velocity

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Abstract: The purpose of this work is to study the Mathemetcal modeling on unsteady MHD Oscillatory blood flow in an inclined tapered stenosed artery with permeable wall the effets of slip velocity. The governing equation are solved by analytically by using Bessel function. The analytic explicit expressions of axial velocity, volumetric flow rate and wall shear stress are given. The effects of slip condition, magnetic field, permeable wall and Oscillatory flow have been discussed. The obtained results for different values of parameters in to the problem under consideration, show that the flow is appreciably influenced by the presence of Reynolds number (R_e), Slip parameter (γ), Froude number (F_r), artery inclination (α), inclination of Magnetic field (θ) , Darcy number (D_a), permeability (k) and Hartmann number (M). Some of the found results show that the flow patterns with slip effects in non-tapered region ($\xi = 0$), converging region ($\xi < 0$) and diverging region ($\xi > 0$) are effectively influenced by the presence of magnetic field and change in learning of artery, with time varying pressure gradient. Results for the effect of permeability on these flow characteristics are shown graphically and discussed briefly.

MSC: 58D30, 92B05, 92C35.

Keywords: Darcy number, Bessel function, permeable wall , Inclined tapered artery, Slip velocity.

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1. Introduction

The investigations of blood flow through inclined tapered artery is considerable importance in many cardiovascular diseases particularly atherosclerosis (medically called stenosis). In developed and developing countries, one of the major health hazards is atherosclerosis, which refers to the narrowing of the arterial lumen. The main disadvantage is using a tapered geometry however, is the much greater energy losses which may leads to diminished blood flow through the tapered grafts. It is important therefore the losses are quantified and taken account in the design of tapered grafts. The arterial MHD pulsatile flow of blood under periodic body acceleration has been studied by Das [8]. A mathematical model for blood flow through inclined stenosed artery is studied by Siddiqui and Geeta (2016). Mathematical modeling of blood flow in an inclined tapered artery under MHD effect through porous medium was analyzed by Ajaykumar [3]. Slip effects on the Unsteady MHD pulsatile blood flow through porous medium in an artery under the effect of body acceleration is studied by Islam [1]. A numerical analysis for the effect of slip velocity and stenosis shaper on non-Newtonian flow of blood is worked by Bhatnagar (2015). Kumar and Kumar [9] gave an idea on oscillatory MHD flow of blood through an artery with mild

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stenosis. Jain et al (2010) have dealt a mathematical modeling of blood flow in an artery under MHD effect through porous medium. Tripathi [10] investigated a mathematical model for blood flow through an inclined artery under the influence of an inclined magnetic field. M. Chitra [5] discussed on Mathematical modeling of pulsatile flow of non-Newtonian fluid through an elastic artery: Effects of elasticity. Again, M. Chitra [6] have studied the Oscillatory flow of blood in porous vessel of a stenosed artery with variable viscosity: Effects of magnetic field. Mekheimer [11] have presented the micro polar fluid model for blood flow through a tapered artery with a stenosis.

In view of the above mentioned fact, we are analyzing the characteristics of the blood flow of an inclined tapered artery with mild stenosis under the influence of an inclined magnetic field through porous medium. The effect of slip condition on unsteady blood flow through a porous medium has been studied under the influence of Oscillatory flow and time varying pressure gradient. The analysis is carried out by employing analytically bessel functions and some important predictions have been made basing upon the unsteady. This unsteady with slip effect can play a big role in the conclusion of axial velocity, wall shear stress and volume flow rate through porous medium. This study is also useful for evaluating the role of porosity. This is also useful for evaluating the role of porosity and slip condition when the body is subjected to magnetic resonance imaging (MRI).

2. Formulation of the Problem

We consider unstudy, laminar and fully developed flow of blood through inclined tapered artery, with the presence of mild stenosis through porous medium under the influence of time varying pressure gradient. It is assume that the formation of stenosis which is symmetrical about the axis, but non symmetrical with respect to radial co-ordinates, and it depends upon the height and location of the constriction, formed at the innermost wall and the axial wall. It is assumed that the wall of the tapered tube is rigid. There is no loss of generally in considering a rigid artery as due to the formation of a stenosis, the elasticity of the arterial wall gets reduced. Further, the artery length is assumed to be large enough as compared to its radius, so that the entrance and special wall effects can be neglected. The geometry of an arterial non-symmetrical stenosis in a tapering wall can be expressed (Mekheiner [11]) as

$$R(z) = \begin{cases} h(z)\left[1-\eta(b^{n-1}(z-a)-(z-a)^2)\right] & ;\ a \le z \le a+b \\ h(z) & ;\ \text{otherwise} \end{cases} \tag{1}$$

with

$$h(z) = h_0 + \xi z \tag{2}$$

[where $R(z)$ is the radius of the stenosed portion of arterial segment and $h(z)$ radius of tapered arterial segment in the stenotic region, h_0 is the radius of the non tapered artery in the non stenotic region, ξ is the tapering parameter, b is taking the length of the stenosis, and $(n \ge 2)$ is being a parameter determining the shape constriction and referred to as a shape parameter]. Here we are using the parameter η which is given by :

$$\eta = \frac{\delta}{h_0 b^n}\left(\frac{\frac{1}{n^{(n-1)}}}{n-1}\right) \tag{3}$$

where δ denotes the maximum height of the stenosis to be found at

$$z = a + \frac{b}{\frac{1}{n^{(n-1)}}} \tag{4}$$

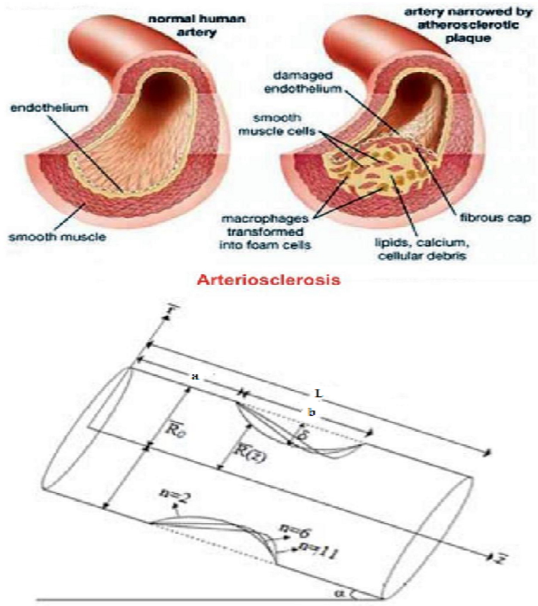

Figure 1. **Geometry in inclined stenosed artery with axially non-symmetrical stenosis**

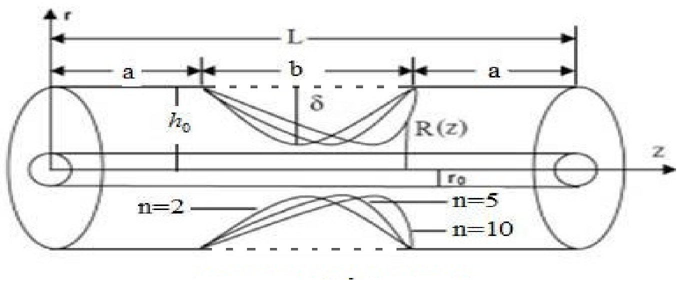

Figure 2. **Geometry of construction**

Here the body fluid is assumed to behave as a Newtonian fluid (Schlitchting and Gerstein 2004) . The equation (as obtained from Navier- Stokes equation of motion for various fluids) describing the unsteady flow of Newtonian fluid is given by (Schlitchting and Gerstein 2004):

$$\frac{\partial p}{\partial r} = 0 \tag{5}$$

$$\frac{\partial p}{\partial \theta} = 0 \tag{6}$$

3. Mathematical Solution

As per the published literature and available physiological data, blood flow in the neighborhood of the vessel wall can be considered as Newtonian, if the shear rate of blood is high enough. However, the shear rate is very small towards the center of the artery (circular tube), the non-Newtonian behavior of blood is more evident (Mishra et al 2007). The unsteady flow of blood through the cylindrical artery inclined at an angle α can write as follows:

$$\nabla.\bar{u} = 0 \tag{7}$$

$$\frac{\partial \bar{u}}{\partial t} + \rho \bar{F}\bar{u} = -\nabla p + \mu\left[\frac{\partial^2 \bar{u}}{\partial \bar{r}^2} + \frac{1}{r}\frac{\partial \bar{u}}{\partial \bar{r}}\right] - \frac{\mu \bar{u}}{\bar{k}} + \bar{J}X\bar{B} \tag{8}$$

Boundary conditions for the problem stated above may be listed as

$$\left.\begin{array}{l}\textbf{Slip condition} \;\; : \; u = u_B \; and \; \frac{\partial u}{\partial r} = \frac{\gamma}{\sqrt{D_a}}(u_B - u_p) \text{ at } r = R(z) \\ \textbf{Symmetry condition} : \; \frac{\partial u}{\partial r} = 0 \text{ at } r = 0\end{array}\right\} \tag{9}$$

The above equation (3) transformed in the term as

$$\frac{\partial \bar{u}}{\partial t} + \rho g \; sin \; \alpha \bar{u} + \mu\left[\frac{\partial^2 \bar{u}}{\partial \bar{r}^2} + \frac{1}{r}\frac{\partial \bar{u}}{\partial \bar{r}}\right] - \frac{\mu \bar{u}}{\bar{k}} + \sigma B_0^2 \bar{u} \tag{10}$$

where g is the acceleration due the gravity, μ is the viscosity of blood, k is the permeability of porous medium, ρ is the fluid density, α is the inclination of an artery, B_0 is an applied magnetic field with an inclination θ. The non-dimensional variables are

$$r = \frac{\bar{r}}{R_0},\; z = \frac{\bar{z}}{R_0},\; R = \frac{\bar{R}}{R_0}, u = \frac{\bar{u}}{u_0} \; p = \frac{\bar{p}R_0^2}{b\mu u_0},\; Re = \frac{\rho u_0 R_0}{\mu},\; k = \frac{\bar{k}}{R_0^2}, F_r = \frac{u_0^2}{gR_0},\; M^2 = \frac{\sigma R_0^2 B_0^2}{\mu} \tag{11}$$

Where $\bar{u}$ is velocity component in the axial $\bar{z}$ and radial $\bar{r}$ directions, $\bar{p}$ the pressure, ρ is the density, R_0 is the radius of the normal artery δ is the maximum height of the stenosis, Re is the Reynolds number, Fr is the Froude number, M is the Hartmann number. Substituting (11) in , we can get a dimensionless form for as follows

$$\frac{\partial u}{\partial t}\left(\frac{R_0^2}{\mu}\right) = -\frac{\partial p}{\partial z}\left(\frac{b}{RR_0}\right) + \left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right] - \left[\frac{1}{k} + \frac{R_e}{F_r} sin\alpha + M^2 cos\theta\right]u \tag{12}$$

As the flow is unsteady and axi-symmetric, let the solution for $u(r,t)$ and p be set in the forms

$$u(r,t) = u(r)e^{i\omega t} \text{ and } \frac{\partial p}{\partial z} = P(z)e^{i\omega t} \tag{13}$$

Were P is a constant. Substitute equation (13) in (12) we can have a second order ordinary differential equations as follows

$$\frac{d^2 u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \beta^2 u = -C \tag{14}$$

where $\beta^2 = \frac{1}{k} + \frac{R_e}{F_r} sin\alpha + M^2 cos\theta - \frac{i\omega R_0^2}{\mu}$ and $C = P(z)\left(\frac{b}{RR_0}\right)$. The solution of second order differential equation (14) by using boundary conditions, we get

$$u(r,t) = \frac{C}{\beta^2}\left(1 + \frac{J_0(i\beta.r)}{\gamma J_1(i\beta.R(z)).i\beta + J_0(i\beta.r)}\right)e^{i\omega t} \tag{15}$$

Where J_0 and J_1 are the modified Bessels function of the zero order. The Volumetric flow rate Q of the blood flow of stenotic analysis

$$Q = 2\pi R_0^2 \int_0^{R(z)} r u dr$$

$$Q = 2\pi R_0^2 \left[\frac{R^2(z)}{2\beta} + \frac{R(z)}{i\beta}\frac{J_1(i\beta.R(z))}{\gamma J_1(i\beta.R(z)).i\beta + J_0(i\beta.R(z))}\right]e^{i\omega t} \tag{16}$$

The Wall shear stress is defined by,

$$\tau = \left(-\mu\frac{du}{dr}\right)$$

$$\tau = \frac{-C^2}{\beta}\left[\frac{J_1(i\beta.R(z))}{\gamma J_1(i\beta.R(z)) + J_0(i\beta.R(z))}\right]e^{i\omega t} \tag{17}$$

4. Results and Discussion

Most of the theoretical result such as the effects of various parameters permeability (k), Froude number (Fr), inclination angle of artery (α), Slip parameter (γ), time (t), Hartmann number (M), the inclination of magnetic field θ on the flow characteristics wall shear stress (τ) , axial velocity (u) and volumetic flow rate (Q) are computed graphically. All graphs are plotted by using mathematical software MATLAB for the fixed value $Re = 0.1, Fr = 0.1, M = 2, 3$ and 4, $k = 1, 2$ and 3, $t = 0.1, 0.2$ and 0.3, $\delta = 0.10, 0.15, 0.20$.

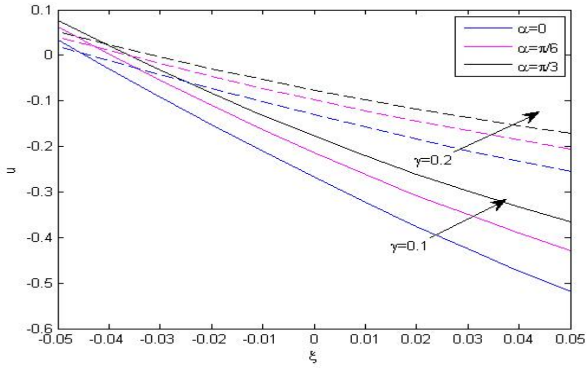

Figure 3. Variation of axial velocity u with tapering angle (ξ) for different values of artery angle (α) with slip parameter (γ).

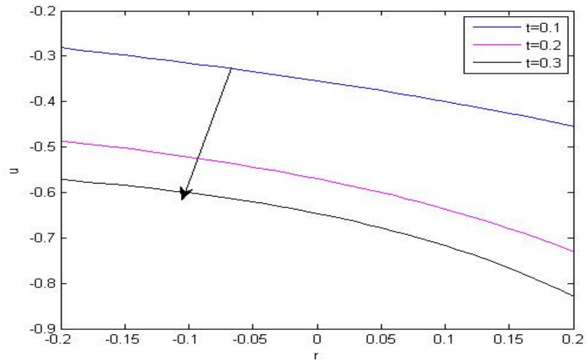

Figure 4. Variation of axial velocity u with r different values of time t

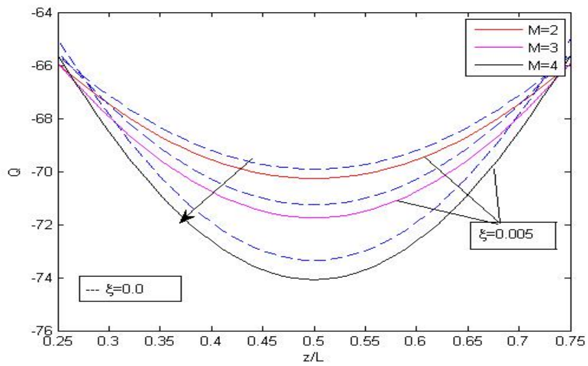

Figure 5. Variation of volume flow rate with z/L and magnetic field for different values of tapering angle ξ

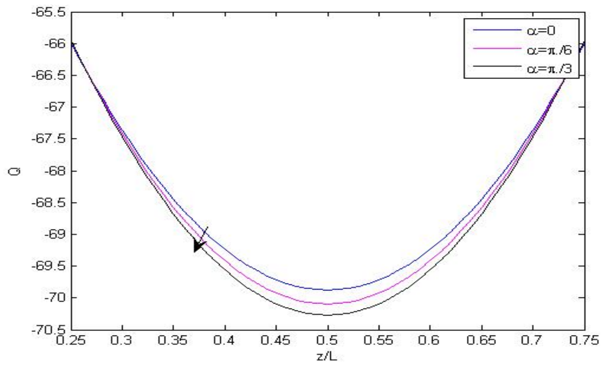

Figure 6. Variation of volume flow rate with z/L for different values of angle of inclination α.

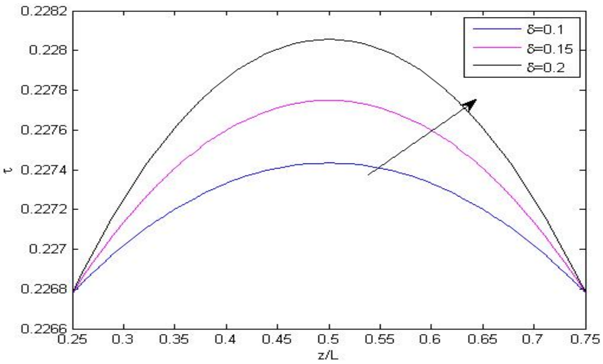

Figure 7. Variation of wall shear stress with z/L for different values thickness of stenosis δ.

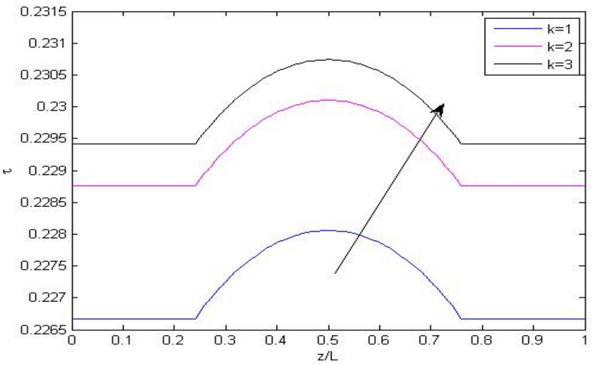

Figure 8. Variation wall shear stress with z/L for different values of permeability k.

♣ To observe from the figure (3) that as the argumentation in the axial velocity shows the remarkable changes with the inclination of artery, It has been observed the with the increase of inclination artery the curve representing the axial flow velocities also increasing.

♣ Figure (4) reveals that the variation of axial velocity with radial position r for different value time t. It has been observed that the axial velocity decreases of increasing the value time t.

♣ In figure (5) with the increase of magnetic field the volume flow rate shows a reverse behavior. It is observed that with the increase of magnetic field, the curve representing the volume flow rate do shift towards the origin for a converging region, while they shift away from the origin for a non tapered and diverging tapered artery.

♣ Variation of Volumetric flow rate with z/L for different values the angle of inclination α, In figure (6) shows that the inclination α increases as the decreases of volumetric flow rate.

♣ It is quite interesting to observe from the figure (7) that as the variation of wall shear stress τ with length of artery z/L for different values thickness of stenosis δ. It has been noticed that with the increase of thickness of stenosis the wall shear stress is also increases.

♣ In figure (8) illustrates that variation of wall shear stress with z for different value permeability (k). It is clear that the wall shear stress increases with the increase of permeability.

5. Conclusion

This study we have developed slip effects on the unsteady MHD Oscillatory blood flow in an inclined tapered artery with mild stenosis through porous medium. Analytical expressions of flow variables are obtained and variations of wall shear stress, volumetric flow rate and axial velocity are examined graphically. Since, this study has been carried out for a situation when the human body is subjected to an external magnetic field. The unstudy is also useful for evaluating the roll of permeability. It is observed that magnetic field reduces the flow characteristics amazingly. Also the height of stenosis significantly affects the wall shear stress, axial velocity and volumetric flow rate. This investigation may be helpful for the practioners to treat hypertension patient through magnetic therapy and to understand the flow of blood under stenotic conditions. The slip condition plays an important role in shear skin, spurt and hysteresis effects. The fluids that exhibit boundary slip have important technological applications such as in polishing valves of artificial heart and internal cavities.

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