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Square Difference Labeling for Pyramid Graph and its Related Graphs

P. Jagadeeswari[1], K. Ramanathan[2,*] and K. Manimekalai[3]

1 Research Scholar, Department of Mathematics, Mother Teresa Women's University, Kodaikanal, Tamil Nadu, India.
2 Department of Mathematics, KCG College of Engineering and Technology, Chennai, Tamil Nadu, India.
3 Department of Mathematics, Bharath Institute of Higher Education and Research, Chennai, Tamil Nadu, India.

Abstract: In this paper, we prove the characterization of pyramid graph ($n \geq 3$), hanging pyramid graph and the associated graphs admits square difference labeling.

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1. Introduction

Simple, undirected, finite graphs are taken for the present work and we refer [4] for terminologies and notations. Gallian [3] updated a dynamic survey on graph labeling. Many graph labeling methods are discussed in detailed by Rosa [2]. Earlier, the square difference labeling was stated in [7] and some of the graphs were discussed in [1, 8]. Square difference labeling of the key graph is discussed in [10]. The above-mentioned graph is also proved for middle graph, total graph, $P_m \cup C_n$ and $P_m \cup S_n$ in [9]. Jude Annie Cythania and Poorani [5] established Circulant Network is square difference graph. Lee and Wang [6] defined pyramid graph and proved it is graceful labeling. Here we examine the pyramid and its related graphs for square difference labeling. We afford a brief summary of definitions, which are mandatory for the existing work.

Definition 1.1. *A function of a graph $G = (p, q)$ is said to be a square difference graph, if it admits a bijective function $f : V(G) \to \{0, 1, 2, \dots p-1\}$ such that the induced function $f^* : E(G) \to N$ given by $f^*(xy) = |[f(x)]^2 - [f(y)]^2|$, $\forall\, xy \in E(G)$ and the edge labels are distinct [7] and is pictorial below.*

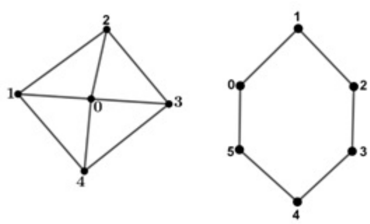

Figure 1. **Square difference graph for W_4 and C_6**

* E-mail: krnath_11@yahoo.com

Definition 1.2 ([6]). *A graph which procured by set out the vertices into a fixed number of rows with i vertices in the i^{th} line and every line the j^{th} apex in that row is joined to the j^{th} and $(j+1)^{th}$ vertex of the next line is called pyramid graph and is denoted by J_n. It is illustrated in Figure 2.*

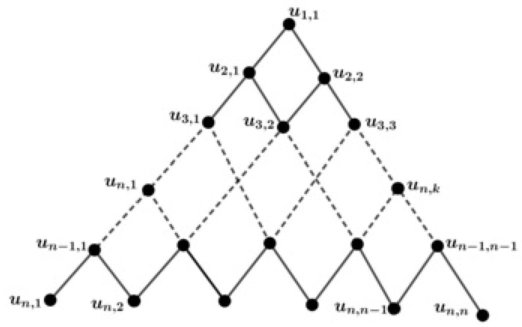

Figure 2. **Pyramid graph J_n**

Definition 1.3. *By given in place of each vertex of the path by the n copies of graph G is called the path union of the graph G and is denoted by $P(n, G)$.*

Definition 1.4. *Hanging pyramid graph obtained by attaching the apex of a pyramid graph to a new pendent edge and is denoted by HJ_n.*

Definition 1.5 ([11]). *A rooted graph is attained by a graph G in which a vertex is noted by other vertices and that apex is said to be the root of G. The graph $G^{(r)}$ obtained by determining the roots of r copies of G is called one point union of r copies of G.*

2. Main Results

In this segment, we obtain some outcomes for pyramid and its associated graphs.

Theorem 2.1. *J_n is a square difference graph.*

Proof. Let G be the pyramid graph with the vertex set $V = \{u_{i,j}/1 \leq i \leq n; 1 \leq j \leq i\}$ and the edge set $E = \{u_{i,j}u_{i+1,j}, u_{i,j}u_{i+1,j+1}/1 \leq i \leq n-1; 1 \leq j \leq i\}$. Then $|V(G)| = \frac{(n^2+n)}{2} = p$ and $|E(G)| = (n^2 - n) = q$. Let the function $f : V \to \{0, 1, 2, \ldots, p-1\}$ defined as $f(u_{i,j}) = \frac{1}{2}i(i-1) + (j-1)$ for $1 \leq i \leq n$; $1 \leq j \leq i$. Clearly, the above given labeling satisfies the condition of square difference. Here all the edge labels are distinct and also in increasing sequence. Hence the theorem is verified. The example of J_5 is shown in Figure 3.

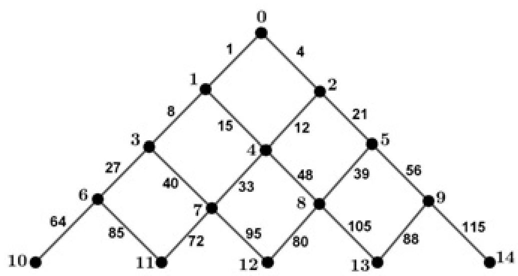

Figure 3. **Pyramid graph J_5**

□

Theorem 2.2. *For $n \geq 3$, HJ_n admits square difference labeling.*

Proof. Let $HJ_n = G(V,E)$ be the Hanging pyramid graph with $V = V_1 \cup V_2$, where, $V_1 = \{u_0\}$ and $V_2 = \{u_{i,j}/1 \leq i \leq n; 1 \leq j \leq i\}$ and $E = E_1 \cup E_2$, where, $E_1 = \{u_0 u_{i,j}/i = j = 1\}$ and $E_2 = \{u_{i,j}u_{i+1,j}, u_{i,j}u_{i+1,j+1}/1 \leq i \leq n-1; 1 \leq j \leq i\}$. Clearly, for $n \geq 3$,

$$|V(G)| = \left[\frac{1}{2}(n^2+n)+1\right] \text{ and } |E(G)| = n^2 - n + 1$$

Now, consider a function $f : V \to \{0, 1, 2 \ldots, \frac{n}{2}(n+1)\}$ be a bijection defined as follows:

$$f(u_0) = 0$$
$$f(u_{i,j}) = \frac{1}{2}(i^2 - i) + j, \ 1 \leq i \leq n; 1 \leq j \leq i$$

By the above function, the induced function f^* is in non-decreasing sequence and are distinct. Therefore the Hanging pyramid graph HJ_n admits a square difference labeling. For instance, HJ_5 for square difference labeling is illustrated below:

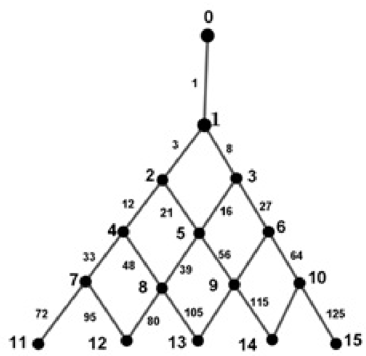

Figure 4. **Hanging pyramid graph HJ_5**

□

Theorem 2.3. *$P(r, J_n)$ is square difference graph.*

Proof. Let $G(V,E) = P(r, J_n)$ be the path union of 'r; copies of pyramid graph. Define $V = V_1 \cup V_2$, where,

$$V_1 = \{u_{1,1}^{(k)}/1 \leq k \leq r\}$$
$$V_2 = \{u_{i,j}^{(k)}/1 \leq i \leq n, 1 \leq j \leq i; 1 \leq k \leq r\} \text{ and } E = E_1 \cup E_2, \text{ where,}$$
$$E_1 = \{u_{1,1}^{(k)}u_{1,1}^{(k+1)}/1 \leq k \leq r-1\};$$
$$E_2 = \{u_{i,j}^{(k)}u_{i+1}^{(k)}, u_{i,j}^{(k)}u_{i+1,j+1}^{(k)}/1 \leq i \leq n-1; 1 \leq j \leq i; 1 \leq k \leq r\}$$

It is obvious that,

$$|V(G)| = r\left[\frac{(n^2+n)}{2}\right] = p \text{ and } |E(G)| = r(n^2-n) + r - 1 = q, \text{ for } n \geq 3.$$

We define a bijection $f : V \to \{0, 1, 2, \ldots, p-1\}$ as follows:

$$f(u_{i,j}^{(k)}) = \frac{r(i^2-i)}{2} + (k-1)i + (j-1), 1 \leq i \leq n; 1 \leq j \leq i, 1 \leq k \leq r.$$

By the above labeling pattern, it is easily observed that the function satisfies the condition of square difference labeling and also $f^*(uv)$ is an increasing function. Here, all the edge labeling are distinct. Hence, the graph $P(r, J_n)$ is square difference graph. □

Example 2.4. *The graph $P(3, J_5)$ is square difference graph.*

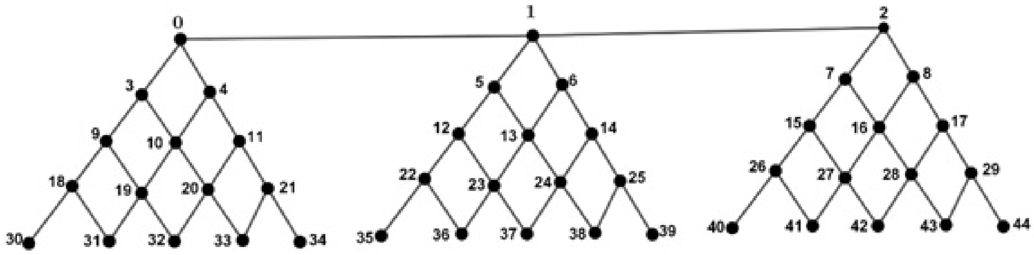

Figure 5. **Sdl of** $P(3, J_5)$

Theorem 2.5. *For $n \geq 3$, the graph $P(r, HJ_n)$ admits square difference labeling.*

Proof. Consider the graph $G(V, E) = P(r, HJ_n)$ be the path union of r copies of hanging pyramid graph. Let the vertex set$V = V_1 \cup V_2$, where, $V_1 = \{u_k/1 \leq k \leq r\}$ and $V_2 = \{u_{i,j}^{(k)}/1 \leq i \leq n; 1 \leq j \leq i; 1 \leq k \leq r\}$ and the edge set

$$E = \{u_k u_{k+1}/1 \leq k \leq r-1\} \cup \{u_k u_{1,1}^{(k)}/1 \leq k \leq r\} \cup \{u_{i,j}^{(k)} u_{i+1,j}^{(k)}, u_{i,j}^{(k)} u_{i+1,j+1}^{(k)}/1 \leq i \leq n-1; 1 \leq j \leq i; 1 \leq k \leq r\}$$

In graph G,

$$|V(G)| = r\left[\frac{(n^2+n)}{2} + 1\right] \text{ and } |E(G)| = r(n^2 - n + 1) + (r-1), \text{ for } n \geq 3,$$

Now, define a bijection $f : V \to \left\{0, 1, 2 \ldots, \left(\frac{r(n^2+n+2)}{2}\right) - 1\right\}$ as follows:

$$f(x_k) = k - 1, \ 1 \leq k \leq r$$
$$f(x_{1,1}^{(k)}) = r + k - 1/1 \leq k \leq r$$
$$f(x_{i,j}^{(k)}) = f(x_{i-1,i-1}^{(r)} + (k-1)i + j; \ 2 \leq i \leq n; \ 1 \leq j \leq i,$$

and also define the injective $f^* : E(G) \to N$ is $f^*(xy) = |[f(x)]^2 - [f(y)]^2| \ \forall \ xy \in E(G)$. Clearly, $f^*(xy)$ is an increasing sequence order. Hence all the weights of the edges are distinct and the graph $P(r, HJ_n)$ admits square difference labeling. □

Example 2.6. *The graph $P(3, HJ_5)$ is square difference graph*

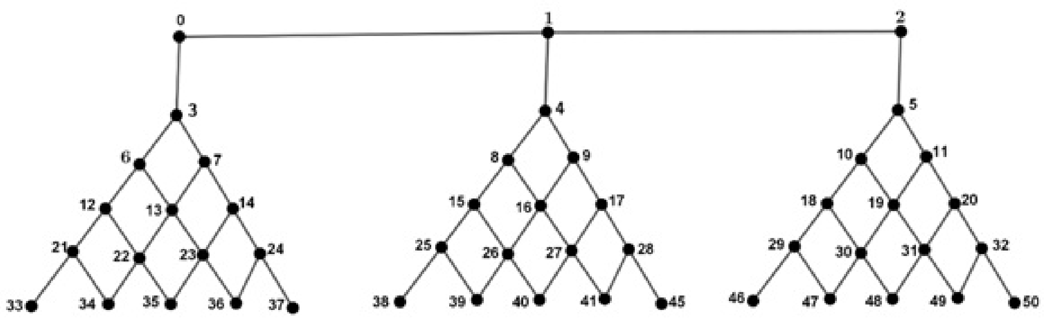

Figure 6. $P(3, HJ_5)$

Theorem 2.7. *The graph $J_n^{(r)}$,$(n \geq 3)$ admits Square difference labeling.*

Proof. Consider the graph G as the one point union of r copies of pyramid graph J_n. Let us define the vertex set $V = V_1 \cup V_2$, where, $V_1 = \{u_{1,1}\}$ and $V_2 = \{u_{i,j}^{(k)}/2 \leq i \leq n; 1 \leq j \leq i; 1 \leq k \leq r\}$ and the edge set $E = \{u_{i,j}^{(k)}u_{i+1,j}^{(k)}, u_{i,j}^{(k)}u_{i+1,j+1}^{(k)}/1 \leq i \leq n-1; 1 \leq j \leq i; 1 \leq k \leq r\}$. Also consider $u_{1,1}$ as the root vertex of G. Then, For $n \geq 3$,

$$|V(G)| = r\left[\frac{n(n+1)}{2}\right] - r + 1 = p \text{ and } |E(G)| = r(n^2 - n) = q$$

Let the bijective function $f : V \to \{0, 1, 2, \ldots, p-1\}$ is given below

$$f(u_{1,1}) = 0$$
$$f(u_{2,j}^{(k)}) = (k-1)i + j, i = 2, 1 \leq j \leq i$$
$$f(u_{i,j}^{(k)}) = f(u_{i-1,i-1}^{(r)} + (k-1)i + j, 3 \leq i \leq n, 1 \leq j \leq i,$$

Since the above defined function satisfies the condition of square difference labeling, the edge labels of graph G are in ascending sequence and distinct. Hence, the graph $J_n^{(r)}$ admits square difference labeling. For instance, the square difference graph of $J_5^{(4)}$ is given in Figure 7.

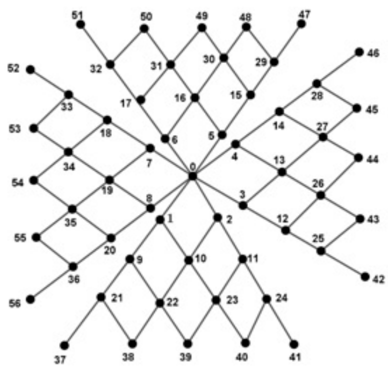

Figure 7. **Sdl of** $J_5^{(4)}$

□

Theorem 2.8. *The graph $HJ_n^{(r)}$ is square difference graph.*

Proof. Let $G(V, E) = HJ_n^{(r)}$ be the one point union of r copies of hanging pyramid graph with

$$V = \{u_0\} \cup \{u_{i,j}^{(k)}\}, \quad i = 1, 2, \ldots n; \; j = 1, 2, \ldots i \; \textit{ and}$$
$$E = \{u_0u_{1,1}^{(k)}/1 \leq k \leq r\} \cup \{u_{i,j}^{(k)}u_{i+1,j}^{(k)}, u_{i,j}^{(k)}u_{i+1,j+1}^{(k)}/1 \leq i \leq n-1; \; 1 \leq j \leq i; \; 1 \leq k \leq r\},$$

Also consider u_0 be the root vertex of G. Then, For $n \geq 3$,

$$|V(G)| = r\left[\frac{n(n+1)}{2}\right] + 1 \text{ and } |E(G)| = r(n^2 - n + 1)$$

Now, the bijective function $f : V \to \left\{0, 1, 2, \ldots \left[\frac{r(n^2+n)}{2}\right]\right\}$ is given as follows:

$$f(u_0) = \frac{r(n(n+1))}{2},$$
$$f(u_{i,j}^{(k)}) = \frac{r(i-1)i}{2} + (k-1)i + (j-1); 1 \leq i \leq n; 1 \leq j \leq i; i \leq k \leq r,$$

It has seen clear, that the defined function comes under the condition of sdl. Thus the edge labels are non-decreasing and not repeated. Hence the graph $HJ_n^{(r)}$ is a square difference graph. □

Example 2.9. *The graph $HJ_5^{(4)}$ is square difference graph.*

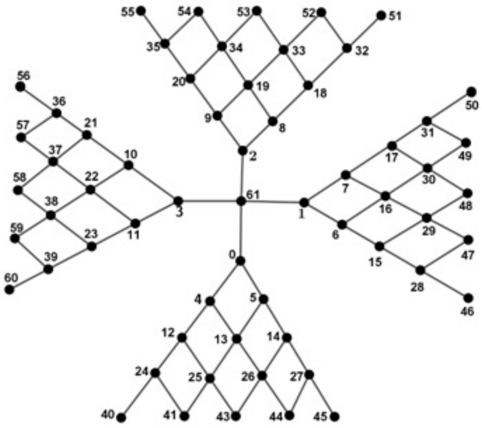

Figure 8. **Square difference graph of $HJ_5^{(4)}$**

3. Conclusion

In this paper we studied some characterization of the pyramid graph, hanging pyramid graph, path union of pyramid graph ($n \geq 3$), path union of hanging pyramid graph, one point union of pyramid graph, one point union of hanging pyramid graph are square difference graph. Some other labeling is also verified for these graphs.

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