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Difference Cordial Labeling of Arrow Graph

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Abstract: In this work we establish the difference cordial labeling of arrow graphs A_n^2, DA_n^2, A_n^m and DA_n^m.

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1. Introduction

We deal here with finite, simple and undirected graphs. Graph labeling connects many branches of mathematics and is consider one of the important blocks of graph theory. Labeled graphs plays an important role in communication network addressing and models for constraint programming over finite domains [2]. Cordial was first introduced in 1987 by Cahit [1], and then there has been a major effort in this area made this topic growing steadily and widely [2]. Initially the difference cordial labeling (DCL) was stated in [5] and some of the standard graphs were proved for the same in [6–9]. Seoud and Salman [10, 11] studied some characterization of ladder graphs for DCL. V.J. Kaneria et al [4] introduced a new graphs called arrow graphs.We examine the difference cordial labeling of A_n^2, DA_n^2, A_n^m and DA_n^m. Terms are used as defined in [3].

Definition 1.1 ([5]). *Let a bijective function f, mapping from $V \to \{1, 2, \dots, p\}$ in a graph G satisfies the condition $|f(u) - f(v)|$ for assigning the edge labeling and $|e_f(0) - e_f(1)| \leq 1$ where $e_f(1)$ and $e_f(0)$ denote the number of edges labeled with 1 and not labeled with 1 respectively is called a difference cordial labeling graph.*

Example 1.2. *The labeling of diagonal ladder is shown in figure 1*

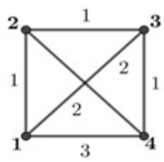

Figure 1. Difference cordial labeling for DL_2

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Definition 1.3 ([4])**.** *In graph, $P_m \times P_n$ (grid graph on mn vertices) vertices $v_{1,1}, v_{2,1}, v_{3,1}, \ldots, v_{m,1}$ and vertices $v_{1,n}, v_{2,n}, v_{3,n}, \ldots, v_{m,n}$ are known as superior vertices from both the ends as in figure 2.*

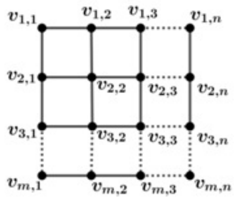

Figure 2. **Grid graph**

Definition 1.4 ([4])**.** *An arrow graph A_n^m with width m and length n is obtained by joining a vertex v with superior vertices of $P_m \times P_n$ by m new edges from one end is shown in figure 3.*

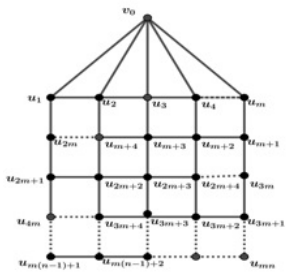

Figure 3. **Arrow graph A_n^m**

Definition 1.5 ([4])**.** *A Double arrow graph DA_n^m with width m and length n is obtained by joining two vertices v and m with superior vertices of $P_m \times P_n$ by $m + m$ new edges from both ends in figure 4.*

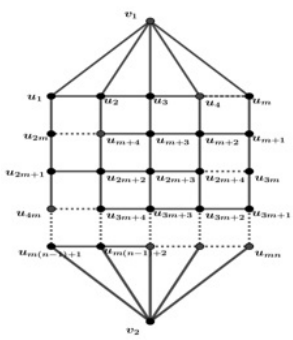

Figure 4. **Double arrow graph A_n^m**

2. Main Results

In this section, we present some characterization of arrow graphs

Theorem 2.1. *A_n^2 is difference cordial graph.*

Proof. Let A_n^2 be an arrow graph combined by a vertex v with the superior vertices of $P_2 \times P_n$ by 2 new edges. Define, $V = \{v_0, v_i \mid 1 \leq i \leq 2n\}$ and $E = \{v_i v_{i+1} \mid 1 \leq i \leq 2n-1\} \cup \{v_0 v_1, v_{2n} v_0\} \cup \{v_{i+1} v_{2n-i} \mid 0 \leq i \leq n-1\}$. Then $|V(G)| = 2n+1$ and $|E(G)| = 3n$. Define a function $f : V(G) \to \{1, 2, \ldots, 2n+1\}$ as, for $r = 1, 2, \ldots, N$

Case 1:

$$f(v_0) = 1$$
$$f(v_i) = i+1,\ 1 \le i \le \frac{3n-2}{2}$$
$$f\left(v_{\frac{3n+2i-2}{2}}\right) = \frac{3n}{2} + 2i,\ 1 \le i \le \left\lfloor \frac{n+2}{4} \right\rfloor$$

If $n = 4r$

$$f\left(v_{\frac{3n+2i-2}{2}+\lfloor \frac{n+2}{4} \rfloor}\right) = 2(n-i)+3,\ 1 \le i \le \left\lceil \frac{n+2}{4} \right\rceil$$

If $n = 4r+2$

$$f\left(v_{\frac{3n+2i-2}{2}+\lfloor \frac{n+2}{4} \rfloor}\right) = 2(n-i+1),\ 1 \le i \le \left\lceil \frac{n+2}{4} \right\rceil$$

Case 2:

$$f(v_0) = 1$$
$$f(v_i) = i+1,\ 1 \le i \le \frac{3n-1}{2},$$
$$f\left(v_{\frac{3n+2i-1}{2}}\right) = \frac{3n+1}{2} + 2i,\ 1 \le i \le \left\lfloor \frac{n+1}{4} \right\rfloor$$

If $n = 4r+1$

$$f\left(v_{\frac{3n+2i-1}{2}+\lfloor \frac{n+1}{4} \rfloor}\right) = 2(n-i)+3,\ 1 \le i \le \left\lceil \frac{n+1}{4} \right\rceil$$

If $n = 4r+3$

$$f\left(v_{\frac{3n+2i-1}{2}+\lfloor \frac{n+1}{4} \rfloor}\right) = 2(n-i+1),\ 1 \le i \le \left\lceil \frac{n+1}{4} \right\rceil$$

Then it is easily observed that the above labeling is difference cordial labeling of A_n^2. □

Example 2.2. *Difference cordial labeling for A_8^2 and A_7^2 are shown in Figure 5(a) and Figure 5(b)*

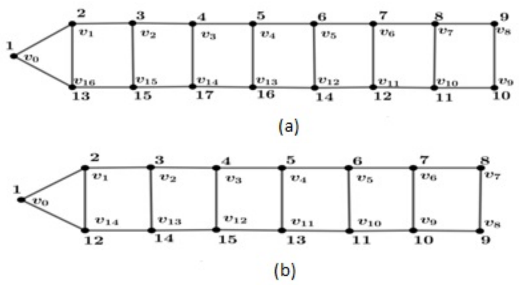

Figure 5. **(a) Arrow graph A_8^2 (b) Arrow graph A_7^2**

Theorem 2.3. *For $n > 3$, DA_n^2 is difference cordial graph.*

Proof. Consider DA_n^2 be a double arrow graph incurred by connecting two vertices u, v with $P_2 \times P_n$ by 2+2 new edges both sides and with vertex set $V = \{v, v_i \mid 1 \le i \le 2n\}$ and the edge set $E = \{v_i v_{i+1} \mid 1 \le i \le 2n-1\} \cup \{v_1 u_1, v_{2n} u_1\} \cup \{v_n u_2, v_{n+1} u_2\} \cup \{v_{i+1} v_{2n-i} \mid 0 \le i \le n-1\}$. Obviously, $|V(G)| = 2n+2$ and $|E(G)| = 3n+2$. Now, a bijective function is given by

$$f(u_i) = \left\lceil \frac{3n+2}{2} \right\rceil + 2i - 1,\ 1 \le i \le 2$$

$$f(v_i) = i,\ 1 \le i \le \left\lceil \frac{3n+2}{2} \right\rceil$$

When $3 < n \le 7$

$$f\left(v_{\lceil \frac{3n+2}{2} \rceil} + i\right) = \left\lceil \frac{3n+2}{2} \right\rceil,\ 1 \le i \le \left\lfloor \frac{n-2}{2} \right\rfloor$$

When $n > 7$. In particular, when $n \equiv 0, 1 \pmod 4$

$$f\left(v_{\lceil \frac{3n+2}{2} \rceil + i}\right) = \left\lceil \frac{3n+2}{2} \right\rceil + 2i,\ i = 1, 2, \ldots \left\lfloor \frac{n-2}{4} \right\rfloor + 1$$
$$f\left(v_{\lceil \frac{3n+2}{2} \rceil + \lceil \frac{n-2}{4} \rceil + i}\right) = 2(n-i+2),\ 1 \le i \le \left\lfloor \frac{n-2}{4} \right\rfloor - 1$$

In particular, when $n \equiv 2, 3 \pmod 4$

$$f\left(v_{\lceil \frac{3n+2}{2} \rceil + i}\right) = \left\lceil \frac{3n+2}{2} \right\rceil + 2i,\ 1 \le i \le \left\lfloor \frac{n-2}{4} \right\rfloor + 1$$
$$f\left(v_{\lceil \frac{3n+2}{2} \rceil + \lceil \frac{n-2}{4} \rceil + i}\right) = 2(n-i) + 3,\ 1 \le i \le \left\lfloor \frac{n-2}{4} \right\rfloor - 1$$

It is observed that the above function satisfies the condition of difference cordial labeling. Therefore the double arrow graph DA_n^2 admits difference cordial labeling. □

Example 2.4. *Difference cordial labeling for DA_7^2 and DA_9^2 are shown in Figure 6(a) and Figure 6(b)*

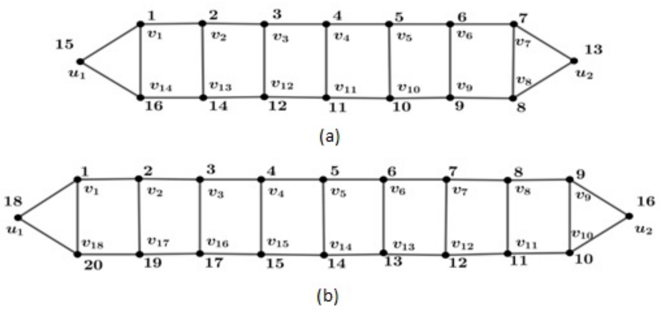

Figure 6. **(a) Double Arrow graph DA_7^2 (b) Double Arrow graph DA_9^2**

Theorem 2.5. *Arrow graph A_n^m is difference cordial graph.*

Proof. Let G be an arrow graph A_n^m obtained by joining a vertex v with the superior vertices of $P_m \times P_n$ by m new edges as shown in figure 3. With the vertex set $V(G) = \{v_0, u_i/1 \le i \le mn\}$ and edge set $E(G) = \{v_0u_i/1 \le i \le m\} \cup \{u_iu_{i+1}/1 \le i \le mn-1\} \cup \left\{u_{(2k-1)m-j}u_{(2k-1)m+1-j}/1 \le j \le m-1; 1 \le k \le \lceil \frac{n}{2} \rceil\right\} \cup \left\{u_{(2k-1)m+j}u_{(2k+1)m+1-j}/1 \le j \le m-1; 1 \le k \le \lceil \frac{n}{2} \rceil - 1\right\}$. Obviously $p = |V(G)| = mn+1$ and $q = |E(G)| = n(2m-1)$. We define a bijection $f : V \to \{1, 2, \ldots, (mn+1)\}$ as follows

$$f(v_0) = 1$$
$$f(u_i) = i+1,\ 1 \le i \le \left\lceil \frac{q}{2} \right\rceil - 1,$$
$$f\left(u_{\lceil \frac{q}{2} \rceil - 1 + i}\right) = \left\lceil \frac{q}{2} \right\rceil + 2i,\ 1 \le i \le \left\lfloor \frac{n+2}{4} \right\rfloor,$$

When $q \equiv 0 \pmod 4$

$$f\left(u_{\lceil \frac{q}{2} \rceil - 1 + \lfloor \frac{n+2}{4} \rfloor + i}\right) = p - 2(i-1),\ 1 \le i \le \left\lceil \frac{n+2}{4} \right\rceil$$

When $q \equiv 1 \pmod 4$. For $1 \leq i \leq \lceil \frac{n+2}{4} \rceil$

$$f\left(u_{\lceil \frac{q}{2} \rceil - 1 + \lfloor \frac{n+2}{4} \rfloor + i}\right) = p - (2i-1), \quad \text{When } p \text{ is odd}$$
$$f\left(u_{\lceil \frac{q}{2} \rceil - 1 + \lfloor \frac{n+2}{4} \rfloor + i}\right) = p - 2(i-1), \quad \text{When } p \text{ is even}$$

When $q \equiv 2 \pmod 4$

$$f\left(u_{\lceil \frac{q}{2} \rceil - 1 + \lfloor \frac{n+2}{4} \rfloor + i}\right) = p - (2i-1), \ 1 \leq i \leq \left\lceil \frac{n+2}{4} \right\rceil$$

When $q \equiv 3 \pmod 4$. For $1 \leq i \leq \lceil \frac{n+2}{4} \rceil$

$$f\left(u_{\lceil \frac{q}{2} \rceil - 1 + \lfloor \frac{n+2}{4} \rfloor + i}\right) = p - 2(i-1), \quad \text{When } p \text{ is odd}$$
$$f\left(u_{\lceil \frac{q}{2} \rceil - 1 + \lfloor \frac{n+2}{4} \rfloor + i}\right) = p - (2i-1), \quad \text{When } p \text{ is even}$$

Then it is easily observed that $e_f(1) = e_f(0) = \frac{n(2m-1)}{2}$, if p is odd and $e_f(1) = e_f(0) + 1$, if p is even. Hence the arrow graph A_n^m admits difference cordial labeling and the example for A_5^6 is illustrated in Figure 7

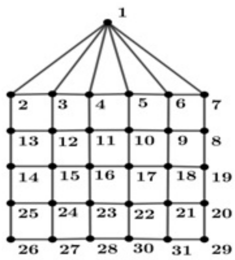

Figure 7. **Arrow graph A_5^6**

□

Theorem 2.6. *Double Arrow graph DA_n^m is difference cordial graph.*

Proof. Choose DA_n^m be adouble arrow graph procured by combining two vertices u, v with $P_m \times P_n$ by $m + m$ new edges from both ends as shown in figure 4 and $V = \{v_0, u_i/1 \leq i \leq mn\}$, $E = \{v_1 u_i/1 \leq i \leq m\} \cup \{v_2 u_{(mn-n)+i}/1 \leq i \leq m\} \cup \{u_i u_{i+1}/1 \leq i \leq mn-1\} \cup \left\{u_{(2k-1)m-j} u_{(2k-1)m+1+j}/1 \leq j \leq m-1; 1 \leq k \leq \lceil \frac{n}{2} \rceil\right\} \cup \left\{u_{(2k-1)m+j} u_{(2k+1)m+1-j}/1 \leq j \leq m-1; 1 \leq k \leq \lceil \frac{n}{2} \rceil - 1\right\}$. Then $|p| = mn + 2$ and $|q| = 2mn$

$$f(v_1) = 1$$
$$f(v_2) = mn - 1$$
$$f(u_i) = i + 1, \ 1 \leq i \leq mn - 3$$
$$f(u_{mn-3+i}) = mn - 2 + 2i, \ i = 1, 2$$
$$f(u_{mn-1+i}) = mn + 2 - i, \ i = 1$$

Then it is clearly observed that $e_f(1) = e_f(0) = mn$ whenever n, m is odd or even. Hence the double arrow graph DA_n^m admits difference cordial graph. □

Example 2.7. *Difference cordial labeling for DA_5^7 are shown in Figure 8.*

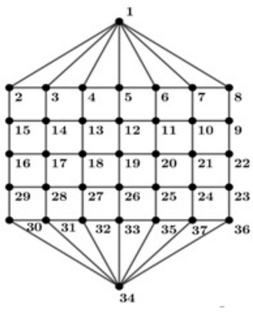

Figure 8. **Double arrow graph** DA_5^7

3. Conclusion

Here we studied the arrow graphs A_n^2, double arrow graphs DA_n^2, Arrow graph A_n^m and double arrow graphs DA_n^m are difference cordial graphs.

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