Int. J. Math. And Appl., 6(1)(2018), 125–130. (Special Issue)

ISSN: 2347-1557

Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Product Cordial Labeling of n-chain Aztec Diamond Graphs

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Abstract: A function $f : V$ to $\{0, 1\}$ of a graph G is known to be a product cordial labeling if each edge uv is given the label $f(u)f(v)$ the resulting number of vertices with labels 0 and the number of vertices with labels 1 vary to the maximum of 1, and the number of edge labels with 0 and the number of edge labels with 1 vary also to the maximum of 1. A graph that satisfies the conditions of product cordial labeling is named product cordial. In this paper product cordial labeling is proved for n-chain Aztec diamond graph for even positive integer n.

MSC: 05C78.

Keywords: Product cordial labeling, Aztec diamond, Aztec diamond graph, n-chain Aztec diamond graph.

 Accepted on: 13th April 2018

1. Introduction

A graph labeling is an allotment of integers to the points or links, or both, subject to certain specified rules. Graph labeling was first introduced in the late 1960s. Much of the graph labeling methods that are used today, point their origin to Rosa [8]. Labeling of graphs is an interesting and fast developing field in graph theory. For a detailed survey on labeling of graphs, one can always refer to the dynamic survey by J.Gallian [5]. Labelings is a major sub field of research in graph theory. Research in Labeling of graphs has increased so rapidly, mainly because of its sheer interest and the vast fields in which labelings are applied. Applications in Computer field, Mathematical models in Mechanics, Electronic, Electrical, and Communication Networking are innumerable and vast. This branch of graph theory contributes greatly to the mathematical modeling of day-to-day problems faced in the industries. Acharya and Germina [1] introduced set valuations in graph labeling and A.Rosa [8] put forth graceful labeling while Cahit initiated cordial labeling as a weaker version of graceful labeling. A survey of set valued graphs can also be seen in Abhishek [2]. A new family of staircase graphs were introduced and various labelings were defined on them by A.Solairaju and M.Antony Arockiasamy [9].

The broad fields of research in graph labeling are two in number. First among the two is finding new labeling techniques to apply on the existing set of graphs and the second one is generating new graphs and using the available graph labeling techniques on these new graphs. The first field is chosen for this research project. The graph chosen is Aztec diamond graphs. Aztec diamond graphs [6] are known for tilings and its related properties. But in this paper the Aztec diamond graphs are seen through a different lens. It is subjected to the test of cordial labeling. A brief preliminary definitions are

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Research partially supported by UGC-SERO, MRP-5873/15.

given in this section and main results are derived in the second section. Throughout this paper edges and links are identically used, while vertices and points have equal meaning.

Definition 1.1. *Let n be a positive integer. The Aztec diamond of order n is the union Aztec diamond of all the unit squares with integral vertices (x, y) satisfying $|x| + |y| \leq n + 1$. The Aztec diamond of order 1 consists of 4 unit squares which have the origin $(0, 0)$ as one of their vertices. The dual graph obtained from an Aztec diamond of order n, where each square is a vertex and if two squares are adjacent in Aztec diamond then their corresponding vertices are linked by and edge in the dual graph; is known as Aztec diamond graph of order n. It is denoted by $G(A, k)$. The corner vertex of an Aztec diamond graph is defined as two degree vertex whose adjacent vertices are either degree 2 or degree 4 only.*

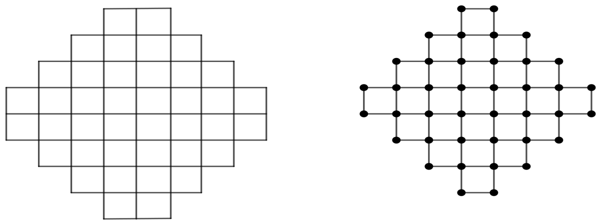


Figure 1: Aztec Diamond and Aztec Diamond Graph of order 4

Definition 1.2. *Let $G_1(A, k)$ and $G_2(A, k)$ be any two Aztec diamond graphs of same order. The graph $G(2A, k)$ is known to be twin Aztec diamond graph [3] if the graph is obtained by linking an edge with any one of the corner vertices of $G_1(A, k)$ to any one of the corner vertices of $G_2(A, k)$. It is shown in Figure 2. It contains $(4k^2 + 4k)$ vertices and $(8k^2 + 1)$ edges. Similarly triple and n-chain Aztec diamond graphs [3] can be defined as shown in Figures 3 and 4. They are accordingly denoted by $G(3A, k), G(nA, k)$ respectively.*

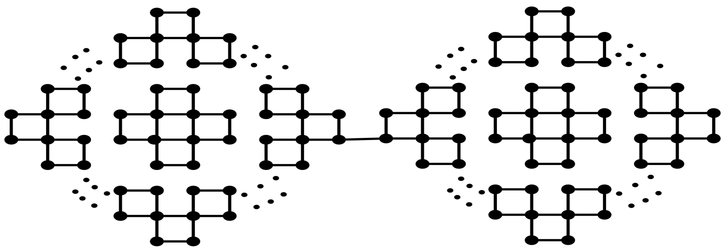


Figure 2: Graph $G(2A, k)$

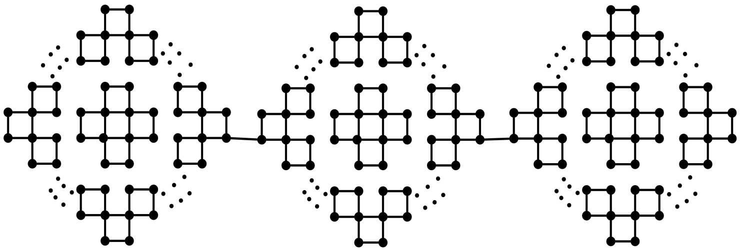


Figure 3: Graph $G(3A, k)$

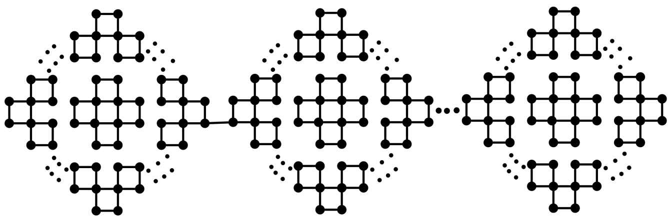


Figure 4: Graph $G(nA, k)$

Definition 1.3. *A graph G along with a mapping $g : V(G) \to \{0, 1\}$ on the vertex set V that induces an edge labeling function $g^* : E(G) \to \{0, 1\}$ defined by $g*(uv) = |g(u) - g(v)|$ for every vertex u and v in V, is known as cordial labeling of G if $|v_g(0) - v_g(1)| \leq 1$ and $|e_g(0) - e_g(1)| \leq 1$ where $v_g(i)$ is the number of points of G labeled i under g and $e_g(i)$is the number of links of G labeled i under g^* for $i = 1, 2$. A graph G that admits cordial labeling is called cordial graph [4].*

Definition 1.4. *A cordial graph with a variation $g^*(e = uv) = g(u)g(v)$ in induced function results in product cordial labeling . A graph G along with a mapping $g : V(G) \to \{0, 1\}$ on the vertex set V that induces an edge labeling function $g^* : E(G) \to \{0, 1\}$ defined by $g^*(e = uv) = g(u)g(v)$ for every vertex u and v in V, is known as product cordial labeling of G if $|v_g(0) - v_g(1)| \leq 1$ and $|e_g(0) - e_g(1)| \leq 1$ where $v_g(i)$ is the number of points of G labeled i under g and $e_g(i)$ is the number of links of G labeled i under g^* for $i = 1, 2$.A graph G is called product cordial graph if it admits a product cordial labeling [7, 9, 10]. In this paper product cordial labeling is verified for n-chain Aztec diamond graph.*

2. Main Results

In this section results derived on twin-Aztec diamond graphs, triple Aztec diamond graphs and n-chain Aztec diamond graphs are presented.

Theorem 2.1. *Graph $G(2A, k)$ is product cordial.*

Proof. Let $G(2A, k)$ be a twin Aztec diamond graph of order k, it contains $(4k^2 + 4k)$ points $(8k^2 + 1)$ links. Let G_1 and G_2 be two Aztec diamond graphs in $G(2A, k)$. The pattern of labeling follows a simple procedure. For each of the $2(k^2 + k)$ points of the $G_1(A, k)$ labels 1 is given. And for each of the $2(k^2 + k)$ points of the second diamond $G_2(A, k)$ the label 0 is allocated. Let $f : V(G) \to \{0, 1\}$ induces an edge labeling function $f^* : E(G) \to \{0, 1\}$ defined as $f^*(e = uv) = f(u)f(v)$. The induced the number of points labeled as 0 and the number of points labeled as 1 differ at most by one. Similarly, the number of links labeled as 0 and the number of links labeled as 1 and they differ at most by one. Therefore the graph $G(2A, k)$ is product cordial.

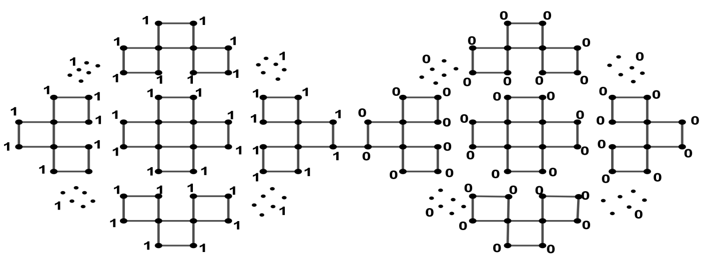
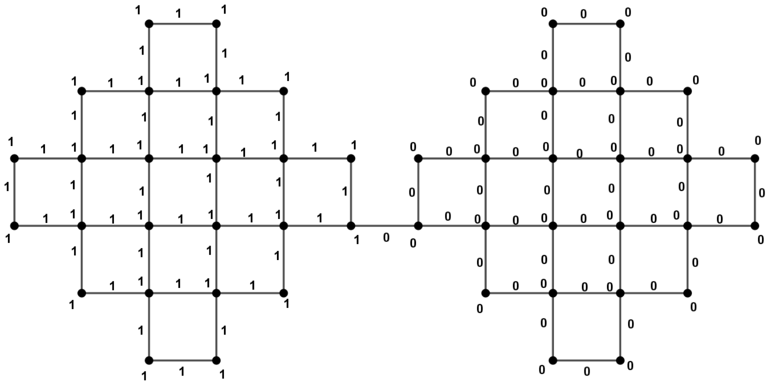


Figure 5: Product Cordial Labeling of $G(2A, k)$

□

Example 2.2. *Twin Aztec diamond graph $G(2A, 3)$ is product cordial.*



p		q	
0	1	0	1
24	24	37	36

Figure 6: Product Cordial Labeling of $G(2A, 3)$

Theorem 2.3. *The graph $G(3A, k)$ is not product cordial.*

Proof. Let $G(3A, k)$ be a triple Aztec diamond graph of order k. It contains $(6k^2 + k)$ points and $(12k^2 + 2)$ links. Let G_1, G_2 and G_3 be triple Aztec diamond graphs in $G(3A, k)$. For each of the $2(k^2 + k)$ points of G_1 all possible combinations $\binom{2(k^2+k)}{k}$ are analysed and tried for the cordial labeling. But none has resulted in the satisfaction of conditions of cordial labeling. Similarly for G_2 and G_3 all possible combinations using the vertex labeling functions and the induced edge labeling functions do not result in the cordial labeling. Hence the graph $G(3A, K)$ is not product cordial. $\square$

Theorem 2.4. *Any 4-chain Aztec diamond graph is product cordial.*

Proof. Let $G(4A, k)$ be 4-chain Aztec diamond graph consisting of $G_1(A, k), G_2(A, k), G_3(A, k), G_4(A, k)$ Aztec diamond graphs. For each of G_1, G_2, G_3, G_4 labels 1 is allotted for each of the points of G_1 and G_2 and labels 0 is allotted for each of the points of G_3 and G_4. Define an edge labeling function $h^*(e = uv) = h(u)h(v)$. The resultant links labels with vertex labels satisfy the conditions of product cordial labeling function. Hence $G(4A, k)$ is product cordial.

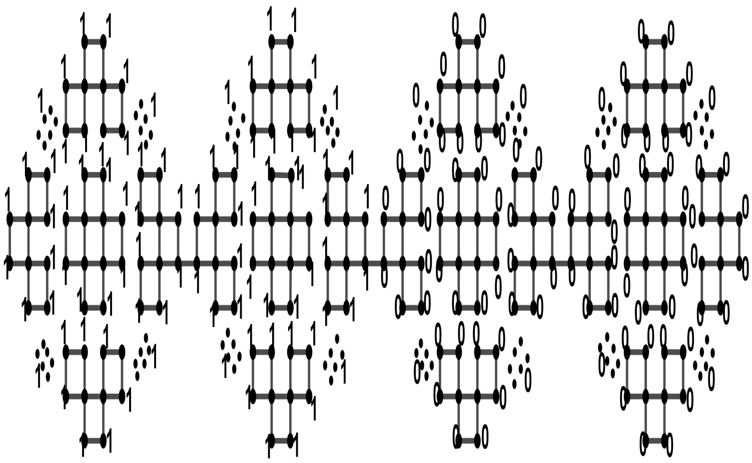
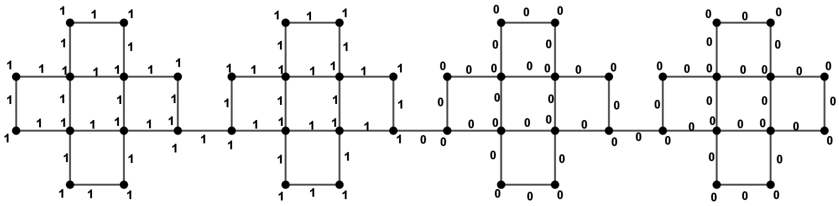


Figure 7: Product Cordial Labeling of $G(4A, k)$

$\square$

Example 2.5. *4-chain Aztec diamond graph $G(4A, 2)$ is product cordial.*



p		q	
0	*1*	*0*	*1*
24	*24*	*34*	*33*

Figure 8: Product Cordial Labeling of $G(4A, 2)$

Theorem 2.6. *Any n-chain Aztec diamond graph is product cordial for even n.*

Proof. The n-chain Aztec diamond graph contains n- Aztec diamond graphs. The proof is analogous to proof given for twin and 4-chain Aztec diamond graphs. Let $G_1(A, k), G_2(A, k), G_3(A, k), ..., G_n(A, k)$ be n-Aztec diamonds in $G(nA, k)$. Allocate labels 1 for each of the points of G_i , For $i = 1, 3, 5, ..., n-1$. Allocate labels 0 for each of the points of G_i , For $i = 2, 4, 6, ..., n$. The induced links labels fulfill the conditions of product cordial labeling. Thus n-chain Aztec diamond graph becomes cordial for even n. □

3. Future Directions

Aztec diamond graphs of different orders can be combined to create twin, triple and n-chain Aztec diamond graphs of different orders. There are many labeling methods still not tested on the existing set of Aztec diamond graphs and the new ones to be generated. These graphs offer a wide range of future opportunities of research.

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