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Cordial Labeling of n-Chain Aztec Diamond Graphs

M. Antony Arockiasamy[1]*, S. Kalaiarasi[1], and P. Tamizharasi[1]

1 Department of Mathematics, Sacred Heart College, Tirupattur, Tamil Nadu, India.

Abstract: A binary vertex labeling $f: V(G) \to \{0,1\}$ of a graph G is called a cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is cordial if it admits cordial labeling.. The dual graph obtained from an Aztec diamond of order n, where each square is a vertex and if two squares are adjacent in Aztec diamond then their corresponding vertices are linked by and edge in the dual graph; is known as Aztec diamond graph of order n. In this paper n-chain Aztec diamond graphs are proved to be cordial.

MSC: 05C78.

Keywords: Cordial labeling, Aztec diamond, Aztec diamond graph. Disjoint union of graphs

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1. Introduction

Applications of graph labeling in fields such as network analysis, communication network, transport efficiency etc., generate not only a vast array of opportunities in the field of graph theory but also create a great amount of interest in the minds of young researchers who intend to do their work in this field. Graph labeling is a mapping that allocates labels to vertices and edges in accordance with a specified rule. Finding this unique function is the main task of graph labeling. Various kinds of graph labeling that are found so far can be easily accessed in dynamic survey by Gallian [4] which is reviewed annually. The enormous amount of researches done in graph labeling indicates not only the interest but also the creative ideas that this field of knowledge emanates in the minds of readers and scholars. The broad fields of research in graph labeling are two in number. First among the two is finding new labeling techniques to apply on the existing set of graphs and the second one is generating new graphs and using the available graph labeling techniques on these new graphs. The first field is chosen for this research project. The graph chosen is Aztec diamond graphs. Aztec diamond graphs [5] are known for tilings and its related properties. But in this paper the Aztec diamond graphs are seen through a different lens. It is subjected to the test of cordial labeling. Moreover the second field of research in graph labeling is also applied, because new disjoint union of n Aztec diamond graphs are generated and the same labeling is verified for these new set of graphs, apart from that the n- chain of Aztec diamond graphs generated by [1] are also taken for testing cordial labeling. Throughout this paper edges and links are identically used, while vertices and points have equal meaning.

Definition 1.1 ([3, 8]). *A binary vertex labeling $f: V(G) \to \{0,1\}$ of a graph G is called a cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is cordial if it admits cordial labeling. The idea of cordial labeling was introduced by Cahit as a weaker version of graceful and harmonious labeling.*

* *E-mail: arockiaanto2008@gmail.com*
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Definition 1.2 ([7])**.** *For a graph $G = (V(G), E(G))$, a vertex labeling function $f : V(G) \to \{0, 1\}$ induces an edge labeling function $f^* : E(G) \to \{0, 1\}$ defined as $f^*(e = uv) = f(u)f(v)$. Then $f$is called a product cordial labeling of graph G if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.*

Definition 1.3. *Let n be a positive integer, The Aztec diamond of order n is the union of all the unit squares with integral vertices (x, y) satisfying $|x| + |y| \leq n + 1$. The Aztec diamond of order 1 consists of 4 unit squares which have the origin $(0, 0)$ as one of their vertices. The Aztec diamond of order n is the dual graph to the Aztec diamond of order n in which the vertices are the squares and an edge joins two vertices if and only if the corresponding squares are adjacent in the Aztec diamond [5]. The corner vertex of an Aztec diamond graph is defined as two degree vertex whose adjacent vertices are either degree 2 or degree 4.*

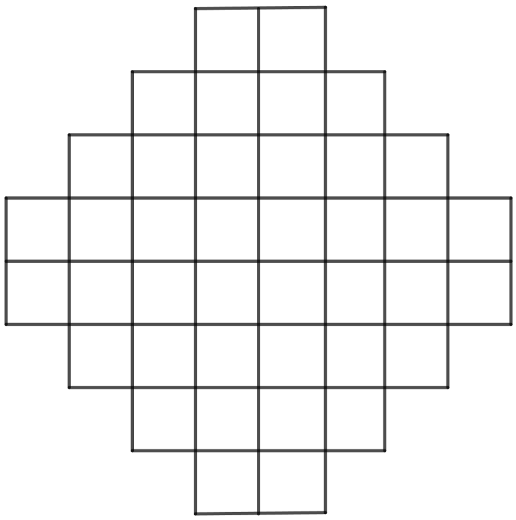


Figure 1: 1a: Aztec diamond of order 4

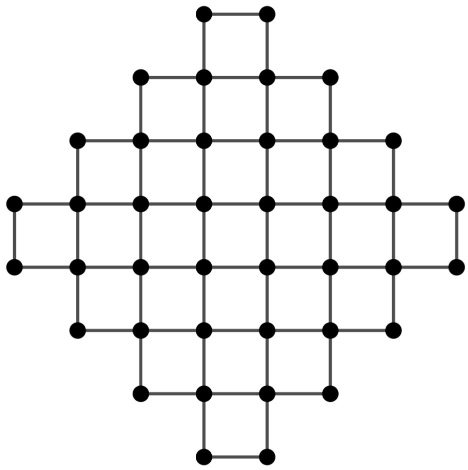


Figure 2: 1b: Aztec diamond graph of order 4

Definition 1.4. *Let $G_1(A, k)$ and $G_2(A, k)$ be any two Aztec diamond graphs of same order. The graph $G(2A, k)$ is said to be Twin Aztec diamond graph if the graph is obtained by linking an edge with any one of the corner vertices of $G_1(A, k)$ to any one of the corner vertices of $G_2(A, k)$. It is shown in Figure 1. It contains $4(k^2 + k)$ vertices and $(8k^2 + 1)$ edges [1]. Similarly triple and n-chain Aztec diamond graphs [1] can be defined as shown in Figures 2 and 3.*

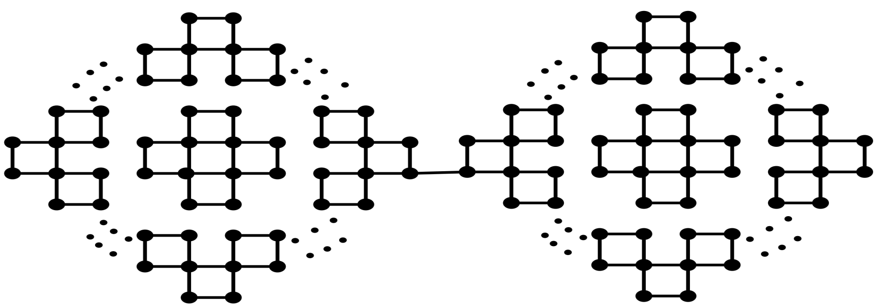


Figure 3: Twin Aztec Diamond Graph

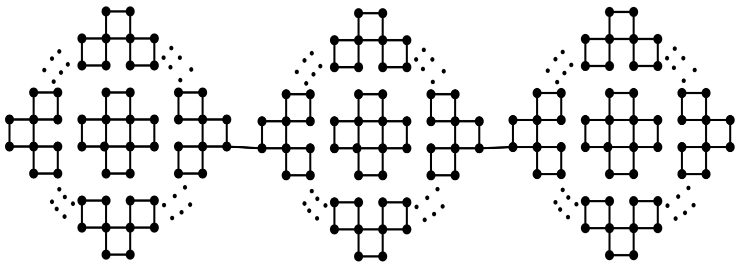


Figure 4: Triple Aztec Diamond Graph

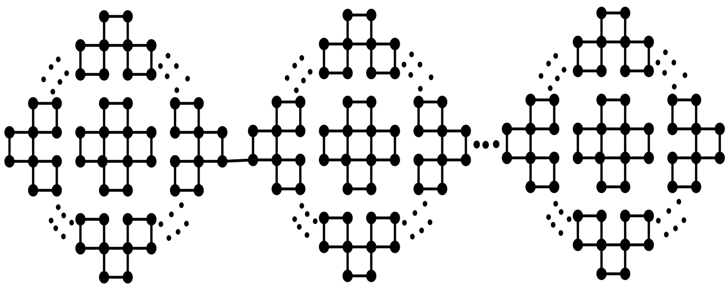


Figure 5: n-Chain Aztec Diamond Graph

Definition 1.5 ([6]). *Let G be a graph and $G_1, G_2, ..., G_n, n \geq 2$ be n copies of graph G. The graph obtained by adding an edge from G_i to $G_(i+1)(for i = 1, 2, ..., n-1)$ is called path union of G.*

Definition 1.6. *Let G_1 and G_2 be subgraphs of G then G_1 and G_2 are disjoint if they have no common vertex, and edge disjoint if they have no edge in common. And it is denoted by $G_1 + G_2$ [2]. Let $G_1(A, k), G_2(A, k), ..., G_n(A, k)$ be n Aztec diamond graphs of same order k. The disjoint union of n Aztec diamond graphs is denoted as $G_1(A, k) + G_2(A, k) + ... + G_n(A, k)$. It contains $(2nk^2 + 2nk)$ vertices and $(4nk^2)$ edges. It is denoted by $G_d(nA, k)$.*

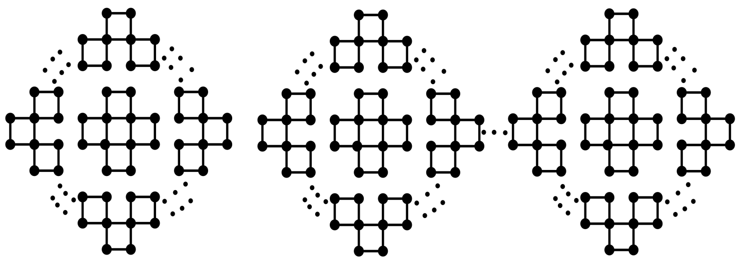


Figure 6: Disjoint Union of n Aztec diamond graphs $G_d(nA, k)$

In this paper cordial labeling are investigated for n-Chain Aztec diamond graphs and disjoint union of n Aztec diamond graphs. Throughout the paper k indicates the order of the Aztec diamond graphs, while n denotes the number of Aztec diamond graphs of order k. The notations p and q are used to represent the number of vertices and number of edges in a graph G respectively.

2. Main Results

In this section single, double, triple Aztec diamond graphs are proved to be cordial and the result is extended for n chain Aztec diamond graphs.

Theorem 2.1. *Aztec diamond graph $G(A, k)$ is cordial.*

Proof. Let $G(A, k)$ be a Aztec diamond graph of order k . The number of points is $2(k^2 + k)$ and the number of links is $(4k^2)$. The vertex labeling pattern is as given below. The Aztec diamond graph $G(A, k)$ is split into $(2k)$ horizontal paths $P_1, P_2, P_3, ., P_{2k}$. where P_i represents the shortest path between V_{ij} and V_{ij} ,$1 \leq i \leq 2k, 1 \leq j \leq 2k$. For each $P_i, 1 \leq i \leq 2k$,the label 0 is assigned to all the points of P_i if i is odd , and the label 1 is assigned to all the points of P_i , if i is even. Let $f : V \to \{0, 1\}$ be a mapping from the set of points of G to $\{0, 1\}$ and for each edge $(uv) \in E$ assign the label $|f(u) - f(v)|$. The resulting number of points labeled as 0 and the number of points labeled as 1 differ at most by 1. Similarly, the induced number of links labeled as 1 and the number of links labeled as 0 also differ at most by 1. Hence the graph $G(A, k)$ is cordial. □

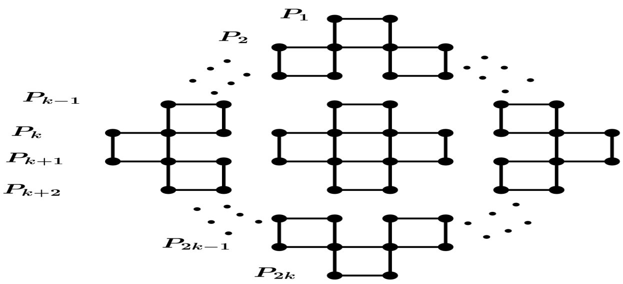
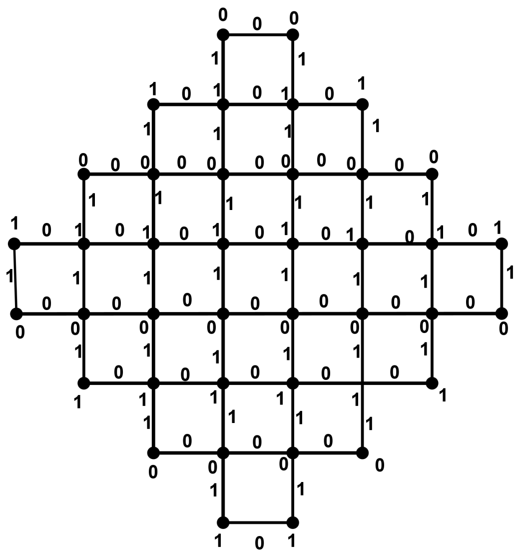


Figure 7: Cordial Labeling Pattern of G (A,k)

Example 2.2. *Aztec diamond graph $G(A, 4)$ is proved cordial in figure 6. The inner box indicates the number of points and links labeled as* 1 *or* 0*.*



p		q	
0	1	0	1
20	20	32	32

Figure 8: Cordial G(k,4)

Theorem 2.3. *Twin Aztec diamond graph $G(2A, k)$ is cordial.*

Proof. Let $G(2A, k)$ be a twin Aztec diamond graph of order k, It contains $(4k^2 + 4k)$ points $(8k^2 + 1)$ links. Assume $G_1(A, k)$ and $G_2(A, k)$ as two Aztec diamond graphs in $G(2A, k)$. The $G(2A.k)$ is split into $4k$ horizontal paths $P_1, P_2, ..., P_{4K}$, where P_i represents the shortest horizontal path between V_{ij} and V_{ij}, $1 \leq i \leq 4k, 1 \leq j \leq 4k$. Further the graph $G_1(A, k)$ contains $2k$ horizontal paths $P_1, P_2, ..., P_{2k}$ and the graph $G_2(A, k)$ contains $2k$ horizontal paths $P_{2k+1}, P_{2k+2}, ...P_{4k}$. For each $Pi, 1 \leq i \leq 4k$, the label 0 is assigned to all the points of P_i if i is odd, and the label 1 is assigned to all the points of P_i if i is even. Let $f : v \to \{0, 1\}$ be a mapping from the set of points of G to $\{0, 1\}$ and for each edge $(uv)E$ assign the label$|f(u) - f(v)|$. The above Pattern of labeling is modified near the link edge in such a way the resulting labels are as follows. The induced number of points labeled as 0 and the number of points labeled as 1 differ at most by one. Similarly, the number of links labeled as 0 and the number of links labeled as 1 differ also at most by one. Therefore the graph $G(2A, k)$ is cordial. $\square$

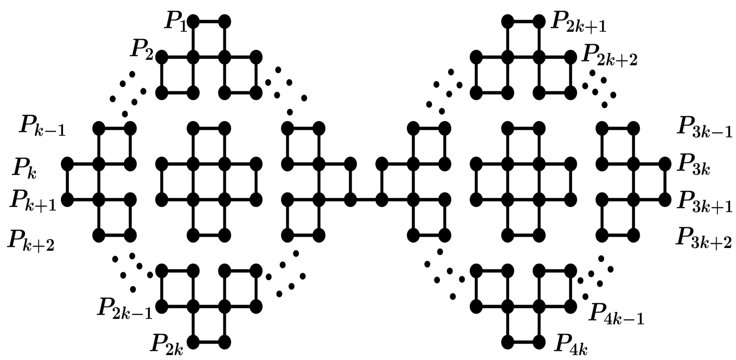
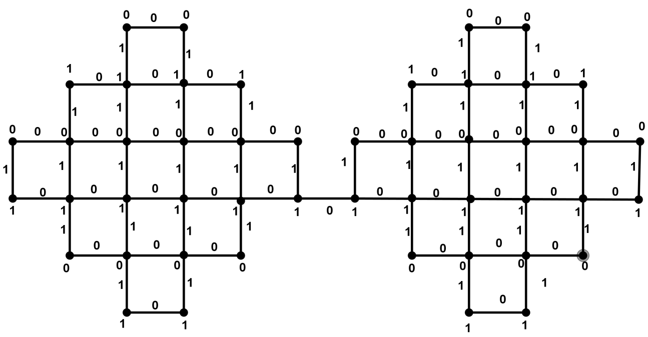


Figure 9: Cordial Labeling Pattern of $G(2A, k)$.

Example 2.4. *$G(2A, 3)$ is shown to be cordial in figure 9.*



p		*q*	
0	*1*	*0*	*1*
18	*18*	*25*	*25*

Figure 10: Cordial $G(2A, 3)$.

Theorem 2.5. *$G(3A, k)$ satisfies cordial labeling.*

Proof. Let $G(3A, k)$ be a triple Aztec diamond graph of order k. It contains $6(k^2 + k)$ points and $(12k^2 + 2)$ links. Let $G_1(A, k), G_2(A, k) and G_3(A, k)$ be three Aztec diamond graphs in $G(3A, k)$. Here the Triple Aztec diamond graph $G(3A, k)$ is split into $6k$ horizontal paths $P_1, P_2, ..., P_{6k}$, where p_i represents the shortest horizontal path between $V_{ij} and V_{ij}, 1 \leq i \leq 6k, 1 \leq j \leq 6k$. The graph $G_1(A, k)$ contains $2k$ horizontal paths $P_1, P_2, ..., P_{2k}$. and the graph $G_2(A, k)$ contains $2k$

horizontal paths $P_{2k+1}, P_{2k+2}, ..., P_{4k}$. For each $P_i, 1 \leq i \leq 2k$, The label 0 is assigned to all the points of P_i if i is odd, and the label 1 is assigned to all the points of P_i if i is even For each P_i , $2k+1 \leq i \leq 4k$, The label 0is assigned to all the points of P_i if i is odd, and the label 1 is assigned to all the points of P_i if i is even. Further the graph $G_3(A, k)$ contains $6k$ horizontal path, $P_{4k+1}, ..., P_{6k}$ for each P_i ,$4k+1 \leq i \leq 6k$, The label 0 is assigned to all the points of P_i if i is even ,and the label 1 is assigned to all the points of P_i if i is odd. Let $f : v \rightarrow \{0, 1\}$ be a mapping from the set of points of G to $\{0, 1\}$ and for each edge $(uv)E$ assign the label $|f(u) - f(v)|$. The above labeling patterns need to be altered for the link and its adjacent points to achieve the desired result. The number of points labeled as 0 and the number of points labeled as 1 differ at most by one. Similarly, The number of links labeled as 0 and the number of links labeled as 1 also differ at most by one. Hence $G(3A, k)$ is cordial. □

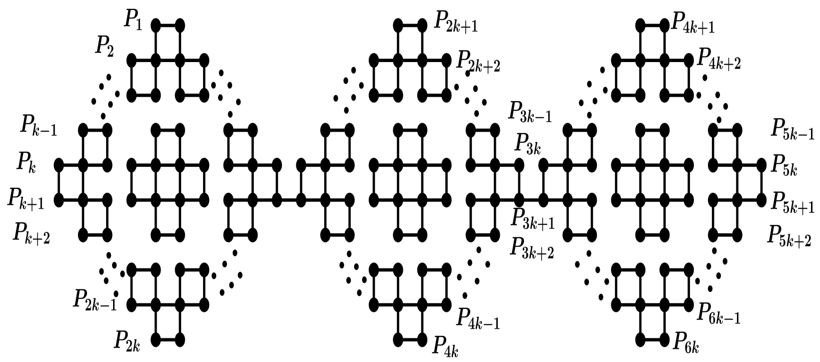
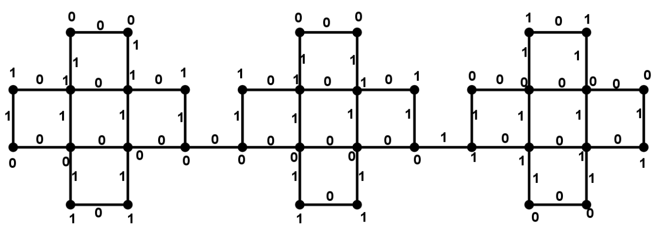


Figure 11: Cordial Labeling Pattern of G(3A,k)

Example 2.6. *Triple Aztec diamond graph $G(3A, 2)$ is verified to be cordial in figure 11.*



p		q	
0	*1*	*0*	*1*
24	*24*	*37*	*36*

Figure 12: Cordial Triple Aztec Diamond Graph G(3A,2)

Theorem 2.7. *Any n-Chain Aztec diamond graph $G(nA, k)$ is cordial.*

Proof. The n-Chain Aztec diamond graph consists of n Aztec diamond graphs. For odd n , the cordial labeling pattern is similar to triple Aztec diamond graph. For even n, the labeling is similar to double Aztec diamond graphs. Hence for any finite positive integer n, the n chain Aztec diamond graph is cordial. □

Theorem 2.8. *Finite disjoint union of n-Aztec diamond graph $G_d(nA, k)$ of order k is cordial, for any positive integer n and k.*

Proof. Let $G_d(nA, k)$ be a disjoint union of n-Aztec diamond graphs of order k. It contains $(2nk^2 + 2nk)$ points and $(4nk^2)$ links. If $G_1(A, k), G_2(A, k), ..., G_n(A, k)$ are n- Aztec diamond graphs of same order in $G(nA, k)$. By applying the cordial labeling conditions to each of the Aztec diamond graphs, it is easy to verify that $G_d(nA, k)$ is cordial $G_d(2A, 3)$ and $G_d(3A, 2)$ are proved to be cordial in Example 2.9 and 2.10.

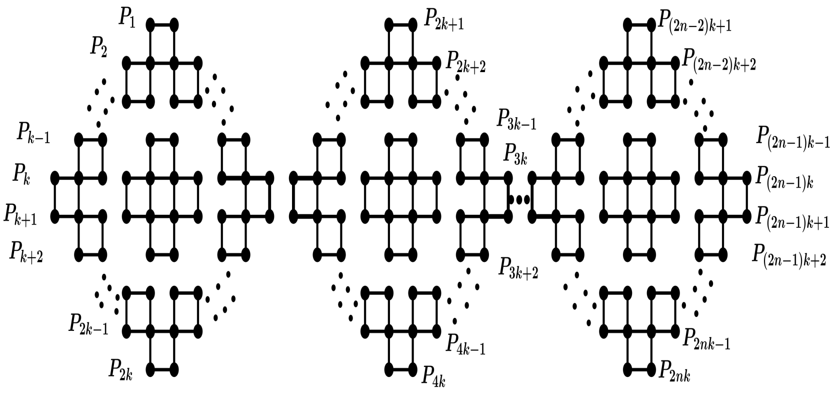
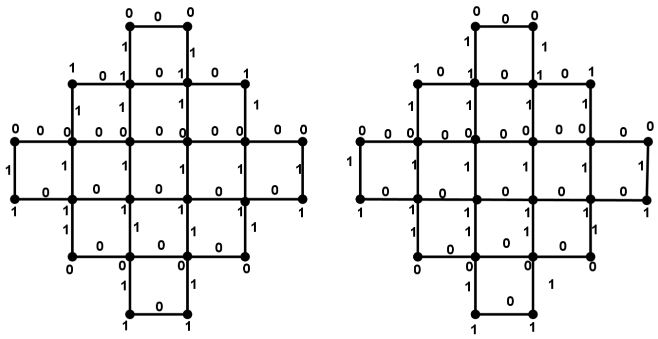


Figure 13: Cordial Labeling Pattern of $G_d(nA, k)$

Extending the result to any finite n results in Theorem 2.8. □

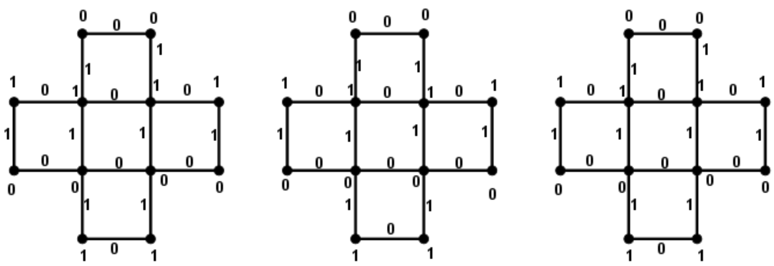
Example 2.9. *Disjoint union of two Aztec diamond graph $G_d(2A, 3)$ is cordial.*



p		*q*	
0	*1*	*0*	*1*
18	*18*	*25*	*25*

Figure 14: Cordial Labeling of $G_d(2A, 3)$

Example 2.10. *Disjoint union of three Aztec diamond graph $G_d(3A, 2)$ is cordial.*



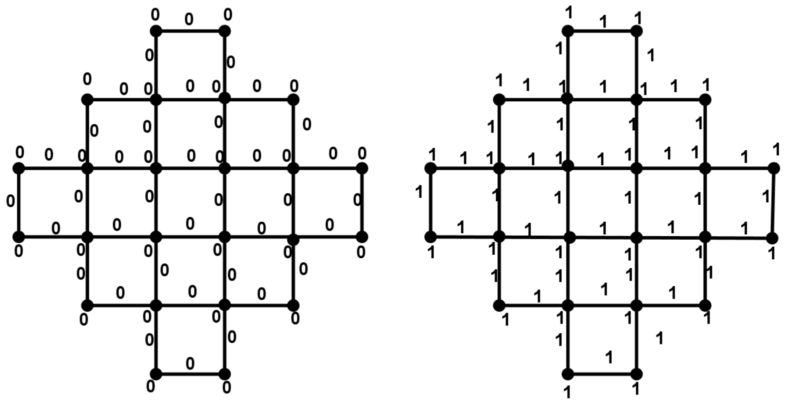
p		*q*	
0	*1*	*0*	*1*
18	*18*	*24*	*24*

Figure 15: Cordial labeling of $G_d(3A, 2)$

Theorem 2.11. *Finite disjoint union of Aztec diamond graph $G_d(nA, k)$ of order k is product cordial for even n.*

Proof. Let $G(nA, k)$ be a disjoint union of n-Aztec diamond graph of order k. It contains $(2nk^2 + 2nk)$ points and $(4nk^2)$ links. Let $G_1(A, k), G_2(A, k), ..., G_n(A, k)$ be n Aztec diamond graphs of same order in $G(nA, k)$. The vertex labeling pattern is simple. For each of points of alternative Aztec diamond graphs labels either 0 or 1 is allocated completely. The resultant edge labels satisfy the condition of product cordial labeling. □

Example 2.12. *Disjoint union of two Aztec diamond graph $G_d(2A, 3)$ is product cordial.*



p		*q*	
0	*1*	*0*	*1*
24	*24*	*36*	*36*

Figure 16: Product Cordial Labeling of $G_d(2A, 3)$

3. Scope of Further Research

Aztec diamond graphs of different order can be combined to get a n-chain Aztec diamond graph of different order and labeling techniques such as mean labeling, harmonious labeling, elegant labeling can be tested on the newly created Aztec diamond graphs.

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