Int. J. Math. And Appl., 6(1)(2018), 165–174. (Special Issue)
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Balanced Intuitionistic Triple Layered Fuzzy Graph

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Abstract: A new balanced intuitionistic triple layered fuzzy graph is defined using intuitionistic triple layered fuzzy graph by modifying its conditions. Also some properties of BITLFG are verified in this paper.

MSC: 05C90

Keywords: Intuitionistic fuzzy graph, triple layered fuzzy graph, complete fuzzy graph, balanced fuzzy graph, balanced intuitionistic triple layered fuzzy graph.

 Accepted on: 13th April 2018

1. Introduction

Fuzzy graph theory has many applications in operations research, computer sciences, system analysis etc. in which it deals with the uncertainty. Many crisp graph concepts have been extended to fuzzy graph theory. Fuzzy relation was introduced by Zadeh in his definitive work fuzzy sets in 1965, which has wide spread application [15]. Azriel Rosenfeld introduced fuzzy graph theory in 1975 and also he discussed various graph theoretical concepts [16]. Yeh and Bang have contributed various concepts in connectedness in fuzzy graphs [17]. Operations on fuzzy graph were first studied by Mordeson and Peng [12]. Sunitha and Vijayakumar developed the concept of complement of a fuzzy graph and also discussed about the operation union, join, cartesian product and composition on two fuzzy graphs [14]. The concept of intuitionistic fuzzy sets was introduced by Atanassov as a generalization of fuzzy sets [18]. M.G. Karunambigai, M. Akram, S. Sivasankar and K. Palanivel have discussed on Balanced Intuitionistic Fuzzy Graphs [10]. M. Akram and B. Davvaz have discussed on strong intuitionistic fuzzy graph [8]. The triple layered fuzzy graph was introduced by T. Pathinathan and J. Jesintha Roseline in 2014 [1]. They developed the concept of intuitionistic triple layered fuzzy graph in the year 2015, and discussed about the Cartesian product of intuitionistic double layered fuzzy graph and its vertex degree [6]. They developed structural core graph of TLFG in the same year [3]. In this paper we introduced balanced intuitionistic triple layered fuzzy graph based on intuitionistic triple layered fuzzy graph by modifying its conditions. Also we have justified our approach by investigating the properties related to BITLFG.

2. Preliminaries

Definition 2.1. *An intuitionistic fuzzy graph (IFG) is of the form*

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(i). $G:\langle V,E\rangle$ *where* $V=\{v_1,v_2,...,v_n\}$ *such that* $\mu_1 : V\times V \to [0,1]$ *and* $\gamma_1 : V \to [0,1]$ *denote the degree of membership and non membership of the element* $v_i \in V$ *respectively and*

$$0 \leq \mu_1(v_i) + \gamma(v_j) \leq 1 \tag{1}$$

for every $v_i \in V,\ (i = 1,2,..,n)$,

(ii). $E \subseteq V \times V$ *where* $\mu_2 : v \to [0,1]$ *and* $\gamma_2 : v \to [0,1]$, *are such that*

$$\mu_2(v_i,v_j) \leq \mu(v_i) \wedge \mu_1(v_j) \tag{2}$$

$$\gamma_2(v_i,v_j) \leq \gamma(v_i) \vee \gamma_1(v_j) \tag{3}$$

and $0 \leq \mu_2(v_i,v_j) + \gamma_2(v_i,v_j) \leq 1$ *for every* $(v_i,v_j) \in E,\ (i,j = 1,2,..,n)$.

Note 2.2.

(i). *The triple* $(v_i,\mu_{1i},\gamma_{1i})$ *denotes the degree of membership and non-membership of the vertex* v_i. *The triple* $(e_{ij},\mu_{2ij},\gamma_{2ij}$ *denotes the degree of membership and non-membership of the edge* $e_{ij} = (v_i,v_j)$ *on* V *, i.e.* $\mu_{1i} = \mu_1(v_i),\ \gamma_{1i} = \gamma_1(v_i)$ *and* $\mu_{2ij} = \mu_2(v_i,v_j),\ \gamma_{2ij} = \gamma_2(v_i,v_j)$.

(ii). *When* $\mu_{2ij} = 0 = \gamma_{2ij}$, *for some* i *and* j, *then there is no edge between* v_i *and* v_j.

Definition 2.3. *An intuitionistic fuzzy graph,* $G = \langle V,E\rangle$ *is said to be a complete intuitionistic fuzzy graph if* $\mu_{2ij} = \min(\mu_{1i},\mu_{1j})$ *and* $\gamma_{2ij} = \max(\gamma_{1i},\gamma_{1j})$ *for every* $v_i,v_j \in V$.

Definition 2.4. *The density of a fuzzy graph* $G:(\mu_1,\mu_2)$ *is*

$$D(G) = 2\left[\frac{\sum\limits_{u,v\in E} \mu_2(u,v)}{\sum\limits_{u,v\in E} \mu_1(u) \wedge \mu_2(u)}\right]$$

Definition 2.5. *An intuitionistic fuzzy graph* $G:\langle(v_i,\mu_1,\gamma_1),\ (e_{ij},\mu_2,\gamma_2)\rangle$ *is balanced if* $D_\mu(H) \leq D_\mu(G)$ *and* $D_\gamma(H) \leq D_\gamma(G)$ *for all fuzzy non-empty subgraphs* H *of* G.

Definition 2.6. *A complete intuitionistic fuzzy graph* $G:\langle(v_i,\mu_1,\gamma_1),(e_{ij},\mu_2,\gamma_2)\rangle$ *is balanced if* $D_\mu(H) = D_\mu(G)$ *and* $D_\gamma(H) = D_\gamma(G)$ *for all fuzzy non-empty subgraphs* H *of* G.

Example 2.7. *Consider an intuitionistic triple layered fuzzy graph* $G :< (v_i,\mu_1,\gamma_1),(e_{ij},\mu_2,\gamma_2) >$.

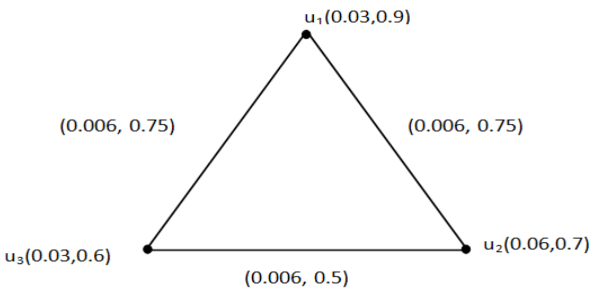

Figure 1. **Complete balanced intuitionistic fuzzy graph**

The density of the given graph is $D_\mu(G) = 0.4$ *and* $D_\gamma(G) = 1.7$ *let consider the subgraph* $H_1 = (u_1,u_2), H_2 = (u_1,u_2), H_3 = (u_1,u_2)$, *now* $D_\mu(H_1) = 0.4, D_\gamma(H_1) = 1.7$. *Since* $D_\mu(H) = D_\mu(G)$ *and* $D_\gamma(H) = D_\gamma(G)$ *the given intuitionistic fuzzy graph is complete.*

Definition 2.8. *Let $G : \langle(v_i, \mu_1, \gamma_1), (e_{ij}, \mu_2, \gamma_2)\rangle$ be an intuitionistic fuzzy graph with the underlying crisp graph $G^* : (\sigma^*, \mu^*)$. The pair $ITL(G) : \langle(v_i, \mu_{TL1}, \gamma_{TL1}), (e_{ij}, \mu_{TL2}, \gamma_{TL2})\rangle$ is called the intuitionistic fuzzy graph and is defined as follows. The node set of ITL(G) be $\langle\mu_{TL1}, \gamma_{TL1}\rangle$. The fuzzy subset $\langle\mu_{TL1}(u), \gamma_{TL1}(u)\rangle$ is defined as*

$$\langle\mu_{TL1}(u), \gamma_{TL1}(u)\rangle = \begin{cases} \langle\mu_1(u), \gamma_1(u)\rangle & \text{if } u \in \sigma_* \\ \text{where } 0 \le \langle\mu_{TL1} + \gamma_{TL1}\rangle \le 1 & \\ \langle\mu_2(uv), \gamma_2(uv)\rangle & \text{if } uv \in \mu_* \end{cases}$$

The fuzzy relation $\langle\mu_{TL2} + \gamma_{TL2}\rangle$ on $\sigma^* \cup \mu^* \cup \mu^*$ is defined as

$$\langle\mu_{TL2}(uv) + \gamma_{TL2}(uv)\rangle = \begin{cases} \langle\mu_2(u,v), \gamma_2(u,v)\rangle \\ \text{if } uv \in \mu_* \\ \langle\mu_2(e_i) \wedge \mu_2(e_j),\ \gamma_2(e_i) \wedge \gamma_2(e_j)\rangle \\ \text{if the edge } e_i \\ \text{and have a node incommon between them.} \\ \langle\mu_1(u_i) \wedge \mu_2(e_i),\ \gamma_1(u_i) \wedge \gamma_2(e_i)\rangle \\ \text{if } \ u_i \in \sigma^* \text{ and } e_i \in \mu^* \text{ each } \ e_i \text{is incident} \\ \ \text{with single } u_i \text{ either clockwise or anticlockwise} \\ 0 \text{ otherwise} \end{cases}$$

3. Balanced Intuitionistic Triple Layered Fuzzy Graphs

Let $G : \langle(v_i, \mu_1, \gamma_1),\ (e_{ij}, \mu_2, \gamma_2)\rangle$ be an intuitionistic fuzzy graph with the underlying crisp graph $G^* : (\sigma^*, \mu^*)$. The pair BITL(G):$\langle(v_i, \mu_{BITL1}, \gamma_{BITL1}), (e_{ij}, \mu_{BITL2}, \gamma_{BITL2})\rangle$ is called the balanced intuitionistic fuzzy graph and is defined as follows. The node set of BITL(G) be $\langle\mu_{BTL1}, \gamma_{BTL1}\rangle$. The fuzzy subset $\langle mu_{BTL1}, \gamma_{BTL1}\rangle$ is defined as

$$\langle\mu_{BITL1}(u), \gamma_{BITL1}(u)\rangle = \begin{cases} \langle\mu_1(u), \gamma_1(u)\rangle & \text{if } u \in \sigma^* \\ \text{where } 0 \le \langle\mu_{TL1} + \gamma_{TL1}\rangle \le 1 & \\ \langle\mu_2(uv), \gamma_2(uv)\rangle & \text{if } uv \in \mu^* \end{cases}$$

Case (i):

(1). $\langle\mu_{BITL2}(uv), \gamma_{BITL}(uv)\rangle = \langle\mu_2(u,v), \gamma_2(u,v)\rangle$ if $uv \in \sigma^*$.

(2). $\langle\mu_{BITL2}(uv), \gamma_{BITL}(uv)\rangle = \langle\mu_2(e_i) \wedge \mu_2(e_j),\ \gamma_2(e_i) \wedge \gamma_2(e_j)\rangle$ if the edge e_i and e_j have a node in common between them.

(3). $\langle\mu_{BITL2}(uv), \gamma_{BITL}(uv)\rangle = \langle\mu_1(e_i) \wedge \mu_2(e_j),\ \gamma_1(e_i) \wedge \gamma_2(e_j)\rangle$ if $u_i \in \sigma^*$ and $e_i \in \mu^*$ each e_i is incident with single u_i either clockwise or anticlockwise.

(4). $\mu_{BITL2} = 0$ otherwise.

Case (ii):

(1). $\langle \mu_{BITL2}(uv), \gamma_{BITL}(uv)\rangle < \langle \mu_2(u,v), \gamma_2(u,v)\rangle$ if $uv \in \sigma^*$.

(2). $\langle \mu_{BITL2}(uv), \gamma_{BITL}(uv)\rangle = \langle \mu_2(e_i) \wedge \mu_2(e_j),\ \gamma_2(e_i) \wedge \gamma_2(e_j)\rangle$ if the edge e_i and e_j have a node in common between them.

(3). $\langle \mu_{BITL2}(uv), \gamma_{BITL}(uv)\rangle < \langle \mu_1(e_i) \wedge \mu_2(e_j),\ \gamma_1(e_i) \wedge \gamma_2(e_j)\rangle$ if $u_i \in \sigma^*$ and $e_i \in \mu^*$ each e_i is incident with single u_i either clockwise or anticlockwise.

(4). $\mu_{BITL2} = 0$ otherwise.

Case (iii):

(1). $\langle \mu_{BITL2}(uv), \gamma_{BITL}(uv)\rangle < \langle \mu_2(u,v), \gamma_2(u,v)\rangle$ if $uv \in \sigma^*$.

(2). $\langle \mu_{BITL2}(uv), \gamma_{BITL}(uv)\rangle < \langle \mu_2(e_i) \wedge \mu_2(e_j),\ \gamma_2(e_i) \wedge \gamma_2(e_j)\rangle$ if the edge e_i and e_j have a node in common between them.

(3). $\langle \mu_{BITL2}(uv), \gamma_{BITL}(uv)\rangle = \langle \mu_1(e_i) \wedge \mu_2(e_j),\ \gamma_1(e_i) \wedge \gamma_2(e_j)\rangle$ if $u_i \in \sigma^*$ and $e_i \in \mu^*$ each e_i is incident with single u_i either clockwise or anticlockwise.

(4). $\mu_{BITL2} = 0$ otherwise.

Case (iv):

(1). $\langle \mu_{BITL2}(uv), \gamma_{BITL}(uv)\rangle < \langle \mu_2(u,v), \gamma_2(u,v)\rangle$ if $uv \in \sigma^*$.

(2). $\langle \mu_{BITL2}(uv), \gamma_{BITL}(uv)\rangle < \langle \mu_2(e_i) \wedge \mu_2(e_j),\ \gamma_2(e_i) \wedge \gamma_2(e_j)\rangle$ if the edge e_i and e_j have a node in common between them.

(3). $\langle \mu_{BITL2}(uv), \gamma_{BITL}(uv)\rangle < \langle \mu_1(e_i) \wedge \mu_2(e_j),\ \gamma_1(e_i) \wedge \gamma_2(e_j)\rangle$ if $u_i \in \sigma^*$ and $e_i \in \mu^*$ each e_i is incident with single u_i either clockwise or anticlockwise.

(4). $\mu_{BITL2} = 0$ otherwise.

4. Theoretical Concept on BTLFG

Theorem 4.1. *Every complete balanced intuitionistic triple layered fuzzy graph is balanced.*

Proof. Let $BITL(G) : \langle (v_i, \mu_{BITL1}, \gamma_{BITL1}),\ (e_{ij}, \mu_{BITL2}, \gamma_{BITL2})\rangle$ be a balanced intuitionistic complete triple layered fuzzy graph, then by the definition we have,

(1). $\langle \mu_{BITL2}(uv), \gamma_{BITL}(uv)\rangle = \langle \mu_2(u,v), \gamma_2(u,v)\rangle$ if $uv \in \sigma^*$.

(2). $\langle \mu_{BITL2}(uv), \gamma_{BITL}(uv)\rangle = \langle \mu_2(e_i) \wedge \mu_2(e_j),\ \gamma_2(e_i) \wedge \gamma_2(e_j)\rangle$ if the edge e_i and e_j have a node in common between them.

(3). $\langle \mu_{BITL2}(uv), \gamma_{BITL}(uv)\rangle = \langle \mu_1(e_i) \wedge \mu_2(e_j),\ \gamma_1(e_i) \wedge \gamma_2(e_j)\rangle$ if $u_i \in \sigma^*$ and $e_i \in \mu^*$ each e_i is incident with single u_i either clockwise or anticlockwise.

(4). $\mu_{BITL2} = 0$ otherwise.

Now $D_{\mu}(G) = 2\left[\frac{\sum\limits_{u,v\in V}\mu(u,v)}{\sum\limits_{u,v\in V}\sigma(u)\wedge\sigma(v)}\right]$, where $\sum\limits_{u,v\in V}\mu(u,v) = \sum\limits_{u,v\in V}\sigma(u)\vee\sigma(v)$. We have, $D_{\mu}(G) = 2$ and now $D_{\gamma}(G) = 2\left[\frac{\sum\limits_{u,v\in V}\mu(u,v)}{\sum\limits_{u,v\in V}\sigma(u)\vee\sigma(v)}\right]$, where $\sum\limits_{u,v\in V}\mu(u,v) = \sum\limits_{u,v\in V}\sigma(u)\wedge\sigma(v)$. We have, $D_{\gamma}(G) = 2 \Rightarrow$ Density of all the subgraphs of a complete intuionistic triple layered fuzzy graph equals 2. $D_{\gamma}(H) = 2, D_{\gamma}(H_n) = 2, \Rightarrow D_{\mu}(H) = D_{\mu}(G)$ and $D_{\gamma}(H) = D_{\gamma}(G)$. Hence the graph BITL(G): $\langle(v_i,\mu_{BITL1},\gamma_{BITL1}),\ (e_{ij},\mu_{BITL2},\gamma_{BITL2})\rangle$ is complete. □

Theorem 4.2. *Let BITL(G): $\langle(v_i,\mu_{BITL1},\gamma_{BITL1}),(e_{ij},\mu_{BITL2},\gamma_{BITL2})\rangle$ be a balanced intuitionistic triple layered fuzzy graph, if $\langle\mu_{BITL2}(uv),\gamma_{BITL2}(uv)\rangle = \langle\mu_2(e_i)\wedge\mu_2(e_j),\gamma_2(e_i)\wedge\gamma_2(e_j)\rangle$ and $\langle\mu_{BITL2}(uv),\gamma_{BITL2}(uv)\rangle < \langle\mu_2(ui)\wedge\mu_2(e_i),\gamma_2((ui)\wedge\gamma_2(e_i)\rangle$ the given graph is not balanced.*

Proof. Let $BITL(G) : \langle(v_i,\mu_{BITL1},\gamma_{BITL1}),\ (e_{ij},\mu_{BITL2},\gamma_{BITL2})\rangle$ be a balanced intuitionistic complete triple layered fuzzy graph, then by the definition we have,

(1). $\langle\mu_{BITL2}(uv),\gamma_{BITL}(uv)\rangle = \langle\mu_2(u,v),\gamma_2(u,v)\rangle$ if $uv \in \sigma^*$.

(2). $\langle\mu_{BITL2}(uv),\gamma_{BITL}(uv)\rangle = \langle\mu_2(e_i)\wedge\mu_2(e_j),\ \gamma_2(e_i)\wedge\gamma_2(e_j)\rangle$ if the edge e_i and e_j have a node in common between them.

(3). $\langle\mu_{BITL2}(uv),\gamma_{BITL}(uv)\rangle < \langle\mu_1(u_i)\wedge\mu_2(e_j),\ \gamma_1(u_i)\wedge\gamma_2(e_j)\rangle$ if $u_i \in \sigma^*$ and $e_i \in \mu^*$ each e_i is incident with single u_i either clockwise or anticlockwise.

(4). $\mu_{BITL2} = 0$ otherwise.

Now $D_{\mu}(G) = 2\left[\frac{\sum\limits_{u,v\in V}\mu(u,v)}{\sum\limits_{u,v\in V}\sigma(u)\wedge\sigma(v)}\right]$ and $D_{\gamma}(G) = 2\left[\frac{\sum\limits_{u,v\in V}\mu(u,v)}{\sum\limits_{u,v\in V}\sigma(u)\vee\sigma(v)}\right]$. The density of the graph will be $\mathrm{D}_{\mu}(G) < 2$ and $\mathrm{D}_{\gamma}(G) < 2$. Since $\langle\mu_{BITL2}(uv),\gamma_{BITL2}(uv)\rangle = \langle\mu_2(e_i)\wedge\mu_2(e_j),\gamma_2(e_i)\wedge\gamma_2(e_j)\rangle$ the density of this subgraph that is equivalent to the case defined above will be equal to 2. $\mathrm{D}_{\mu}(\mathrm{Hn}) = 2,\ \ \mathrm{D}_{\gamma}(\mathrm{Hn}) = 2$, because

$$\sum_{u,v\in V}\mu(u,v) = \sum_{u,v\in V}\sigma(u)\wedge\sigma(v)$$

$\Rightarrow \mathrm{D}_{\mu}(\mathrm{H}) > \mathrm{D}_{\mu}(\mathrm{G})$ and $\mathrm{D}_{\gamma}(\mathrm{H}) > \mathrm{D}_{\gamma}(\mathrm{G})$. Hence Case (ii) is not balanced. □

Example 4.3. *Consider a triple layered fuzzy graph G:(μ_1,μ_2) such that $\mu_1 = \{u_1,u_2,u_3,e_1,e_2,e_3,e_4,e_5,e_6\}$; $\mu_2 = \{(u_1,u_2),(u_1,u_3),(u_2,u_3),(u_1,e_1),(u_2,e_2),(u_3,e_3),(e_1,e_2),(e_1,e_3),(e_1,e_4),(e_2,e_3),(e_2,e_5),(e_3,e_6)(e_4,e_5),(e_4,e_6)(e_5,e_6)\}$*

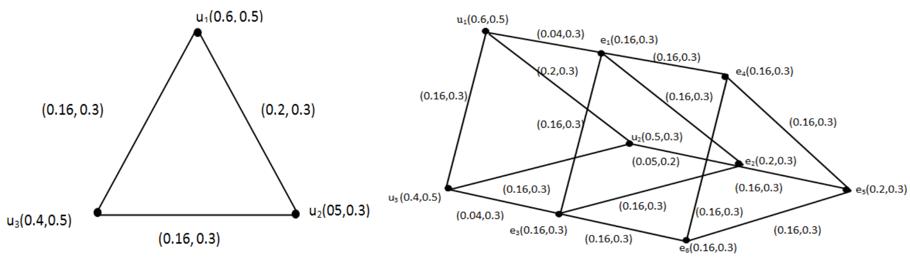

Figure 2. **unbalanced intuitionistic triple layered fuzzy graph**

The density of given triple layered fuzzy graph is $D_{\mu}(G) = 1.3$. and $D\gamma(G) = 1.6$. Let $H_1 = \{u_1,u_2\}$, $H_2 = \{u_1,u_3\}$, $H_3 = \{u_2,u_3\}$, $H_4 = \{e_1,e_2\}$, $H_5 = \{e_1,e_3\}$, $H_6 = \{e_1,e_4\}$, $H_7 = \{e_2,e_3\}$, $H_8 = \{e_2,e_5\}$, $H_9 = \{e_3,e_6\}$, $H_{10} = \{e_4,e_5\}$, $H_{11} = \{e_4,e_6\}$, $H_{12} = \{e_5,e_6\}$ be a non empty subgraphs of $D_{\mu}(G)$. The densities of the subgraphs are $D_{\mu}(H_1) = 0.8$,

$D_\mu(H_2) = 0.8$, $D_\mu(H_3) = 0.8$, $D_\mu(H_4) = 2$, $D_\mu(H_5) = 2$, $D_\mu(H_6) = 2$, $D_\mu(H_7) = 2$, $D_\mu(H_8) = 2$, $D_\mu(H_9) = 2$, $D_\mu(H_{10}) = 2$, $D_\mu(H_{11}) = 2$, $D_\mu(H_{12}) = 2$. *Shows that the density of subgraphs,* H_{10}, H_{11}, H_{12} *are greater than the density of the graph* G, *i.e.* $D_\mu(H) > D_\mu(G)$. *Similarly Let* $H_1 = \{u_1, u_2\}$, $H_2 = \{u_1, u_3\}$, $H_3 = \{u_2, u_3\}$, $H_4 = \{e_1, e_2\}$, $H_5 = \{e_1, e_3\}$, $H_6 = \{e_1, e_4\}$, $H_7 = \{e_2, e_3\}$, $H_8 = \{e_2, e_5\}$, $H_9 = \{e_3, e_6\}$, $H_{10} = \{e_4, e_5\}$, $H_{11} = \{e_4, e_6\}$, $H_{12} = \{e_5, e_6\}$ *be a non empty subgraphs of* $D_\mu(G)$. *The densities of the subgraphs are* $D_\gamma(H_1) = 1.6$, $D_\gamma(H_2) = 1.6$, $D_\gamma(H_3) = 1.6$, $D_\gamma(H_4) = 2$, $D_\gamma(H_5) = 2$, $D_\gamma(H_6) = 2$, $D_\gamma(H_7) = 1.2$, $D_\gamma(H_8) = 2$, $D_\gamma(H_9) = 2$, $D_\gamma(H_{10}) = 2$, $D_\gamma(H_{11}) = 2$, $D_\gamma(H_{12}) = 2$. *Shows that the density of subgraphs,* $H_4, H_5, H_6, H_7, H_8, H_9, H_{10}, H_{11}, H_{12}$ *are greater than the density of the graph* G, *i.e.,* $D_\gamma(H) > D_\gamma(G)$. *Hence the graph is not balanced.*

Theorem 4.4. *Let BITL(G):* $\langle (v_i, \mu_{BITL1}, \gamma_{BITL1}), (e_{ij}, \mu_{BITL2}, \gamma_{BITL2}) \rangle$ *be a balanced intuitionistic triple layered fuzzy graph, if* $\langle \mu_{BITL2}(uv), \gamma_{BITL2}(uv) \rangle = \langle \mu_2(e_i) \wedge \mu_2(e_j), \gamma_2(e_i) \wedge \gamma_2(e_j) \rangle$ *and* $\langle \mu_{BITL2}(uv), \gamma_{BITL2}(uv) \rangle < \langle \mu_2(ui) \wedge \mu_2(e_i), \gamma_2((ui) \wedge \gamma_2(e_i) \rangle$ *the the given graph is not balanced.*

Proof. Let $BITL(G) : \langle (v_i, \mu_{BITL1}, \gamma_{BITL1}),\ (e_{ij}, \mu_{BITL2}, \gamma_{BITL2}) \rangle$ be a balanced intuitionistic complete triple layered fuzzy graph, then by the definition of balanced intuitionistic triple layered fuzzy graph

(1). $\langle \mu_{BITL2}(uv), \gamma_{BITL}(uv) \rangle < \langle \mu_2(u, v), \gamma_2(u, v) \rangle$ if $uv \in \sigma^*$.

(2). $\langle \mu_{BITL2}(uv), \gamma_{BITL}(uv) \rangle < \langle \mu_2(e_i) \wedge \mu_2(e_j),\ \gamma_2(e_i) \wedge \gamma_2(e_j) \rangle$ if the edge e_i and e_j have a node in common between them.

(3). $\langle \mu_{BITL2}(uv), \gamma_{BITL}(uv) \rangle < \langle \mu_1(u_i) \wedge \mu_2(e_j),\ \gamma_1(u_i) \wedge \gamma_2(e_j) \rangle$ if $u_i \in \sigma^*$ and $e_i \in \mu^*$ each e_i is incident with single u_i either clockwise or anticlockwise.

(4). $\mu_{BITL2} = 0$ otherwise.

Now $D_\mu(G) = 2\left[\frac{\sum_{u,v \in V} \mu(u,v)}{\sum_{u,v \in V} \sigma(u) \wedge \sigma(v)}\right]$ and $D_\gamma(G) = 2\left[\frac{\sum_{u,v \in V} \mu(u,v)}{\sum_{u,v \in V} \sigma(u) \vee \sigma(v)}\right]$. The density of the given graph D(G) will be less than 2, since i.e. $\langle \mu_{BITL2}(uv), \gamma_{BITL2}(uv) \rangle < \langle \mu_2(e_i) \wedge \mu_2(e_j), \gamma_2(e_i) \wedge \gamma_2(e_j) \rangle$, $\langle \mu_{BITL2}(uv), \gamma_{BITL2}(uv) \rangle < \langle \mu_1(ui) \wedge \mu_2(e_i), \gamma_1(ui) \vee \gamma_2(e_j) \rangle$. $D_\mu(G) = 2\left[\frac{\sum_{u,v \in V} \mu_2(u,v)}{\sum_{u,v \in V} \mu_1(u) \wedge \mu(v)}\right] < 2$ and $D_\gamma(G) = 2\left[\frac{\sum_{u,v \in V} \mu_2(u,v)}{\sum_{u,v \in V} \mu_1(u) \vee \mu_1(v)}\right] < 2$ all the subgraphs defined in the above condition will be less than 2 which makes the graph to be balanced i.e., $D_\mu(H) < D_\mu(G)$ and $D_\gamma(H) < D_\gamma(G)$. Hence the graph is balanced. □

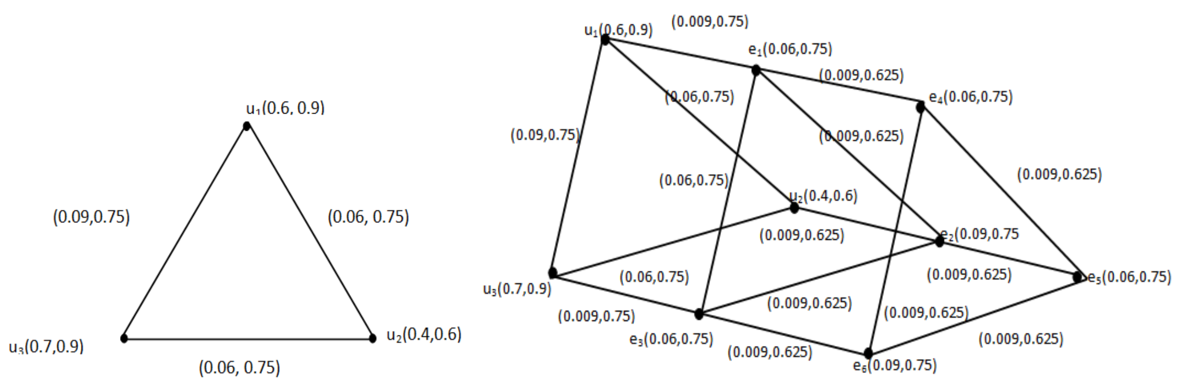

Figure 3. **Balanced Intuitionistic Triple Layered Fuzzy Graph with density .** $D(G) = 0.3$ **and** $D\gamma(G) = 1.7$

Example 4.5. *The density of the given graph is calculated using the formulae. Now* $D_\mu(G) = 2\left[\frac{\sum_{u,v\in V}\mu(u,v)}{\sum_{u,v\in V}\sigma(u)\wedge\sigma(v)}\right]$ *and* $D_\gamma(G) = 2\left[\frac{\sum_{u,v\in V}\mu(u,v)}{\sum_{u,v\in V}\sigma(u)\vee\sigma(v)}\right]$; $D_\mu(G) = 0.3$ *and* $D_\gamma(G) = 1.7$. *The density of the given graph is calculated using the same formulae and the values are listed in the following table (Table no.1), where we get* $D_\mu(H) = D_\mu(G)$ *and* $D_\gamma(H) = D_\gamma(G)$. *Hence the given intuitionistic triple layered fuzzy graph is balanced which satisfies the condition* $\langle\mu_{BITL2}(uv), \gamma_{BITL2}(uv)\rangle = \langle\mu_2(e_i)\wedge\mu_2(e_j), \gamma_2(e_i)\wedge\gamma_2(e_j)\rangle$ *and* $\langle\mu_{BITL2}(uv), \gamma_{BITL2}(uv)\rangle < \langle\mu_2(ui)\wedge\mu_2(e_i), \gamma_2((ui)\wedge\gamma_2(e_i)\rangle$. *Consider the sub graphs* H_1, H_8, *We get* $D_\mu(H_1) = \{u_1, u_2\} = 0.3$, *and for* H_8 *we get* $D_\mu(H_8) = \{u_1, e_3\} = 0$. *The same is true for* $D_\gamma(H_1) = \{u_1, u_2\} = 1.7$ *for* H_8 *we get* $D_\gamma(H_8) = \{u_1, e_8\} = 0$. *The density of the subgraphs are less than or equal to the density of the graph G i.e.,* $D_\mu(H) = D_\mu(G)$ *and* $D_\gamma(H) = D_\gamma(G)$.

Table 1. **The density of the subgraphs of D(G) are listed below**

S.NO	Sub graph of G(Hn)	DBITL(H)	S.NO	Sub graph of G(Hn)	DBITL(H)
1.	$H_1 = \{u_1, u_2\}$	0.3	42.	$H_{42} = \{e_6, e_4, e_5\}$	0.3
2.	$H_2 = \{u_1, u_3\}$	0.3	43.	$H_{43} = \{u, e_1, e_2\}$	0.3
3.	$H_3 = \{u_1, e_1\}$	0.3	44.	$H_{44} = \{u_1, e_1, e_3\}$	0.3
4.	$H_4 = \{u_1, e_4\}$	0	45.	$H_{45} = \{u_2, e_2, e_3\}$	0.3
5.	$H_5 = \{u_1, e_5\}$	0	46.	$H_{46} = \{u_2, e_2, e_1\}$	0.3
6.	$H_6 = \{u_1, e_6\}$	0	47.	$H_{47} = \{u_3, e_3, e_1\}$	0.3
7.	$H_7 = \{u_2, e_2\}$	0.3	48.	$H_{48} = \{u_3, e_{3,2}\}$	0.3
8.	$H_8 = \{u_2, e_3\}$	0	49.	$H_{49} = \{e_4, e_1, e_5\}$	0.3
9.	$H_9 = \{u_2, e_4\}$	0	50.	$H_{50} = \{e_1, u_1, e_4\}$	0.3
10.	$H_{10} = \{u_2, e_5\}$	0	51.	$H_{51} = \{e_2, u_2, e_5\}$	0.3
11.	$H_{11} = \{u_2, e_6\}$	0	52.	$H_{52} = \{e_6, e_3, u_3\}$	0.3
12.	$H_{12} = \{u_1, e_2\}$	0	53.	$H_{53} = \{u_1, u_2, u_3, e_2\}$	0.3
13.	$H_{13} = \{u_3, e_1\}$	0	54.	$H_{54} = \{u_2, u_1, u_3, e_2\}$	0.3
14.	$H_{14} = \{u_3, e_2\}$	0	55.	$H_{55} = \{u_2, u_3, u_1, e_1\}$	0.3
15.	$H_{15} = \{u3, e_3\}$	0.3	56.	$H_{56} = \{u_1, u_3, u_2, e_2\}$	0.3
16.	$H_{16} = \{u_3, e_4\}$	0	57.	$H_{57} = \{u_3, u_1, u_2, e_2\}$	0.3
17.	$H_{17} = \{u_3, e_5\}$	0	58.	$H_{58} = \{u_3, u_2, u_1, e_1\}$	0.3
18.	$H_{18} = \{u_3, e_6\}$	0	59.	$H_{59} = \{e_1, e_2, e_3, u_3\}$	0.3
19.	$H_{19} = \{e_3, e_4\}$	0	60.	$H_{60} = \{e_3, e_2, e_1, u_1\}$	0.3
20.	$H_{20} = \{e_3, e_5\}$	0	61.	$H_{61} = \{e_1, e_3, e_2, u_2\}$	0.3
21.	$H_{21} = \{e_3, e_6\}$	0.3	62.	$H_{62} = \{e_2, e_1, e_3, u_3\}$	0.3
22.	$H_{22} = \{e_1, e_3\}$	0.3	63.	$H_{63} = \{u_1, e_1, e_2, u_2\}$	0.3
23.	$H_{23} = \{e_1, e_2\}$	0.3	64.	$H_{64} = \{u_2, e_2, e_1, u_1\}$	0.3
24.	$H_{24} = \{e_1, e_4\}$	0.3	65.	$H_{65} = \{u_2, e_2, e_3, u_3\}$	0.3
25.	$H_{25} = \{e_4, e_2\}$	0	66.	$H_{66} = \{u_2, e_1, e_2, e_3\}$	0.3
26.	$H_{26} = \{e_4, e_3\}$	0	67.	$H_{67} = \{u_2, e_1, e_3, e_2\}$	0.3
27.	$H_{27} = \{u_2, e_6\}$	0	68.	$H_{68} = \{u_2, e_2, e_1, e_3\}$	0.3
28.	$H_{28} = \{u_2, e_1\}$	0	69.	$H_{69} = \{u_2, e_2, e_3, e_1\}$	0.3
29.	$H_{29} = \{u_2, e_3\}$	0	70.	$H_{70} = \{u_3, e_3, e_2, e_1\}$	0.3
30.	$H_{30} = \{u_2, e_4\}$	0	71.	$H_{71} = \{u_3, e_3, e_1, e_2\}$	0.3
31.	$H_{31} = \{u_2, e_5\}$	0	72.	$H_{72} = \{e_3, e_1, e_2, u_2\}$	0.3
32.	$H_{32} = \{u_2, e_6\}$	0	73.	$H_{73} = \{e_1, u_1, u_3, e_3\}$	0.3
33.	$H_{33} = \{e_3, e_6\}$	0	74.	$H_{74} = \{e_1, u_1, u_2, e_2\}$	0.3
34.	$H_{34} = \{u_1, u_2, u_3\}$	0.3	75.	$H_{75} = \{e_1, u_1, e_1, e_2\}$	0.3
35.	$H_{35} = \{u_2, u_3, u_1\}$	0.3	76.	$H_{76} = \{e_1, u_1, u_2, e_2\}$	0.3
36.	$H_{36} = \{u_3, u_1, u_2\}$	0.3	77.	$H_{77} = \{e_1, e_2, u_2, e_5\}$	0
37	$H_{37} = \{e_1, e_2, e_3\}$	0.3	78.	$H_{78} = \{e_2, e_3, u_3, e_3\}$	0
38.	$H_{38} = \{e_2, e_3, e_1\}$	0.3	79.	$H_{79} = \{e_2, u_2, e_5, e_3\}$	0
39.	$H_{39} = \{e_3, e_1, e_2\}$	0.3	80.	$H_{80} = \{e_2, u_2, e_5, e_1\}$	0
40.	$H_{40} = \{e_4, e_5, e_6\}$	0.3	81.	$H_{81} = \{e_6, e_5, e_2, u_2\}$	0.3
41.	$H_{41} = \{e_5, e_6, e_4\}$	0.3	82.	$H_{82} = \{e_2, u_2, u_3, e_3\}$	0.3

S.NO	Sub graph of G(Hn)	DBITL(H)	S.NO	Sub graph of G(Hn)	DBITL(H)
83.	$H_{83} = \{e_2, u_2, u_1, e_1\}$	0.3	108.	$H_{108} = \{e_3, e_2, u_2, u_3\}$	0.3
84.	$H_{84} = \{e_6, u_3, e_3, e_1\}$	0	109.	$H_{109} = \{u_1, u_2, u_3, e_6, e_1\}$	0
85.	$H_{85} = \{e_6, e_1, u_1, e_1\}$	0	110.	$H_{110} = \{u_1, u_3, u_2, e_2, e_5\}$	0.3
86.	$H_{86} = \{e_6, u_3, e_3, e_2\}$	0	111.	$H_{111} = \{u_3, u_2, e_2, e_1\}$	0.3
87.	$H_{87} = \{e_6, u_3, u_2, e_2\}$	0	112.	$H_{112} = \{u_1, e_1, e_2, e_5, e_6\}$	0.3
88.	$H_{88} = \{e_1, e_2, e_3, u_3\}$	0	113.	$H_{113} = \{u_1, e_1, e_6, e_2, u_2\}$	0
89.	$H_{89} = \{e_1, u_1, u_2, u_3\}$	0.3	114.	$H_{114} = \{u_1, e_1, e_6, u_3, u_2\}$	0
90.	$H_{90} = \{e_1, u_1, u_2, e_6\}$	0	115.	$H_{115} = \{u_2, u_3, e_6, e_2, e_1\}$	0
91.	$H_{91} = \{e_1, e_2, u_2, u_3\}$	0.3	116.	$H_{116} = \{u_2, u_3, e_6, e_1, u_1\}$	0
92.	$H_{92} = \{e_1, e_2, u_2, u_1\}$	0.3	117.	$H_{117} = \{u_2, u_1, u_3, e_6, e_1\}$	0
93.	$H_{93} = \{e_1, e_3, u_3, u_2\}$	0.3	118.	$H_{118} = \{u_2, e_2, e_3, e_1, u_1\}$	0.3
94.	$H_{94} = \{e_1, e_3, e_2, u_2\}$	0.3	119.	$H_{119} = \{u_3, e_3, e_1, u_1, u_2\}$	0.3
95.	$H_{95} = \{e_1, e_3, u_3, u_6\}$	0	120.	$H_{120} = \{u_3, u_1, u_2, e_2, e_5\}$	0.3
96.	$H_{96} = \{e_2, e_3, e_1, u_1\}$	0.3	121.	$H_{121} = \{u_3, u_2, e_2, e_1, e_4\}$	0.3
97.	$H_{97} = \{e_2, e_3, u_3, e_6\}$	0	122.	$H_{122} = \{e_1, u_1, u_2, u_3, e_3\}$	0.3
98.	$H_{98} = \{e_2, u_2, e_5, e_1\}$	0	123.	$H_{123} = \{e_1, e_4, e_6, e_5, e_2\}$	0.3
99.	$H_{99} = \{e_2, u_2, u_3, e_6\}$	0.3	124.	$H_{124} = \{e_1, e_3, e_6, e_4, e_1, e_2\}$	0.3
100.	$H_{100} = \{e_2, u_2, e_5, e_6\}$	0.3	125.	$H_{125} = \{e_2, e_5, e_6, e_3, u_3\}$	0.3
101.	$H_{101} = \{e_2, u_2, u_3, u_1\}$	0.3	126.	$H_{126} = \{e_1, e_3, u_3, u_2, u_1\}$	0.3
102.	$H_{102} = \{e_3, e_2, e_1, u_1\}$	0.3	127.	$H_{127} = \{e_6, e_3, u_3, u_2, u_1\}$	0.3
103.	$H_{103} = \{e_3, u_3, e_6, e_1\}$	0	128.	$H_{128} = \{e_6, e_5, e_4, e_1, e_2\}$	0.3
104.	$H_{104} = \{e_3, u_3, u_2, e_2\}$	0.3	129.	$H_{129} = \{u_2, u_3, e_3, e_2, e_1, u_1, u_2, e_2, e_5\}$	0.3
105.	$H_{105} = \{e_3, e_2, u_2, u_1\}$	0.3	130.	$H_{130} = \{e_1, e_2, e_3, e_6, e_5, e_4, e_1, e_2, e_3\}$	0.3
106.	$H_{106} = \{e_3, e_2, u_2, e_1\}$	0	131.	$H_{131} = \{u_1, u_2, u_3, e_3, e_2, e_1, e_4, e_5, e_6\}$	0.3
107.	$H_{107} = \{e_3, e_1, u_1, u_3\}$	0.3	132.	$H_{132} = \{e_3, e_2, e_1, e_4, e_5, e_6, e_4, e_1, u_1\}$	0.3

Table 2. **The density of the subgraphs of Dγ(G) are listed below**

S.NO	Sub graph of Gγ(Hn)	DBITLγ(H)	S.NO	Sub graph of Gγ(Hn)	DBITLγ(H)
1.	$H_1 = \{u_1, u_2\}$	1.7	29.	$H_{29} = \{u_2, e_3\}$	0
2.	$H_2 = \{u_1, u_3\}$	1.7	30.	$H_{30} = \{u_2, e_4\}$	0
3.	$H_3 = \{u_1, e_1\}$	1.7	31.	$H_{31} = \{u_2, e_5\}$	0
4.	$H_4 = \{u_1, e_4\}$	0	32.	$H_{32} = \{u_2, e_6\}$	0
5.	$H_5 = \{u_1, e5\}$	0	33.	$H_{33} = \{e_3, e_6\}$	0
6.	$H_6 = \{u_1, e_6\}$	0	34.	$H_{34} = \{u_1, u_2, u_3\}$	1.7
7.	$H_7 = \{u_2, e_2\}$	1.7	35.	$H_{35} = \{u_2, u_3, u_1\}$	1.7
8.	$H_8 = \{u_2, e_3\}$	0	36.	$H_{36} = \{u_3, u_1, u_2\}$	1.7
9.	$H_9 = \{u_2, e_4\}$	0	37.	$H_{37} = \{e_1, e_2, e_3\}$	1.7
10.	$H_{10} = \{u_2, e_5\}$	0	38.	$H_{38} = \{e_2, e_3, e_1\}$	1.7
11.	$H_{11} = \{u_2, e_6\}$	0	39.	$H_{39} = \{e_3, e_1, e_2\}$	1.7
12.	$H_{12} = \{u_1, e_2\}$	0	40.	$H_{40} = \{e_4, e_5, e_6\}$	1.7
13.	$H_{13} = \{u_3, e_1\}$	0	41.	$H_{41} = \{e_5, e_6, e_4\}$	1.7
14.	$H_{14} = \{u_3, e_2\}$	0	42.	$H_{42} = \{e_6, e_4, e_5\}$	1.7
15.	$H_{15} = \{u_3, e_3\}$	1.7	43.	$H_{43} = \{u_1, e_1, e_2\}$	1.7
16.	$H_{16} = \{u_3, e_4\}$	0	44.	$H_{44} = \{u_1, e_1, e_3\}$	1.7
17.	$H_{17} = \{u_3, e_5\}$	0	45.	$H_{45} = \{u_2, e_2, e_3\}$	1.7
18.	$H_{18} = \{u_3, e_6\}$	0	46.	$H_{46} = \{u_2, e_2, e_1\}$	1.7
19.	$H_{19} = \{e_3, e_4\}$	0	47.	$H_{47} = \{u_3, e_3, e_1\}$	1.7
20.	$H_{20} = \{e_3, e_5\}$	0	48.	$H_{48} = \{u_3, e_3, e_2\}$	1.7
21.	$H_{21} = \{e_3, e_6\}$	0	49.	$H_{49} = \{e_4, e_1, e_5\}$	1.7
22.	$H_{22} = \{e_1, e_3\}$	1.7	50.	$H_{50} = \{e_1, u_1, e_4\}$	1.7
23.	$H_{23} = \{e_1, e_2\}$	1.7	51.	$H_{51} = \{e_2, u_2, e_5\}$	1.7
24.	$H_{24} = \{e_1, e_4\}$	1.7	52.	$H_{52} = \{e_6, e_3, u_3\}$	1.7
25.	$H_{25} = \{e_4, e_2\}$	0	53.	$H_{53} = \{u_1, u_2, u_3, e_2\}$	1.7
26.	$H_{26} = \{e_4, e_3\}$	0	54.	$H_{54} = \{u_2, u_1, u_3, e_2\}$	1.7
27.	$H_{27} = \{u_2, e_6\}$	0	55.	$H_{55} = \{u_2, u_3, u_1, e_1\}$	1.7
28.	$H_{28} = \{u_2, e_1\}$	0	56.	$H_{56} = \{u_1, u_3, u_2, e_2\}$	1.7

S.NO	Sub graph of Gγ(Hn)	DBITLγ(H)	S.NO	Sub graph of Gγ(Hn)	DBITLγ(H)
57.	$H_{57} = \{u_3, u_1, u_2, e_2\}$	1.7	95.	$H_{95} = \{e_1, e_3, u_3, u_6\}$	0
58.	$H_{58} = \{u_3, u_2, u_1, e_1\}$	1.7	96.	$H_{96} = \{e_2, e_3, e_1, u_1\}$	1.7
59.	$H_{59} = \{e_1, e_2, e_3, u_3\}$	1.7	97.	$H_{97} = \{e_2, e_3, u_3, e_6\}$	0
60.	$H_{60} = \{e_3, e_2, e_1, u_1\}$	1.7	98.	$H_{98} = \{e_2, u_2, e_5, e_1\}$	0
61.	$H_{61} = \{e_1, e_3, e_2, u_2\}$	1.7	99.	$H_{99} = \{e_2, u_2, u_3, e_6\}$	1.7
62.	$H_{62} = \{e_2, e_1, e_3, u_3\}$	1.7	100.	$H_{100} = \{e_2, u_2, e_5, e_6\}$	1.7
63.	$H_{63} = \{u_1, e_1, e_2, u_2\}$	1.7	101.	$H_{101} = \{e_2, u_2, u_3, u_1\}$	1.7
64.	$H_{64} = \{u_2, e_2, e_1, u_1\}$	1.7	102.	$H_{102} = \{e_3, e_2, e_1, u_1\}$	1.7
65.	$H_{65} = \{u_2, e_2, e_3, u_3\}$	1.7	103.	$H_{103} = \{e_3, u_3, e_6, e_1\}$	0
66.	$H_{66} = \{u_2, e_1, e_2, e_3\}$	1.7	104.	$H_{104} = \{e_3, u_3, u_2, e_2\}$	1.7
67.	$H_{67} = \{u_2, e_1, e_3, e_2\}$	1.7	105.	$H_{105} = \{e_3, e_2, u_2, u_1\}$	1.7
68.	$H_{68} = \{u_2, e_2, e_1, e_3\}$	1.7	106.	$H_{106} = \{e_3, e_2, u_2, e_1\}$	0
69.	$H_{69} = \{u_2, e_2, e_3, e_1\}$	1.7	107.	$H_{107} = \{e_3, e_1, u_1, u_3\}$	1.7
70.	$H_{70} = \{u_3, e_3, e_2, e_1\}$	1.7	108.	$H_{108} = \{e_3, e_2, u_2, u_3\}$	1.7
71.	$H_{71} = \{u_3, e_3, e_1, e_2\}$	1.7	109.	$H_{109} = \{u_1, u_2, u_3, e_6, e_1\}$	0
72.	$H_{72} = \{e_3, e_1, e_2, u_2\}$	1.7	110.	$H_{110} = \{u_1, u_3, u_2, e_2, e_5\}$	1.7
73.	$H_{73} = \{e_1, u_1, u_3, e_3\}$	1.7	111.	$H_{111} = \{u_3, u_2, e_2, e_1\}$	1.7
74.	$H_{74} = \{e_1, u_1, u_2, e_2\}$	1.7	112.	$H_{112} = \{u_1, e_1, e_2, e_5, e_6\}$	1.7
75.	$H_{75} = \{e_1, u_1, e_1, e_2\}$	1.7	113.	$H_{113} = \{u_1, e_1, e_6, e_2, u_2\}$	0
76.	$H_{76} = \{e_1, u_1, u_2, e_2\}$	1.7	114.	$H_{114} = \{u_1, e_1, e_6, u_3, u_2\}$	0
77.	$H_{77} = \{e_1, e_2, u2, e_5\}$	0	115.	$H_{115} = \{u_2, u_3, e_6, e_2, e_1\}$	0
78.	$H_{78} = \{e_2, e_3, u_3, e_3\}$	0	116.	$H_{116} = \{u_2, u_3, e_6, e_1, u_1\}$	0
79.	$H_{79} = \{e_2, u_2, e_5, e_3\}$	0	117.	$H_{117} = \{u_2, u_1, u_3, e_6, e_1\}$	0
80.	$H_{80} = \{e_2, u_2, e_5, e_1\}$	0	118.	$H_{118} = \{u_2, e_2, e_3, e_1, u_1\}$	1.7
81.	$H_{81} = \{e_6, e_5, e_2, u_2\}$	1.7	119.	$H_{119} = \{u_3, e_3, e_1, u_1, u_2\}$	1.7
82.	$H_{82} = \{e_2, u_2, u_3, e_3\}$	1.7	120.	$H_{120} = \{u_3, u_1, u_2, e_2, e_5\}$	1.7
83.	$H_{83} = \{e_2, u_2, u_1, e_1\}$	1.7	121.	$H_{121} = \{u_3, u_2, e_2, e_1, e_4\}$	1.7
84.	$H_{84} = \{e_6, u_3, e_3, e_1\}$	0	122.	$H_{122} = \{e_1, u_1, u_2, u_3, e_3\}$	1.7
85.	$H_{85} = \{e_6, e_1, u_1, e_1\}$	0	123.	$H_{123} = \{e_1, e_4, e_6, e_5, e_2\}$	1.7
86.	$H_{86} = \{e_6, u_3, e_3, e_2\}$	0	124.	$H_{124} = \{e_1, e_3, e_6, e_4, e_1, e_2\}$	1.7
87.	$H_{87} = \{e_6, u_3, u_2, e_2\}$	0	125.	$H_{125} = \{e_2, e_5, e_6, e_3, u_3\}$	1.7
88.	$H_{88} = \{e_1, e_2, e_3, u_3\}$	0	126.	$H_{126} = \{e_1, e_3, u_3, u_2, u_1\}$	1.7
89.	$H_{89} = \{e_1, u_1, u_2, u_3\}$	1.7	127.	$H_{127} = \{e_6, e_3, u_3, u_2, u_1\}$	1.7
90.	$H_{90} = \{e_1, u_1, u_2, e_6\}$	0	128.	$H_{128} = \{e_6, e_5, e_4, e_1, e_2\}$	1.7
91.	$H_{91} = \{e_1, e_2, u_2, u_3\}$	1.7	129.	$H_{129} = \{u_2, u_3, e_3, e_2, e_1, u_1, u_2, e_2, e_5\}$	1.7
92.	$H_{92} = \{e_1, e_2, u_2, u_1\}$	1.7	130.	$H_{130} = \{e_1, e_2, e_3, e_6, e_5, e_4, e_1, e_2, e_3\}$	1.7
93.	$H_{93} = \{e_1, e_3, u_3, u_2\}$	1.7	131.	$H_{131} = \{u_1, u_2, u_3, e_3, e_2, e_1, e_4, e_5, e_6\}$	1.7
94.	$H_{94} = \{e_1, e_3, e_2, u_2\}$	1.7	132.	$H_{132} = \{e_3, e_2, e_1, e_4, e_5, e_6, e_4, e_1, u_1\}$	1.7

5. Conclusion

In this paper we have introduced the balanced intuitionistic triple layered fuzzy graph. We have derived the condition for intuitionistic triple layered fuzzy graph. Justified our definitions and results through illustrating few examples.

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