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Structural Core Graph of Triple Layered Fuzzy Graph

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Abstract: The Triple Layered Fuzzy Graph (TLFG) gives the $3-D$ structure to fuzzy graph. In this paper, we constructed the structural core graph for the given TLFG using a new algorithm and also the structural core graph for the union of two TLFG is also constructed using the same algorithm. Some of its diagrammatic properties are studied.

MSC: 15B15.

Keywords: Fuzzy graph, Triple layered fuzzy graph, face value, structural core graph.

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1. Introduction

Fuzzy graph theory was introduced by Rosenfeld in 1975 [5]. The degree of a vertex in some fuzzy graphs was discussed by Nagoorgani and Radha [7]. Nagoorgani and Malarvizhi have defined different types of fuzzy graphs and discussed its relationships with isomerism in fuzzy graphs [3]. The double layered fuzzy graph was introduced by Pathinathan and Jesintha Rosline, they have examined some of the properties of DLFG [4]. In this paper, Mrs. L.Jethruth Emelda Mary and P. Amutha introduced the structural core graph of Triple Layered Fuzzy Graph Using new algorithm.

2. Preliminaries

We start with some basic definitions.

Definition 2.1. *A fuzzy graph G is a pair of functions $G:(\sigma,\mu)$ where σ is a fuzzy subset of a non empty set S and μ is a symmetric fuzzy relation on σ The underlying crisp graph of $G:(\sigma,\mu)$ is denoted by $G^*:(\sigma^*,\mu^*)$*

Definition 2.2. *Let $G:(\sigma,\mu)$ be a fuzzy graph, The order of G is defined as $O(G)=\sum\sigma(u)\ u\in V$.*

Definition 2.3. *Let $G:(\sigma,\mu)$ be a fuzzy graph, The size of G is defined as $S(G)=\sum\mu(u,v)\ u\in V$.*

Definition 2.4. *Let $G:(\sigma,\mu)$ be a fuzzy graph the degree of a vertex u in G is defined as $d(u)=\sum\mu(u,v)$ and is denoted as $d_G(u)$.*

Definition 2.5. *Let G be a fuzzy graph, The μ- complement of G is denoted as $G^\mu:(\sigma^\mu,\mu^\mu)$ where $\sigma^*\cup\mu^*$ and*

$$\mu^\mu(u,v)=\begin{cases}\sigma(u)\sigma(v)-\mu(u,v) & \text{if } \mu(u,v)>0\\ 0, & \text{if } \mu(u,v)=0\end{cases}$$

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Definition 2.6. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs with $G_1^* : (V_1, E_1)$ and $G_2^* : (V_2, E_2)$, with the union of G_1^* and G_2^*. Then the union of two fuzzy graphs G_1 and G_2 is a fuzzy graphs $G = G_1 \cup G_2 : (\sigma_1 \cup \sigma_2, \mu_1 \cup \mu_2)$ defined by*

$$(\sigma_1 \cup \sigma_2)(u) = \begin{cases} \sigma_1(u) & \text{if } u \in V_1 - V_2 \\ \sigma_2(u) & \text{if } u \in V_1 - V_2 \end{cases}$$

$$\textit{and} \quad (\mu_1 \cup \mu_2)(uv) = \begin{cases} \mu_1(uv) & \text{if } uv \in E_1 - E_2 \\ \mu_2(uv) & \text{if } uv \in E_1 - E_2 \end{cases}$$

Definition 2.7. *Let $G : (\sigma, \mu)$ be a fuzzy graph with the underlying crisp graph $G^* : (\sigma^*, \mu^*)$. The pair $DL : (\sigma_{DL}, \mu_{DL})$ is defined as follows. The node set of $DL(G)$ be $\sigma^* \cup \mu^*$. The fuzzy subset σ_{DL} is defined as*

$$\sigma_{DL} = \begin{cases} \sigma(u) & \text{if } u \in \sigma^* \\ \mu(uv) & \text{if } uv \in \mu^* \end{cases}$$

The fuzzy graph relation μ_{DL} on $\sigma^* \cup \mu^*$ is defined as

$$\sigma_{DL} = \begin{cases} \mu(uv) & \text{if } u, v \in \sigma^* \\ \mu(e_i) \wedge \mu(e_j) & \text{if the edge } e_i \text{ and } e_j \text{ have a node in common between them} \\ \sigma(\mu_i) \wedge \mu(e_i) & \text{if } \mu_i \in \sigma^* \text{ and } e_i \in \mu^* \text{ and each } e_i \text{ is incident single } \mu_i \text{ either clockwise or anticlockwise} \\ 0, & \text{otherwise} \end{cases}$$

By definition $\mu_{DL}(u, v) \leq \sigma_{DL}(u) \wedge \sigma_{DL}(v)$ for all u, v in $\sigma^* \cup \mu^*$. Here μ_{DL} is a fuzzy relation on the fuzzy subset σ_{DL}. Hence the pair $DL(G) : (\sigma_{DL}, \mu_{DL})$ is defined as Double Layered Fuzzy Graph (DLFG).

3. Definition of Triple Layered Fuzzy Graph

Let $G : (\sigma, \mu)$ be a fuzzy graph with the underlying crisp graph $G^* : (\sigma^*, \mu^*)$ the pair $TL(G) : (\sigma_{TL}, \mu_{TL})$ is defined as follows. The node set of $TL(G)$ be $\sigma^* \cup \mu^* \cup \mu^*$. The fuzzy subset σ_{TL} is defined as

$$\sigma_{TL} = \begin{cases} \sigma(u) & \text{if } u \in \sigma^* \\ 2\mu(uv) & \text{if } uv \in \mu^* \end{cases}$$

The fuzzy relation μ_{TL} on $\sigma^* \cup \mu^*$ is defined as

$$\mu_{Tl} = \begin{cases} \mu(u, v) & \text{if } u, v \in \sigma^* \\ \mu(e_i) \wedge \mu(e_j) & \text{if the edge } e_i \text{ and } e_j \text{ have a node in common them} \\ \sigma(u_i) \wedge \mu(e_i) & \text{if } u_i \in \sigma^* \text{ and } e_i \in \mu^* \text{ and each } e_i \text{ is incident with } u_i \text{ in clockwise direction} \\ \sigma(u_i) \wedge \mu(e_i) & \text{if } u_i \in \sigma^* \text{ and } e_i \in \mu^* \text{and each } e_i \text{ is incident with } u_i \text{ in anticlockwise direction} \\ 0 & \textit{otherwise} \end{cases}$$

By definition, $\mu_{TL}(u, v) = \sigma_{TL}(u) \leq \sigma_{TL}(v)$ for all u, v in $\sigma^* \cup \mu^*$. Here μ_{TL} is a fuzzy relation on the fuzzy subset σ_{TL}. Hence the pair $TL(G) : (\sigma_{TL}, \mu_{TL})$ is defined as Triple Layered Fuzzy Graph (TLFG).

3.1. Structural Core Graph

In this section, we have introduced new algorithm to construct a structural core graph of Triple Layered Fuzzy Graph (i.e) To obtain a spanning tree for the given Triple Layered Fuzzy Graph.

Algorithm 3:

(1). Construct a $TLFG$ with $3n$ vertices and $5n$ edges where n is the number of vertex in the base graph whose crisp graph is cycle.

(2). Calculate face values using the formulae $\min\left\{\dfrac{\mu(a,b)}{\sigma(a)\wedge\sigma(b)}\right\}$, where $\mu(a,b)$ is the weight of the edge (a,b) and $\sigma(a)$ & $\sigma(b)$ are membership value of vertices a and b in TLFG.

(3). Select a face with least value. If two(or) more faces are there with least value, choose face with least order value.

(4). Choose a vertex with least value in the selected face.

(5). Select the smallest fuzzy distance, fuzzy distance edge from the selected vertex and include that in T. If two (or) more edges are there with the same value choose an edge with least adjacent vertex value, where T is a tree of TLFG.

(6). If two (or) more vertices are there with same value then choose the edge with least intersecting face value.

(7). Repeat this procedure till we covers all the vertices of $TLFG$.

(8). Stop, when T becomes Spanning tree of TLFG.

Example 3.1. *Consider a fuzzy graph $G:(\sigma,\mu)$ with $n=3$ vertices whose crisp graph is a cycle C_3.*

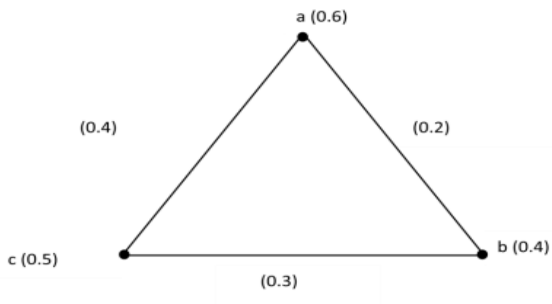

Figure 1. **Fuzzy Graph** (C_3)

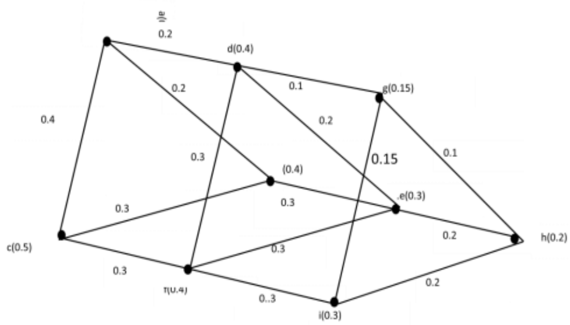

Figure 2. **TLEG of $n=3$ vertices.**

Face value calculation:

$$
\begin{aligned}
F_1\ (a\ b\ c) &\to \min\{0.5, 0.75, 0.8\} &= 0.5\\
F_2\ (a\ c\ d\ f) &\to \min\{0.8, 0.5, 0.5, 0.75\} &= 0.5\\
F_3\ (d\ e\ f) &\to \min\{0.67, 1, 0.5\} &= 0.5\\
F_4\ (a\ b\ e\ d) &\to \min\{0.5, 1, 0.67, 0.5\} &= 0.5\\
F_5\ (g\ h\ i) &\to \min\{1, 1, 1\} &= 1\\
F_6\ (d\ e\ g\ h) &\to \min\{0.67, 1, 0.67, 1\} &= 0.67\\
F_7\ (d\ g\ f\ i) &\to \min\{0.5, 0.5, 0.65, 1\} &= 0.5\\
F_8\ (c\ d\ e\ f) &\to \min\{0.75, 1, 1, 0.75\} &= 0.75\\
F_9\ (e\ f\ i\ h) &\to \min\{1, 1, 1, 075\} &= 0.75
\end{aligned}
$$

Reached node	Edge	Membership Value	Iteration
e	ed	0.2	1
ed	dg	0.1	2
edg	gh	0.5	3
edgh	he (form Cycle)	-	No
edgh	hi	0.2	4
edghi	ig (form Cycle)	-	No
edghi	if	0.3	5
edghif	fe (form Cycle)	-	No
edghif	fc	0.3	6
edghifc	cb	0.3	7
edghifcb	ba	0.2	8

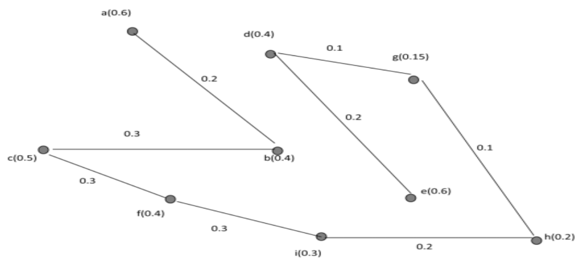

Figure 3. **Structural core graph of TLFG of $n = 3$ vertices**

Figure 4. **Fuzzy graph (C_4)**

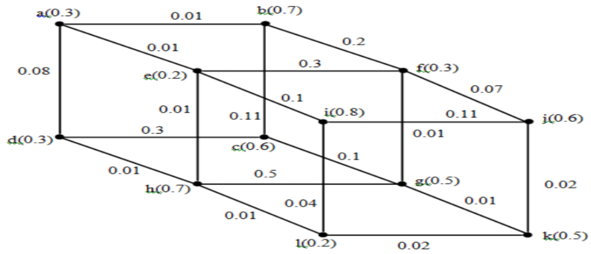

Figure 5. **TLFG of n = 4 vertices**

Example 3.2. ***Face value calculation:***

$$F_1\ (a\ b\ c\ d) \rightarrow \min\{0.03, 0.18, 1, 0.26\} = 0.03$$

$$F_2\ (e\ f\ g\ h) \rightarrow \min\{1, 0.03, 1, 0.02\} = 0.02$$

$$F_3\ (l\ i\ j\ k) \rightarrow \min\{0.2, 0.18, 0.04, 0.02\} = 0.04$$

$$F_4\ (a\ e\ h\ d) \rightarrow \min\{0.03, 0.02, 0.03, 0.26\} = 0.02$$

$$F_5\ (a\ e\ f\ b) \rightarrow \min\{0.03, 1, 0.66, 0.03\} = 0.03$$

$$F_6\ (f\ j\ l\ g) \rightarrow \min\{1, 0.4, 1, 0.33\} = 0.33$$

$$F_7\ (h\ g\ k\ l) \rightarrow \min\{1, 0.02, 0.25, 0.1\} = 0.02$$

$$F_8\ (a\ b\ f\ e) \rightarrow \min\{0.03, 0.66, 1, 0.33\} = 0.03$$

$$F_9\ (e\ f\ j\ i) \rightarrow \min\{1, 0.23, 0.18, 0.14\} = 0.14$$

$$F_{10}\ (b\ c\ g\ f) \rightarrow \min\{0.18, 0.4, 0.03, 0.66\} = 0.03$$

$$F_{11}\ (f\ j\ k\ g) \rightarrow \min\{0.23, 0.04, 0.03, 0.03\} = 0.03$$

Reached node	Edge	Membership Value	Iteration
l	lk	0.05	1
lk	kg	0.01	2
lkg	gh	0.01	3
lkgh	hl(form cycle)	-	No
lkgh	hd	0.01	4
lkghd	dc	0.3	5
lkghdc	cg(form cycle)	-	No
lkghdc	cb	0.11	6
lkghdcb	bf	0.2	7
lkghdcbf	fg(form cycle)	-	No
lkghdcbf	fe(form cycle)	-	No
lkghbcdfe	ei	0.3	8
lkghdcbfei	ij	0.1	9
lkghdcbfeij	jf(form cycle)	-	No
lkghdcbfeij	jk(form cycle)	-	No
lkghdcbfeij	Ji	0.11	10
lkghdcbfeiji	il(form cycle)	-	No
lkghdcbfeiji	Ie	0.1	11
lkghdcbfeijie	eh(form cycle)	-	No
lkghdcbfeijie	ea	0.01	12
lkghdcbfeiliea	ad(form cycle)	-	No
lkghdcbfeijiea	ab	0.01	13
lkghdcbfeijieab	bf(form cycle)	-	No
lkghdcbfeijieab	fe(form cycle)	-	No
lkghdcbfeijieab	bc	0.11	14

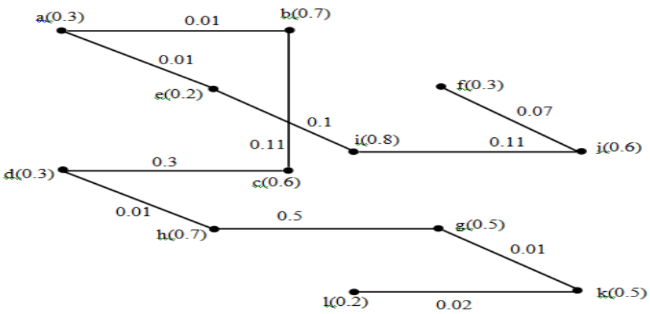

Figure 6. **Sructural core graph of TLFG of $n = 4$ vertices**

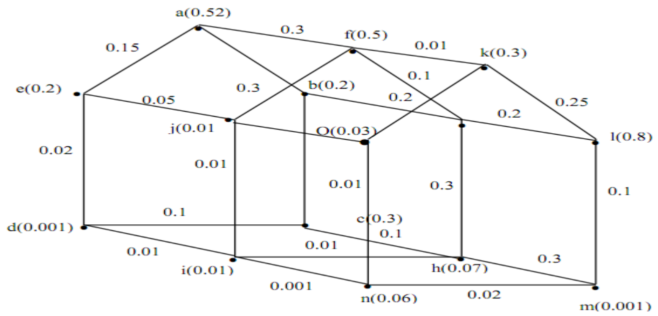

Figure 7. **TLFG of $n = 5$ vertices**

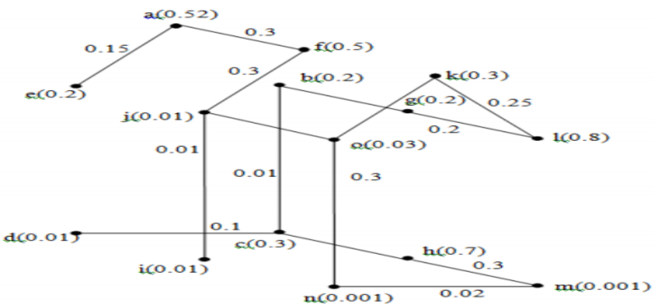

Figure 8. **Structural core graph of TLFG of $n = 5$ vertices**

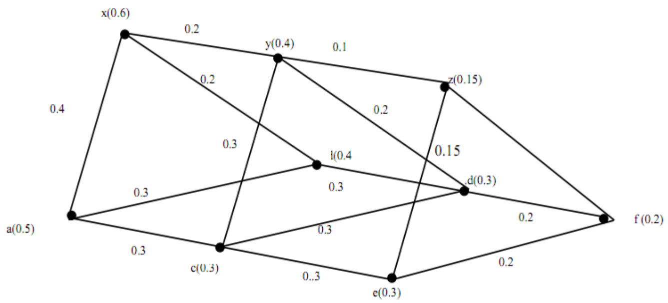

Figure 9. $TL(G_1)$

Example 3.3. ***Face value calculation:***

$$
\begin{aligned}
F_1\ (a\ b\ c\ d\ e) &\rightarrow \min\{0.576, 1, 0.33, 0.1, 0.75\} = 0.33 \\
F_2\ (f\ g\ h\ I\ j) &\rightarrow \min\{0.2, 0.75, 1, 1, 1\} = 0.2 \\
F_3\ (k\ l\ m\ n\ o) &\rightarrow \min\{0.833, 0.2, 0.33, 0.16, 1\} = 0.16 \\
F_4\ (a\ f\ j\ e) &\rightarrow \min\{0.6, 1, 0.25, 0.75, 0.6\} = 0.25 \\
F_5\ (f\ k\ o\ j) &\rightarrow \min\{0.3, 1, 0.66, 1, \} = 0.03 \\
F_6\ (e\ j\ I\ d) &\rightarrow \min\{0.25, 1, 1, 0.1\} = 0.1 \\
F_7\ (j\ o\ n\ i) &\rightarrow \min\{0.66, 0.16, 0.1, 1\} = 0.1 \\
F_8\ (b\ g\ h\ c) &\rightarrow \min\{1, 0.75, 0.33, 1\} = 0.33 \\
F_9\ (g\ l\ m\ h) &\rightarrow \min\{0.28, 0.2, 0.75, 0.75\} = 0.2 \\
F_{11}\ (a\ f\ g\ b) &\rightarrow \min\{0.6, 0.2, 1, 0.5\} = 0.2 \\
F_{12}\ (f\ k\ l\ g) &\rightarrow \min\{0.33, 0.83, 0.28, 0.2\} = 0.2
\end{aligned}
$$

Reached node	Edge	Membership Value	Iteration
i	in	0.001	1
in	nm	0.02	2
inm	mh	0.3	3
inmh	hi(form Cycle)	_	No
inmh	hc	0.1	4
inmhc	cd	0.1	5
inmhcd	di (Form Cycle)	_	No
inmhcdi	ij	0.01	6
inmhcdij	je	0.05	7
inmhcdije	ed(form cycle)	_	No
inmhcdije	ea	0.15	8
inmhcdijea	ab	0.1	9
inmkcdijeab	bc(form cycle)	_	No
inmhcdijeab	bg	0.2	10
inmhcdijeabg	gh(form cycle)	_	No
Inmhcdijeabg	gi	0.2	11
Inmhcdijeabgi	Im(form cycle)	_	No
Inmhcdijeabgi	Ik	0.25	12
Inmhcdijeabgi	Kf	0.01	13
Inmhcdijeabgik	fg(form cycle)	_	No
Inmhcdijeabgikf	fa	0.3	14
Inmhcdijeabgikf	ab(form cycle)	_	No
Inmhcdijeabgikfa	fk(form cycle)	_	No
Inmhcdijeabgikfaf	fj	0.3	15

For different values of n we will get different TLFG and when we apply the algorithm we will get different structures for each graph.

4. Theoretical Concepts

Consider the TLFG from example 2 with different labeling. Consider the TLFG from Example 2 with different labeling

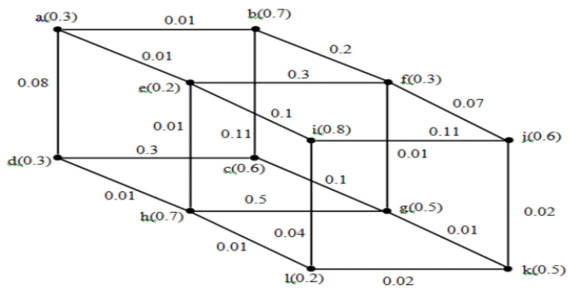

Figure 10. $TL(G_2)$

The union of G_1 and G_2 graph is shown in figure.

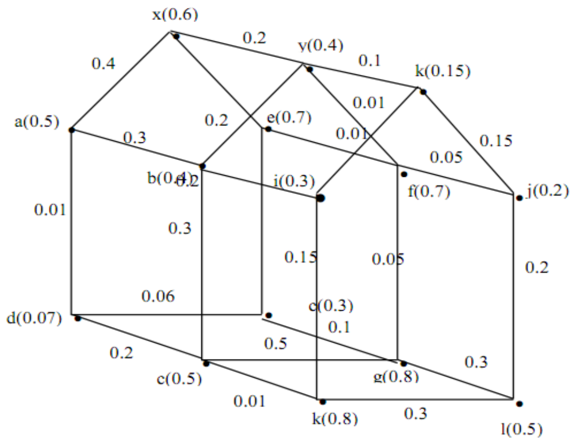

Figure 11. $TL(G_1) \cup TL(G_2)$

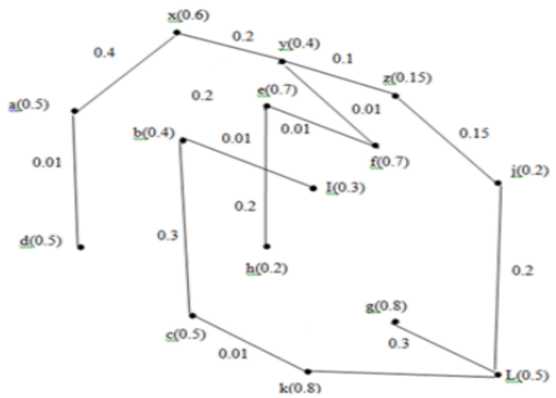

Figure 12. **Structural core graph of** $TL(G1) \cup TL(G2)$

Then the core graph is given by

Face value calculation:

$$F_1\ (a\ b\ c\ d) \rightarrow \min\{0.75, 0.6, 0.4, 0.02\} \quad = 0.02$$

$$F_2\ (a\ x\ y\ b) \rightarrow \min\{0.8, 0.5, 0.5, 0.75\} \quad = 0.5$$

$$F_3\ (b\ I\ k\ c) \to \min\{0.33, 0.5, 0.02, 0.75\} = 0.02$$

$$F_4\ (b\ y\ z\ i) \to \min\{0.4, 0.66, 0.66, 0.33\} = 0.33$$

$$F_5\ (x\ a\ d\ h\ e) \to \min\{0.8, 0.02, 0.3, 1, 0.33\} = 0.02$$

$$F_6\ (y\ f\ g\ c\ b) \to \min\{0.1, 0.5, 1, 0.75, 0.5\} = 0.1$$

$$F_7\ (z\ j\ l\ k\ i) \to \min\{1, 1, 0.6, 0.5, 0.66\} = 0.5$$

$$F_8\ (e\ f\ g\ h) \to \min\{0.1, 0.5, 0.75, 1\} = 0.5$$

$$F_9\ (f\ j\ l\ g) \to \min\{0.5, 1, 0.6, 0.5\} = 0.5$$

$$F_{10}\ (d\ h\ g\ c) \to \min\{0.3, 0.75, 1, 0.4\} = 0.3$$

$$F_{11}\ (c\ g\ l\ k) \to \min\{1, 0.6, 0.6, 0.02\} = 0.024$$

Reached node	Edge	Membership value	Iteration
x	xy	0.2	1
xy	yz	0.1	2
xyz	zj	0.15	3
xyzjl	lk	0.3	4
xyzjlk	ki	0.15	5
xyzjlki	iz (Form Cycle)	-	No
xyzjlki	ib	0.01	6
xyzjlkib	bc	0.3	7
xyzjlkibc	cg (Form Cycle)	-	No
xyzjlkibc	cd	0.2	8
xyzjlkibcd	dh	0.06	9
xyzjlkibcdh	hg(Form Cycle)	-	No
xyzjlkibcdh	he	0.2	10
xyzjlkibcdhe	ef	0.01	11
xyzjlkibcdhef	fg(Form Cycle)	-	No
xyzjlkibcdhef	gl	0.3	12
xyzjlkibcdhefgl	lk	0.15	13
xyzjlkibcdhefglk	kl (Form Cycle)	-	NO
xyzjlkibcdhefgl	kc	0.01	14
xyzjlkibcdhefglkc	cd (Form Cycle)	-	NO
xyzjlkibcdhefglkcd	da	0.01	15
xyzjlkibcdhefglkcda	ab (Form Cycle)	-	NO
xyzjlkibcdhefglkcda	ax	0.4	16
xyzjlkibcdhefglkcdax	xe (Form Cycle)	-	NO
xyzjlkibcdhefglkcdax	xy	0.2	17
xyzjlkibcdhefglkcdax	yf	0.01	18
xyzjlkibcdhefglkcdaxyf	fi (Form Cycle)	-	NO
xyzjlkibcdhefglkcdaxyf	Iz	0.15	19

5. Conclusion

The structural core graph for both the TLFG and the union of two TLFG is constructed in this paper. This graph can be used in different networks to minimize the time for any particular problem whose graphical representation is a triple layered. Further work can be done to apply this concept of Structural core graph in real life situations.

References

[1] J. Jesintha Rosline and T. Pathinathan, *Triple Layered Fuzzy Graph*, Int. J. Fuzzy Mathematical Arhive, 8(1)(2015), 36-42.

[2] A.Nagoorgani and M.Basheed Ahamed, *Order and size in fuzzy graphs*, Bulletin of Pure and Applied Sciences, 22E(1)(2003), 145-148.

[3] A.Nagoorgani and K.Radha, *The degree of a vertex in some fuzzy graphs*, Intern Journal of Algorithms, Computing and Mathematics, 2(3)(2009), 107- 116.

[4] T.Pathinathan and J.Jesintha Rosline, *Double layered fuzzy graph*, Annals of Pure and Applied Mathematics, 8(1)(2014), 135-143.

[5] T.Pathinathan and J.Jesintha Rosline, *Matrix representation of double layered fuzzy graph*, Annals of Pure and Applied Mathematics, 8(2)(2014), 51-58.

[6] A.Rosenfeld, Fuzzy graphs, in: L.A. Zadeh, K.S. Fu, K. Tanaka and M. , (editors), *Fuzzy sets and its application to cognitive and decision process*, Academic press, New York, (1975), 77-95.

[7] R.T.Yeh and S.Y.Bang, Fuzzy relations, *Fuzzy graphs and their applications to clustering analysis*, in: L.A. Zadeh, K.S. Fu, K. Tanaka and M. Shimura, (editors), *Fuzzy sets and its application to cognitive and decision process*, Academic press, New York, (1975), 125-149.