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Edge Domination Number on Graph Variations

Anisha Jean Mathias[1,*]

1 M.Phil. Scholar, Department of Mathematics, CHRIST (Deemed to be University), Bengaluru, Karnataka, India.

Abstract: An edge dominating set D is a set of edges from the edge set E in graph G where every edge of $\langle E - D\rangle$ is adjacent to an element of D. Edge domination number is the cardinality of the smallest edge dominating set and is denoted by $\gamma'(G)$. In this study, results for edge domination number of product graphs have been obtained and their algebraic properties have also been examined. Further, we introduce the concept of edge domination subdivision number and determine its value for some graphs and explore the relationship between edge domination subdivision number and edge domination number.

MSC: 05C69, 05C70.

Keywords: Edge Domination number, independent edge domination number, edge domination subdivision number, matching subdivision number.

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1. Introduction

The concept of domination in graph theory has been one of the favourite areas for the researchers and its related invariants have been generalized in many ways. Among them one such invariant is edge domination introduced by Mitchell and Hedetniemi [1] which is closely related to the concept of matching theory. A matching M in a graph G is a set of edges of G such that no two edges are adjacent to each other. A matching M is a maximum matching if there is no matching in G with greater cardinality. The cardinality of any maximum matching in G is matching number of G and is denoted by $\alpha'(G)$. If every vertex of G is incident with an edge of M, the matching is called a perfect matching. A matching M is called maximal if it cannot be extended to a larger matching in G. The cardinality of any smallest maximal matching in G is the saturation number of G. Saturation number was first studied by Zykov [2] in 1949 and then by Erdős, Hajnal and Moon [3] in 1964. Saturation number $s(G)$ of a graph G is at least one half of the matching number of G, i.e., $s(G) \geq \frac{\alpha'(G)}{2}$ [4]. Problems pertaining to matching has been very useful in the field of chemical graph theory and have already been extensively studied, for example refer [5, 6]. Since for any graph, saturation number is the same as the independent edge domination number or edge domination number, thus in this paper we would be focusing on the concepts of edge domination in graphs.

Formally, a set D of edges of G(V,E) is called an edge dominating set if every edge of $\langle E-D\rangle$ is adjacent to an element of D. Edge domination number is the cardinality of the smallest edge dominating set and is denoted by $\gamma'(G)$. Independent edge dominating set is the an edge dominating set in which no two edges are incident to each other. The cardinality of the smallest independent edge dominating set is independent edge domination number. Mitchell and Hedetniemi [1] had introduced the concept of edge domination for trees. Results on edge domination number for paths, cycles and wheel graph have been

* *E-mail: anisha.mathias@res.christuniversity.in*

already obtained by Alikhani and Soltani [7] and Arumugam and Velammal [8] have characterized connected graphs, trees and unicyclic graph for $\gamma' = \lfloor n/2 \rfloor$. Further, Yannakakis and Gavril [9] have attained results on edge domination number for planar and bipartite graph of maximum degree 3.

Given two graphs G and H, edge domination problems for various product of graphs have been a part of an interesting study and many researchers have already contributed towards this work. In the present study, we attempt to determine the edge domination for the corona product of graphs when the underlying graphs are paths, stars and complete graphs, as an extension of the studies by Anita, Sreedevi and Maheshwari [10, 11]. We would be using the following result:

Lemma 1.1 ([7]). *If P_n and C_n respectively denote the paths and cycle on n vertices, then*

(1). $\gamma'(P_n) = \begin{cases} \lceil \frac{n}{3} \rceil, & \text{if } n \equiv 2(mod3) \\ \lfloor \frac{n}{3} \rfloor, & \text{otherwise} \end{cases}$

(2). $\gamma'(C_n) = \lceil \frac{n}{3} \rceil$ *for* $n \geq 3$.

Another thought provoking study is how domination parameters change on edge subdivision. The problem of domination subdivision number was first defined by Arumugam and Favaron, Haynes and Hedetniemi [12] have presented results on this study for several families of graphs. We would initiating our study in this direction by introducing edge domination subdivision number and matching subdivision number and further attaining results for paths, cycle and star graphs. We would also be exploring the relationship between these parameters. In this paper, we only consider graphs which are finite, connected and undirected.

2. Edge Domination Number of Corona Product of Graphs

Given two graphs G and H, there are different graph products that are already defined in the literature [13]. In this study, we will focus on one such product namely corona product, which was introduced by Frucht [14].

Definition 2.1 ([14]). *Let G_1 and G_2 be the graphs of order n_1 and n_2 respectively. The corona product of two graphs G_1 and G_2 denoted by $G_1 \circ G_2$ obtained by taking one copy of G_1 and n_1 copies of G_2 and then joining the i^{th} vertex of G_1 to every vertex of i^{th} copy of G_2, where $1 \leq i \leq n_1$.*

Remark 2.2. *Note that: $G_1 \circ G_2 \neq G_2 \circ G_1$ as seen in the following example:*

Example 2.3. *Let us take corona product of two graphs P_4 and P_3 where v_i, $i = 1, 2, 3$ be the vertices of graph P_3 and u_i, $i = 1, 2, 3, 4$ be the vertices of graph P_4. The edges colored in red are the edges dominating all the other edges in the graph. $P_3 \circ P_4$ is formed by taking one copy of P_3 with vertices namely v_i, $i = 1, 2, 3$ and three copies of P_4 with vertices namely u_{ij}, $i = 1, 2, 3, 4$ and $j = 1, 2, 3$. The graph so formed is as shown in figure 1. Edge domination for $P_3 \circ P_4 = 5$.*

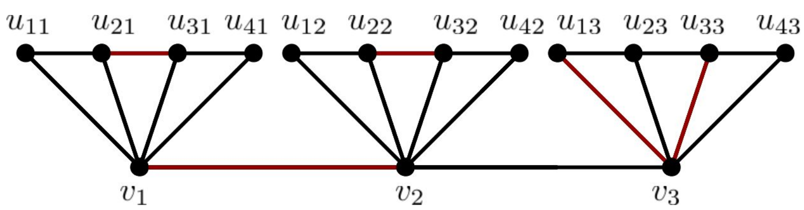

Figure 1. **Graph of $P_3 \circ P_4$**

$P_4 \circ P_3$ is formed by taking one copy of P_3 with vertices namely u_i, $i = 1, 2, 3, 4$ and four copies of P_3 with vertices namely v_{ij}, $i = 1, 2, 3$ and $j = 1, 2, 3, 4$. The graph then formed is as shown in figure 2. Edge domination for $P_4 \circ P_3 = 4$.

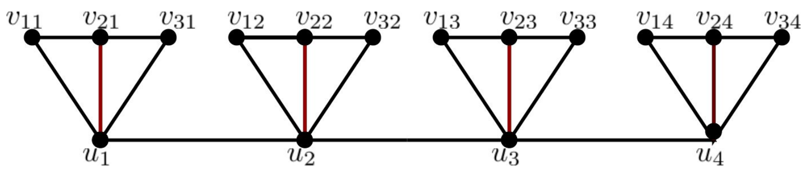

Figure 2. **Graph of $P_4 \circ P_3$**

Thus, we can depict that our remark 2.2 holds true. The equality condition holds true only when the two graphs G and H are isomorphic to each other. We would now be presenting some results on edge domination for corona products with paths, stars and complete graphs.

Theorem 2.4. *If G and H are two graphs where $G = P_n$ with n vertices and $H = P_2$, then $\gamma'(P_n \circ P_2) = \gamma'(P_2 \circ P_n) + 1$.*

Proof. Let P_n be a path of n vertices, namely v_i, $i = 1, 2, 3...n$ and let the vertices of P_2 be u_i, $i = 1, 2$. Then, $P_n \circ P_2$ is formed by taking 1 copy of P_n and n copies of P_2 as shown in figure 3, where the vertices of one copy of P_n be v_i, $i = 1, 2, 3...n$ and each i^{th} copy of P_2 have vertices u_{ij}, $i = 1, 2$; $j = 1, 2, ...n$.

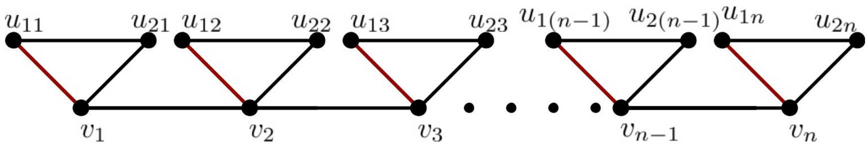

Figure 3. **Graph of $P_n \circ P_2$.**

In figure 3, there are exactly n copies of C_3 been formed. Let us take $v_i v_{i+1}$, $i = 1, 2, ...n-1$ connecting the i^{th} and $i^{th}+1$ copy of C_3 to be the dominated edge. Then clearly, each $u_{1j}u_{2j}$, $j = 1, 2, ...n$ from each copy of C_3 will not be dominated, as no edge connecting the two C_3's can dominate all the edges of C_3 . Hence, if we also choose $u_{1j}u_{2j}$, $j = 1, 2, ...n$ as the dominated edge, then the number of edges dominated will be $2n-1$ edges. Clearly, this is not the edge domination number as we can reduce the number of dominated edges. The only possibility now is to take $u_{1i}v_i$, $i = 1, 2, ...n$ to be the dominated edge. These edges dominates all the other edges of graph C_3 in each copy as well the edge $v_i v_{i+1}$, $i = 1, 2, ...n-1$ connecting the two C_3. Thus, number of edges dominating the graph $P_n \circ P_2$ is equal to $n \leq 2n-1$.
Also, we have already seen there are n copies of C_3 in $P_n \circ P_2$ and by lemma 1.1, $\gamma'(C_3) = 1$. Therefore, $\gamma'(P_n \circ P_2) \geq n$. Hence, $\gamma'(P_n \circ P_2) = n$.

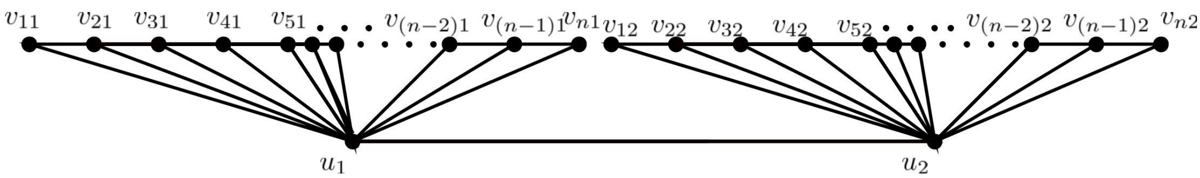

Figure 4. **Graph of $P_2 \circ P_n$**

Now $P_2 \circ P_n$, is formed by taking one copy of P_2 and 2 copies of P_n as shown in figure 4, where the vertices of P_2 be u_i, $i = 1, 2$ and each i^{th} copy of two P_n's have vertices v_{ij}, $i = 1, 2, ...n$; $j = 1, 2$. Proceeding as above, selection of dominated edge depends upon two case as follows:

Case 1. *n is odd.*

If we consider the dominated edge to be the edge u_1u_2 connecting the the two copies of P_n's then this edge u_1u_2 will dominate all the other $2n$ edges. Then, we have two P_n's left and edge domination number of P_n is already been defined in lemma 1.1. In this case, we get number of dominated edges to be n. But, we can still reduce the no of dominated edges. Hence, let us take $v_{ij}u_j$, i=2,4,..(n-1); j=1,2 to be the dominated edges. Then these edges will dominate all the other edges in the graph. Since, there are $(n-1)$ C_3's been formed in each copy of P_n connecting the P_2 and the dominated edge is the edge common to two C_3's. Hence, there are $(n-1)/2$ dominated edges in each copy of P_n. The dominated edge thus in graph $P_2 \circ P_n$ is $(n-1)/2 + (n-1)/2 = n-1 < n$. Thus, $\gamma'(P_2 \circ P_n) = n-1$.

Case 2. *n is even.*

Now if we take $v_{ij}u_j$, $i = 2, 4, ...n$; $j = 1, 2$ as the dominated edges. Then, proceeding as previous case we get $n/2$ dominated edges in each copy of P_n connecting the P_2. Thus, number of dominated edges is $2(n/2) = n$. But, this is not the minimum number of the dominated edges and hence we will now reduce the number of dominated edges. Let us take, the dominated edge to be the edge u_1u_2 connecting the two P_n's then this edge will dominate the other $2n$ edges. Then, again we have two P_n's left and edge domination number of P_n is already been defined in Lemma 1.1. Thus, we get number of dominated edges to be $2\gamma'(P_n)$+1= $n-1$. Thus, $\gamma'(P_2 \circ P_n) = n-1$.

Hence, the theorem holds true for paths and clearly our Remark 2.2 holds true. □

Theorem 2.5. *If G and H are two graphs where G is a is a star graph with $n_1 + 1$ vertices and H is a complete graph with n_2 vertices, then $\gamma'(K_{n_2} \circ K_{1,n_1}) = n_2$.*

Proof. Let the isolated vertices of the star graph K_{1,n_1} be v_i, $i = 1, 2, ...n_1$ and vertices of K_{n_2} be u_i, $i = 1, 2, ...n_2$. The edges colored in red are the dominated edges.

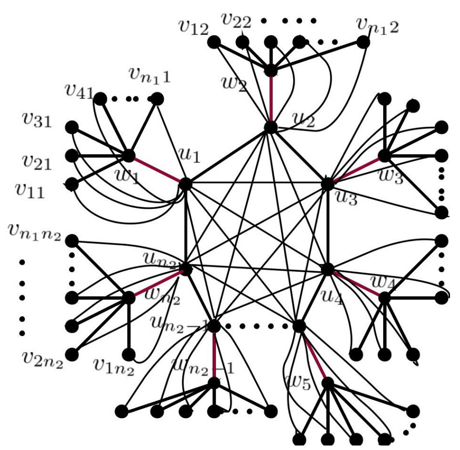

Figure 5. **Graph of $K_{n_2} \circ K_{1,n_1}$**

$K_{n_2} \circ K_{1,n_1}$ is formed by taking one copy of K_{n_2} and n_2 copies of K_{1,n_1} and joining all the vertices of the i^{th} copy of K_{1,n_1} to i^{th} vertex of K_{n_2} as shown in figure 5. Let the vertices of the graph for $K_{n_2} \circ K_{1,n_1}$ be u_i, $i = 1, 2, ...n_2$ for one copy of K_{n_2}, w_i, $i = 1, 2, ...n_2$ be the vertices of the i^{th} vertex of the star graph connecting to all the vertices of the i^{th} copy of K_{1,n_1} and $v_{i,j}$, $i = 1, 2, ...n_1; j = 1, 2, ...n_2$ be the vertices of the remaining vertices of the i^{th} copy of star. The number of dominated edges cannot be less than n_2, since their are n_2 copies of star and $\gamma'(K_{1,n}) = 1$ for n vertices. Also, the edges connecting the vertices u_i and w_i, $i = 1, 2, ...n_2$ are connected to the other edges of the K_{1,n_1} as well as maximum number of edges of K_{n_2}. Hence, we take edges u_iw_i, $i = 1, 2, ...n_2$ to be the dominated edge. Then, number of dominated edges is n_2. Hence, we get the desired result. □

3. Edge Domination Subdivision Numbers

In this section we initiate a study based on the concept of domination subdivision number introduced by Arumugam. We introduce the concept of edge domination subdivision number and matching subdivision number and obtain results for paths, cycles and star graphs. Further, we would be commenting on the relationship between these parameters.

Definition 3.1. *The edge domination subdivision number of a graph G, which we denote as $sd_{\gamma'}(G)$, is equal to the minimum number of edges that must be subdivided (where no edge in G can be subdivided more than once) such that the new graph so formed has the edge domination number greater than the edge domination number of G.*

Definition 3.2. *The matching subdivision number of a graph G, which we denote as $sd_{\alpha'}(G)$, is equal to the minimum number of edges that must be subdivided (where no edge in G can be subdivided more than once) such that the new graph so formed has the matching number greater than the matching number of G.*

Remark 3.3. *Note that: We always take n where n is number of vertices for any graph to be greater than or equal to 3. If $n < 3$, then edge domination number does not change on edge subdivision.*

Theorem 3.4. *If G is a path of n vertices, then* $sd\gamma'(P_n) = \begin{cases} 2, & if\ n \equiv 0(mod3) \\ 1, & if\ n \equiv 1(mod3) \\ 3, & if\ n \equiv 2(mod3). \end{cases}$

Proof. Let G be a path with n vertices. Let v_i, i=1,2,...n be the vertices of P_n as shown in figure 6.

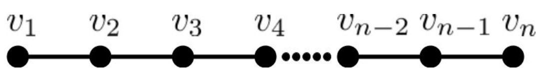

Figure 6. *Graph of P_n*

Then the following are the distinct possibilities for edge domination subdivision number:

Case 1. $n \equiv 0\ (mod\ 3)$.

Let us consider n=3k where k is some positive integer, then by Lemma 1.1, $\gamma'(P_n) = \lfloor \frac{3k}{3} \rfloor = k$. Further, on subdividing the edge v_1v_2 with vertex u_1, our edge set becomes $\{v_1u_1, u_1v_2, v_2v_3, v_3v_4, ..., v_{n-2}v_{n-1}, v_{n-1}v_n\}$, then $n = 3k+1 \equiv 1(mod3)$ and $\gamma'(P_{n+1}) = \lfloor \frac{3k+1}{3} \rfloor = k$. Since edge domination number remains the same, we further subdivide some other edge say $v_{n-1}v_n$ with vertex u_2, our edge set changes to $\{v_1u_1, u_1v_2, v_2v_3, v_3v_4, ..., v_{n-2}v_{n-1}, v_{n-1}u_2, u_2v_n\}$, then $n = 3k+2 \equiv 2(mod3)$ and $\gamma'(P_{n+2}) = \lceil \frac{3k+2}{3} \rceil = k+1$ as shown in figure 7. Hence, $sd\gamma'(P_n) = 2$.

Figure 7. **Graph of P_n when two edges are subdivided**

Case 2. $n \equiv 1\ (mod\ 3)$.

Taking n=3k+1 where k is some positive integer, again by Lemma 1.1, $\gamma'(P_n) = \lfloor \frac{3k+1}{3} \rfloor = k$. Further, on subdividing the edge v_1v_2 with vertex u_1, our edge set now becomes $\{v_1u_1, u_1v_2, v_2v_3, v_3v_4, ..., v_{n-2}v_{n-1}, v_{n-1}v_n\}$, then $n = 3k+1+1 = 3k+2 \equiv 2(mod3)$ and $\gamma'(P_{n+1}) = \lceil \frac{3k+2}{3} \rceil = k+1$ as shown in figure 8. Hence, $sd\gamma'(P_n) = 1$.

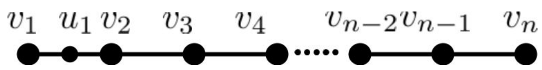

Figure 8. **Graph of P_n when only one edge is subdivided**

Case 3. $n \equiv 2\ (mod\ 3)$.

Consider $n = 3k + 2$ where k is some positive integer and proceeding as above by Lemma 1.1, $\gamma'(P_n) = \lceil \frac{3k+2}{3} \rceil = k + 1$. Further, on subdividing the edge v_1v_2 with vertex u_1, our edge set changes to $\{v_1u_1, u_1v_2, v_2v_3, v_3v_4, ..., v_{n-2}v_{n-1}, v_{n-1}v_n\}$, then $n = 3k + 2 + 1 = 3k \equiv 0(mod3)$ and $\gamma'(P_{n+1}) = \lfloor \frac{3k+3}{3} \rfloor = k + 1$. Since, edge domination number remains the same, we further subdivide some other edge $v_{n-1}v_n$ with vertex u_2, hence our edge set now is $\{v_1u_1, u_1v_2, v_2v_3, v_3v_4, ..., v_{n-2}v_{n-1}, v_{n-1}u_2, u_2v_n\}$, then $n = 3k+2+2 = 3k+4 \equiv 1(mod3)$ and $\gamma'(P_{n+2}) = \lceil \frac{3k+4}{3} \rceil = k+1$. Again, edge domination number remains the same, hence we again subdivide another edge v_2v_3 and get another edge with vertex say u_3. Now the edge set becomes $\{v_1u_1, u_1v_2, v_2u_3, u_3v_3, v_3v_4, ..., v_{n-2}v_{n-1}, v_{n-1}u_2, u_2v_n\}$, then $n = 3k + 2 + 3 \equiv 3(mod3)$ and $\gamma'(P_{n+3}) = \lceil \frac{3k+5}{3} \rceil = k + 2$ as shown in figure 9. Hence, $sd\gamma'(P_n) = 3$.

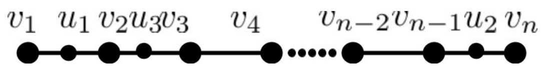

Figure 9. **Graph of P_n when three edges are subdivided**

□

Theorem 3.5. *If G is a cycle on n vertices, then* $sd\gamma'(C_n) = \begin{cases} 1, & \text{if } n \equiv 0(mod3) \\ 3, & \text{if } n \equiv 1(mod3) \\ 2, & \text{if } n \equiv 2(mod3). \end{cases}$

Proof. Let G be a cycle with n vertices. Let v_i, i=1,2,...n where each be the vertices of C_n where $v_i's$ are adjacent to $v_{i+1}, i = 1, ...n-1$ and v_n is adjacent to v_1 as shown in figure 10.

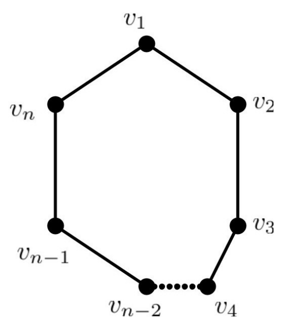

Figure 10. **Graph of C_n**

Then the following are the distinct possibilities for edge domination subdivision number:

Case 1. $n \equiv 0\ (mod\ 3)$.

Taking $n = 3k$ where k is some positive integer, then by Lemma 1.1, $\gamma'(C_n) = \lceil \frac{3k}{3} \rceil = k$. Further, on subdividing the edge v_1v_2 with vertex u_1, our edge set becomes $\{v_1u_1, u_1v_2, v_2v_3, v_3v_4...., v_{n-2}v_{n-1}, v_{n-1}v_n, v_nv_1\}$, then $n = 3k + 1 \equiv 1(mod3)$ and $\gamma'(C_{n+1}) = \lceil \frac{3k+1}{3} \rceil = k$ as shown in figure 11. Hence, $sd\gamma'(C_n) = 1$.

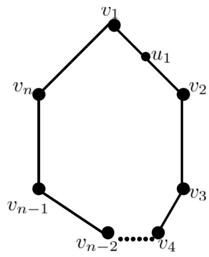

Figure 11. **Graph of C_n when one edge is subdivided**

Case 2. $n \equiv 1\ (mod\ 3)$.

Taking $n = 3k + 1$ where k is some positive integer, by Lemma 1.1, $\gamma'(C_n) = \lceil \frac{3k+1}{3} \rfloor = k$. Further, on subdividing the edge v_1v_2 with vertex u_1, hence our edge set is $\{v_1u_1, u_1v_2, v_2v_3, v_3v_4....., v_{n-2}v_{n-1}, v_{n-1}v_n, v_nv_1\}$, then $n = 3k + 1 + 1 = 3k + 2 \equiv 2(mod3)$ and $\gamma'(C_{n+1}) = \lceil \frac{3k+2}{3} \rceil = k$. Again, on subdividing the edge $v_nv_1$with vertex u_2, our edge set becomes $\{v_1u_1, u_1v_2, v_2v_3, v_3v_4....., v_{n-2}v_{n-1}, v_{n-1}v_n, v_nu_2, u_2v_1\}$, then $n = 3k + 1 + 2 = 3k + 3 \equiv 0(mod3)$ and $\gamma'(C_{n+2}) = \lceil \frac{3k+2}{3} \rceil = k$. Since, edge domination remains the same, we subdivide another edge v_2v_3 with vertex say u_3. Now our edge set is $\{v_1u_1, u_1v_2, v_2u_3, u_3v_3, v_3v_4, ..., v_{n-2}v_{n-1}, v_{n-1}v_n, v_nu_2, u_2v_1\}$, then $n = 3k+1+3 = 3k+4 \equiv 1(mod3)$ and $\gamma'(C_{n+23}) = \lceil \frac{3k+4}{3} \rceil = k + 1$ as shown in figure 12. Hence, $sd\gamma'(C_n) = 3$.

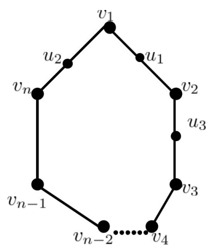

Figure 12. **Graph of C_n when three edges are subdivided**

Case 3. $n \equiv 2\ (mod\ 3)$.

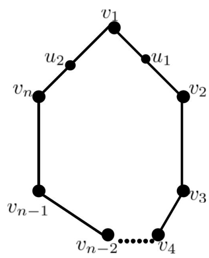

Figure 13. **Graph of C_n when two edges are subdivided**

Taking $n = 3k + 2$ where k is some positive integer and proceeding as above by Lemma 1.1, $\gamma'(C_n) = \lceil \frac{3k+1}{3} \rfloor = k + 1$. Further, on subdividing the edge v_1v_2 with vertex u_1, our edge set becomes $\{v_1u_1, u_1v_2, v_2u_3, u_3v_3, v_3v_4, ..., v_{n-2}v_{n-1}, v_{n-1}v_n, v_nv_1\}$, then $n = 3k + 2 + 1 = 3k + 3 \equiv 0(mod3)$ and $\gamma'(C_{n+1}) = \lceil \frac{3k+3}{3} \rceil = k + 1$. On subdividing the edge v_nv_1 with vertex u_2, our edge set becomes $\{v_1u_1, u_1v_2, v_2u_3, u_3v_3, v_3v_4, ..., , v_{n-2}v_{n-1}, v_{n-1}v_n, v_nu_2, u_2v_1\}$, then $n = 3k + 2 + 2 = 3k + 4 \equiv 1(mod3)$ and $\gamma'(C_{n+2}) = \lceil \frac{3k+4}{3} \rceil = k + 2$ as shown in figure 13. Hence, $sd\gamma'(C_n) = 2$. □

Theorem 3.6. *If G is a star graph of order n, $n \geq 3$, then $sd\gamma'(K_{1,n-1})$=2.*

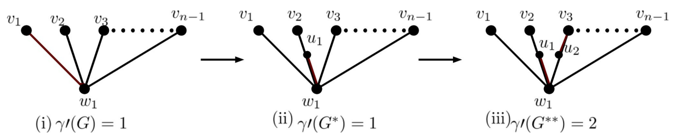

Figure 14. $sd_{\gamma'(G)} = 2$

Proof. Let the isolated vertices of graph $G = K_{1,n-1}$ be v_i, $i = 1, 2, ...n - 1$ and the vertex with maximum degree be w_1 as shown in figure 14 (i). As the greatest distance between any two vertices in graph G is 2 or P_3 and $\gamma'(P_3) = 1$, thus $\gamma'(K_{1,n-1}) = 1$. Let us now subdivide the edge v_2w_1 by adding a vertex u_1 between this edge and let the new graph thus formed be G^* figure 14 (ii). On edge subdivision, the greatest distance between any two vertices of the graph is 3 or P_4 and $\gamma'(P_4) = 1$. Hence, $\gamma'(G^*) = 1$. Since, the value of γ' does not change, we will further subdivide some other edge. We subdivide the edge v_3w_1 with vertex u_2 as shown in figure 14 (iii) and the new graph to be G^{**}. On subdividing this edge, the greatest distance is 4 or P_5 and hence $\gamma'(P_5) = 2$. Hence, $\gamma'(G^{**}) = 2$. Thus, $sd\gamma'(K_{1,n-1})$=2. □

Remark 3.7. *Based on these Theorems 3.4, 3.5, 3.6, we can comment if G is a path, cycle or a star graph of order n, $n \geq 3$ then $1 \leq sd_{\gamma'}(G) \leq 3$.*

Corollary 3.8. *If G is P_n, C_n or $K_{1,n-1}$ for $n \geq 3$, then $sd_{\gamma'}(G) \leq \gamma'(G) + 1$.*

Proof. For a star graph, by Theorem 3.6 $\gamma'(K_{1,n-1}) = 1$ and $sd_{\gamma'}(K_{1,n-1}) = 2$. Hence the theorem holds. If G is P_n, then for $n < 4$, $\gamma'(P_n)$=1 and from Theorem 3.4, $sd_{\gamma'}(P_3) = 2$ and $sd_{\gamma'}(P_4) = 1$. Hence the statement holds. For $n > 4$, $\gamma'(P_n) \geq 2$ and $sd_{\gamma'}(P_n) \leq 3$. Hence, $sd_{\gamma'}(G) \leq \gamma'(G) + 1$. If G is C_n, then for $n = 3$, $\gamma'(C_n)$=1 and from Theorem 3.5, $sd_{\gamma'}(C_n) = 1$. Hence statement holds. For $n > 4$, $\gamma'(C_n) \geq 2$ and $sd_{\gamma'}(C_n) \leq 3$. Hence, $sd_{\gamma'}(G) \leq \gamma'(G) + 1$. □

Remark 3.9. *If G is P_n, C_n or $K_{1,n-1}$ for $n \geq 3$, $sd_{\alpha'}(G) \leq 2$.*

Corollary 3.10. *If G is P_n, C_n or $K_{1,n-1}$ for $n \geq 3$, $sd_{\gamma'}(G) \leq sd_{\alpha'}(G) + 2$.*

Proof. For a star graph, by theorem 3.6 $sd_{\gamma'}(G) = 2$ and $sd_{\alpha'}(G) = 1$. Hence, statement holds. If $G = P_n$ with n vertices, then equality condition holds for $n \equiv 2(mod3)$ where $sd_{\gamma'}(P_n)$=3 and $sd_{\alpha'}(P_n)$=1. For all the other cases, by theorem 3.4 and remark 3.9 holds true. Proceeding in a similar way for $G = C_n$, equality condition holds true for $n \equiv 1(mod3)$ where $sd_{\gamma'}(C_n)$=3 and $sd_{\alpha'}(C_n)$=2 . For all the other cases, by theorem 3.5 and remark 3.9 holds true . Hence the statement holds true for all these three graphs. □

The present study on edge domination subdivision number offers much scope for further investigation. We have already observed that edge domination subdivision number for paths, cycles and star graphs lie between 1 and 3. Further, we would be extending this study for examining more such graphs for which this edge domination subdivision bound holds true. Also, we have already initiated our study by extending this concept for different variants of edge domination number upon edge subdivision.

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