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Cordial And Product Cordial Labeling Of Edge Merged And Vertex Merged n-Chain Aztec Diamond Graphs

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Abstract: A binary vertex labeling function f from the vertices of a graph G to $\{0,1\}$ is called cordial labeling, if each edge xy is given the label $|f(x)-f(y)|$, the resulting number of vertices with labels 0 and the number of vertices with labels 1 differ to the maximum of 1, and the number of edge labels with 0 and the number of edge labels with 1 also differ to the maximum of 1. In this paper n-chain edge merged and vertex merged Aztec diamond graphs are proved to be cordial but not product cordial.

MSC: 05C78

Keywords: Aztec Diamond Graph, Cordial labeling, Product cordial labeling, Chain of vertex merged Aztec diamond graph, Chain of edge merged Aztec diamond graph.

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1. Introduction

Graph theory in its broad sense deals with study of graphs, their properties and applications in different fields of sciences. Labeling of graphs is a branch of graph theory that deals with assignment of labels to vertices or edges or both vertices and edges satisfying certain specified rules. Graph labeling leads to enormous application of opportunities in science and technology in such a way that, in todays context, graph labeling sometimes are considered synonymous with graph theory. This branch of graph theory contributes greatly not only to the mathematical modeling of day-to-day problems faced in the industries but also solution to improve the efficiency of the system components of the industries. For complete survey of results in graph labeling one can refer to dynamic survey of graph labeling updated annually by J.Gallian [5]. Acharya and Germina [1] introduced set valuations in graph labeling and A.Rosa [7] put forth graceful labeling while Cahit initiated cordial labeling as a weaker version of graceful labeling. A survey of set valued graphs can also be seen in Abhishek [2]. A new family of staircase graphs were introduced and various labelings defined on them by A.Solairaju and M.Antony Arockiasamy [8].

Aztec diamond graphs [6] are known for tilings and its related properties. But in this paper the Aztec diamond graphs are seen through a different lens. It is subjected to the test of cordial labeling. From a single Aztec diamond graphs the vertex merged double and triple graphs are generated and extended upto the creation of n-chain vertex and edge merged Aztec diamond graphs for any positive integer n. A brief preliminary definitions are given in this section and main results

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are derived in the second section. Throughout this paper edges and links are identically used, while vertices and points are synonymous.

Definition 1.1 ([4]). *A binary vertex labeling function f from the vertices of a graph G to $\{0,1\}$ is called cordial labeling, if each edge xy is given the label $|f(x)-f(y)|$, the resulting number of vertices with labels 0 and the number of vertices with labels 1 differ to the maximum of 1, and the number of edge labels with 0 and the number of edge labels with 1 also differ to the maximum of 1.*

Definition 1.2 ([9, 10]). *A binary vertex labeling f from the vertices of G to $\{0,1\}$ of graph G with induced edge labeling $f* : E(G) \rightarrow \{0,1\}$ defined by $f*(e = uv) = f(u)f(v)$ is called a product cordial labeling if $|(v_f)f(0) - (v_f)f(1)| \geq 1$ and $|(e_f)(0) - (e_f)(1)| \geq 1$. A graph which admits product cordial labeling is called product cordial.*

Definition 1.3 ([3]). *Let k be a positive integer the Aztec diamond of order k is the union Aztec diamond of all the unit squares with integer vertices (x,y) satisfying $|x|+|y| \leq k+1$ [6]. The dual graph obtained from an Aztec diamond of order k, where each square is a vertex and if two squares are adjacent in Aztec diamond then their corresponding vertices are linked by an edge in the dual graph. This dual graph is known as Aztec diamond graph of order k. It is denoted by $G(A,k)$. The corner vertex of an Aztec diamond graph is defined as two degree vertex whose adjacent vertices are of degree 2 and degree 4. A corner edge joins two adjacent corner vertices of $G(A,k)$.*

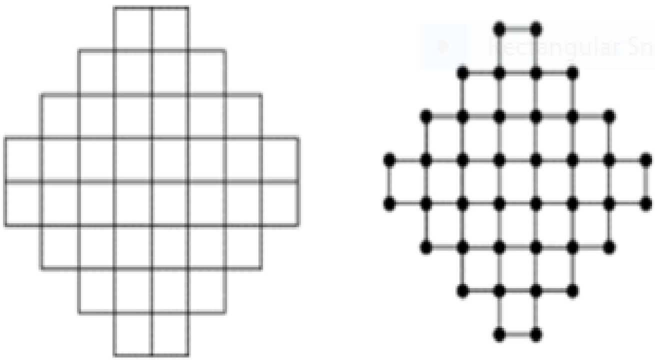

Figure 1. **Aztec diamond and Aztec diamond graph of order 4**

Definition 1.4. *Edge merged twin Aztec diamond graph consists of two Aztec diamond graphs with any one of the corner edges of first diamond graph identified with any one of corner edges of the second diamond graph [3]. Edge merged Aztec diamond graph is denoted by $G(2A_e,k)$ is illustrated in Figure 2. Similarly triple $G(3A_e,k)$ and n-chain edge merged Aztec diamond graphs $G(nA_e,k)$ can be generated as shown in Figures 3 and 4.*

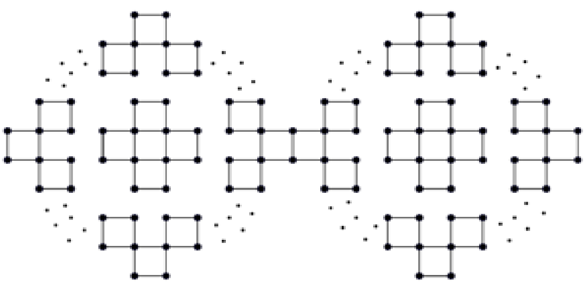

Figure 2. **Graph** $G(2A_e,K)$

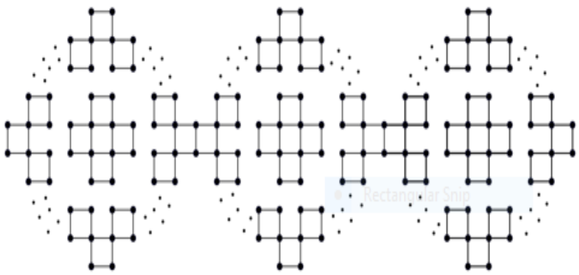

Figure 3. **Graph** $G(3A_e, k)$

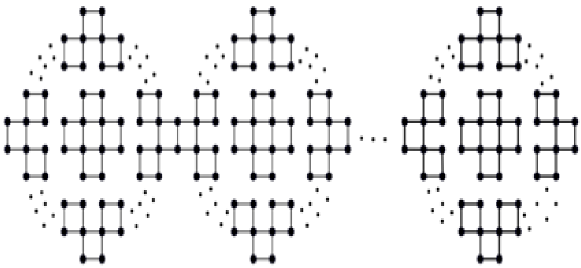

Figure 4. **Graph** $G(nA_e, k)$

Definition 1.5. *Vertex merged twin Aztec diamond graph consists of two Aztec diamond graphs with any one of the corner vertices of first diamond graph identified with any one of corner vertices of the second diamond graph [3]. Vertex merged twin Aztec diamond graph is denoted by* $G(2A_v, k)$ *is illustrated in Figure* 5*. Similarly triple* $G(3A_v, k)$ *and n-chain edge merged Aztec diamond graphs* $G(nA_v, k)$ *can be generated as shown in Figures* 6 *and 7 [3].*

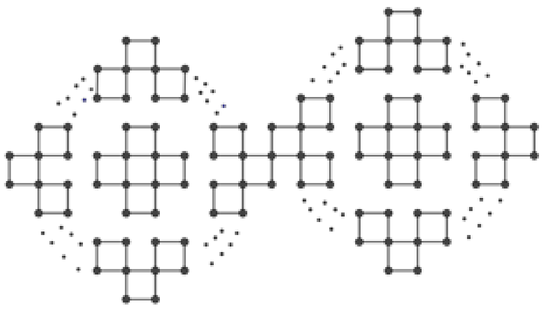

Figure 5. **Graph** $G(2A_v, k)$

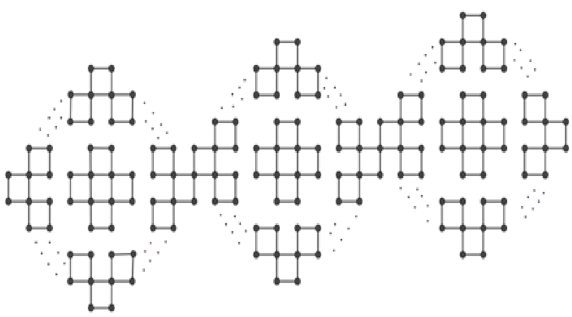

Figure 6. **Graph** $G(3A_v, k)$

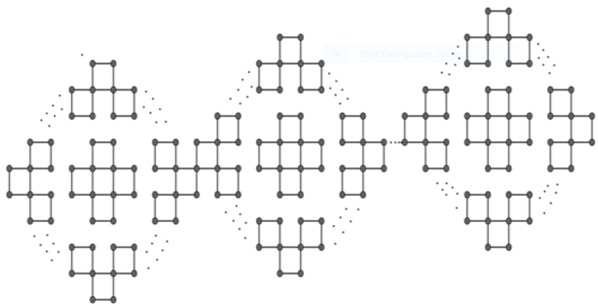

Figure 7. **Graph** $G(nA_v, k)$

In this research paper n-chain edge merged Aztec diamond graph and n-chain vertex merged Aztec diamond graph are proved to be cordial but not product cordial

2. Main Results

Theorem 2.1. *Any twin edge merged Aztec Diamond Graph $G(2A_e, k)$ is Cordial but not Product Cordial.*

Proof. Let $G(2A_e, k)$ be a twin edge merged Aztec Diamond Graph of order k, it contains $(4k^2 + 4k - 2)$ vertices and $(8k^2 + 1)$ edges. Let G_1 and G_2 be two Aztec Diamond Graphs in $G(2A_e, k)$. Set $f : v \to \{0, 1\}$ a mapping from the set of vertices of G to $\{0, 1\}$ and for each edge $(uv) \in E$ assign the label $|f(u) - f(v)|$. Here the twin edge merged Aztec Diamond Graph $G(2A_e, k)$ spilt into $4k$ horizontal paths $P_1, P_2, P_3, ..., P_{4K}$. Where P_i represent the shortest horizontal path between V_{ij} and V_{ij},$1 \le j \le 4k$. Further the graph $G_1(A, k)$ contains $2k$ horizontal paths $P_1, P_2, P_3, ..., P_{2k}$ and the graph $G_2(A, k)$ contains $4k$ horizontal paths $P_{2K+1}, P_{2K+2}, ..., P_{4K}$.For each P_i,$1 \le i \le 4k$,the label 0 is assigned to all the vertices of p_i if i is odd, and the label 1 is assigned to all the vertices of P_i if i is even. The induced number of vertices labeled as 0 and the number of vertices labeled as 1 differ at most by one. Similarly, the number of vertices edges as 0 and the number of edges labeled as 1 differ at most by one. Therefore, the graph $G(2A_e, k)$ is cordial.

To prove the $G(2A_e, K)$ is not product cordial one need to see all possible cases in allocation of labels $0, 1$ for each of the $(4(k^2 + k) - 2)$ vertices. The exhaustive number of possibilities are $2^1x2^1x...((4(k^2 + k) - 2))$ times. In exhausting all possible combinations of labeling the conditions of product cordial labeling is not satisfied. Hence G $(2A_e, K)$ is not product cordial.

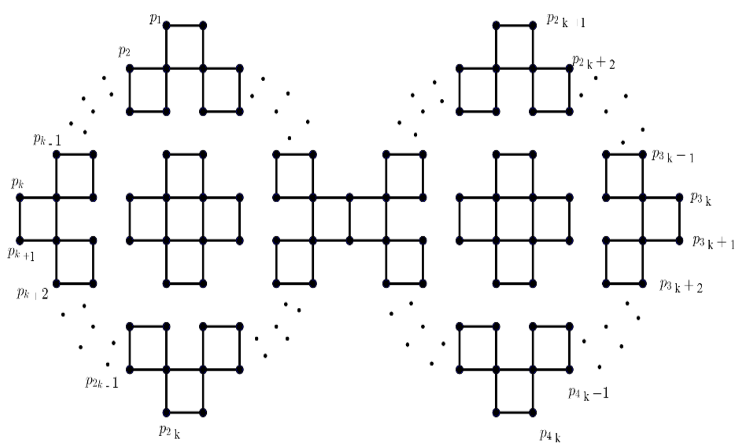

Figure 8. **Cordial labeling of** $G(2A_e, k)$

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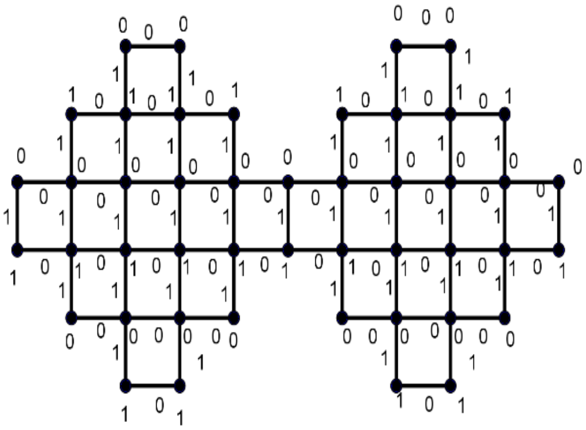
Example 2.2. *Twin Edge Merged Aztec Diamond Graph of order 3 is Cordial*



	0	1
|V|	23	23
|E|	36	35

Figure 9. **Cordial labeling** $G(2A_e, 3)$

Theorem 2.3. *Any triple edge merged Aztec Diamond Graph $G(3A_e, k)$ is cordial but not product cordial.*

Proof. Let $G(3A_e, k)$ be a triple edge merged Aztec Diamond Graph of order k, it contains $(6k^2 + 6k - 4)$ vertices and $(12k^2 - 1)$ edges. Let G_1,G_2 and G_3 be three Aztec Diamond Graphs in $G(3A_e, k)$. Let $f : v \to \{0, 1\}$ be a mapping from the set of vertices of G to $\{0, 1\}$ and for each edge $(uv) \in E$ assign the label $|f(u) - f(v)|$. Here the triple edge merged Aztec Diamond Graph $G(6A_e, k)$ spilt into $6k$ horizontal paths $P_1, P_2, P_3, ..., P_{6K}$ .Where P_i represent the shortest horizontal path between V_{i1} and V_{ij} ,$1 \leq j \leq 6k$. Further the graph $G_1(A, k)$ contains $2k$ horizontal paths $P_1, P_2, P_3, ..., P_{2K}$ and the graph $G_2(A, k)$ contains $2k$ horizontal paths $P_{2K+1}, P_{2K+2}, ..., P_{4K}$,and also the graph $G_3(A, k)$ contains $2k$ horizontal paths $P_{3K+1}, P_{3K+2}, ..., P_{6K}$. For each P_i, $1 \leq i \leq 6k$, the label 0 is assigned to all the vertices of p i if i is odd, and the label 1 is assigned to all the vertices of P i if i is even. The induced, the number of vertices labeled as 0 and the number of vertices labeled as 1 differ at most by one. Similarly, the number of vertices edges as 0 and the number of edges labeled as 1 also differ at most by one. Therefore, the graph $G(3A_e, k)$ is cordial.

To prove the $G(3A_e, K)$ is not product cordial. All possible cases in allocation of labels $0, 1$ for the $(6(k^2 + k) - 4)$ vertices need to be verified. The exhaustive number of possibilities are $2^1 x 2^1 x...(6(k^2 + k) - 4))$ times. In exhausting all possible combinations of labeling the conditions of product cordial labeling is not satisfied. Hence G $(3A_e, K)$ is not product cordial.

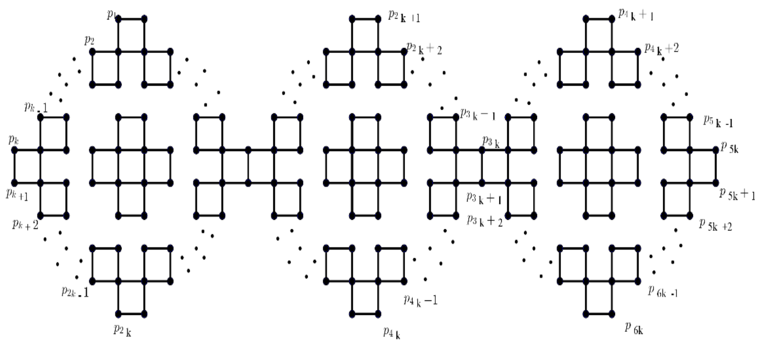


Figure 10. **Cordial labeling** $G(3A_e, k)$

□

Example 2.4. *Triple Edge Merged Aztec Diamond Graph of order 2 is Cordial*

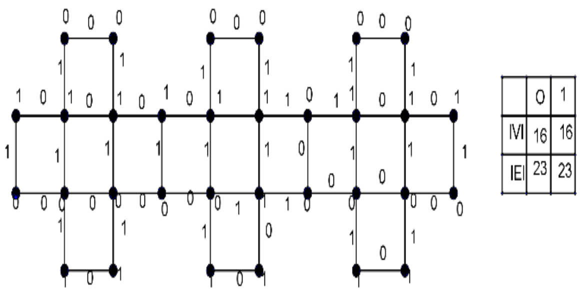


Figure 11. **Cordial labeling** $G(3A_e, 2)$

Theorem 2.5. *Any n-chain edge merged Aztec Diamond Graph $G(nA_e, k)$ is Cordial but not product cordial.*

Proof. The n-chain edge merged Aztec diamond graph consists of n Aztec diamond graph. For $n = odd$ the labeling is similar to $G(3A_e, k)$. For $n = even$ the labeling is similar to $G(2A_e, k)$. Hence for any n finite positive integer n the $n-$chain edge merged Aztec diamond graph is cordial but not product cordial □

Theorem 2.6. *Any vertex merged twin-Aztec diamond graph $G(2Av, k)$is Cordial but not product cordial.*

Proof. Let $G(2Av, k)$ be a twin vertex merged Aztec diamond graph of order k, it contains $(4(k^2 + k) - 1)$ vertices $(8k^2)$ edges. Let G_1 and G_2 be two Aztec diamond graphs in $G(2Av, k)$. Let $f : v \to \{0, 1\}$ be a mapping from the set of vertices of G to $\{0, 1\}$ and for each edge $(uv) \in E$ assign the label $|f(u) - f(v)|$. Here the twin Aztec diamond graph $G(2Av, k)$ is split into $4k$ horizontal paths $P_1, P_2, ...P_4k$ where P_i represents the shortest horizontal path between V_{i1} and V_{ij},$1 \leq j \leq 4k$. Further the graph $G_1(A_v, k)$ contains $2k$ horizontal paths $P_1, P_2, ..., P_{2K}$, and the graph $G_2(A_v, k)$ contains $2k$ horizontal paths $P_{2k+1}, P2k + 2, ...P_{4k}$. For each P_i, $1 \leq i \leq 4k$, the label 0 is assigned to all the vertices of P_i if i is even and the label 1 is assigned to all the vertices of P_i if i is odd.The induced number of vertices labeled as 0 and the number of vertices labeled as 1 differ at most by one. Similarly, the number of edges is labeled as 0 and the number of edges labeled as 1 also differ at most by one. Therefore the graph $G(2A, k)$ is cordial. To prove the $G(2A_v, k)$ is not product cordial, all possible cases in allocation of labels 0, 1 for the $(4(K^2 + 2K) - 1))$ vertices need to be the verified. The exhaustive number of possibilities are $2^1 x 2^1 x...(4(K^2 + K) - 1)$ times. In exhausting all possible combinations of labeling the conditions of product cordial labeling is not satisfied. Hence $G(2A_v, K)$ is not product cordial.

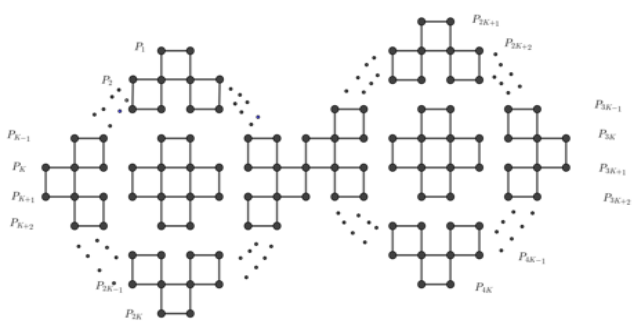
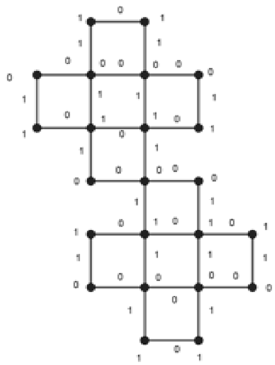


Figure 12. **Cordial labeling** $G(2A_v, k)$

□

Example 2.7. *Twin Vertex Merged Aztec Diamond Graph $G(2A_v, 2)$ is Cordial*



	0	1
|V|	11	12
|E|	16	16

Figure 13. **Cordial labeling** $G(2A_v, 2)$

Theorem 2.8. *Any triple-vertex merged Aztec diamond graph $G(3A_v, k)$ is cordial but not product cordial.*

Proof. Let $G(3A_v, k)$ be a triple vertex merged Aztec diamond graph of order k. it contains $(6(k^2 + k) - 2)$ vertices and $(12k^2)$ edges. Let G_1, G_2 and G_3 be three Aztec diamond graphs in $G(2A_v, k)$. Let $f : v \to \{0, 1\}$ be a mapping from the set of vertices of G to$\{0, 1\}$ and for each edge $(uv) \in E$ assign the label $|f(u) - f(v)|$. Here the triple vertex merged Aztec diamond graph $G(3A_v, k)$ is split into $6k$ horizontal paths $P_1, P_2, ..., P_{6K}$ where P_1 represents the shortest horizontal path between $V_{i1} and V_{iJ}$, $1 \leq j \leq 6k$. Further the graph $G_1(2A_v, k)$ contains $4k$ horizontal paths $P_1, P_2, ..., P_{4K}$.For each P_i , $1 \leq i \leq 4k$, the label 0 is assigned to all the vertices of P_i if i is odd, and the label 1 is assigned to all the vertices of P_i if i is even. And the graph $G(A, k)$ contains $2k$ horizontal paths $P_{4k+1}, P4k + 2, ...P_{6k}$. for each P_i if 1 is even. And the label 1 is assigned to all the vertices of P_i if i is odd. The induced number of vertices labeled as 0 and the number of vertices labeled as 1 differ at most by one. Similarly, the number of edges is labeled as 0 and the number of edges labeled as 1 also differ at most by one. The graph G $(3A_v, k)$ is cordial.

To prove the $G\,(3A_v, k)$ is not product cordial, one need to see all possible cases in allocation of labels $0, 1$ for the $(6(k^2+k)-2)$ vertices. The exhaustive number of possibilities are $2^1 x 2^1 x \cdots (6(K^2+K)-2)$ times. In exhausting all possible combinations of labeling the conditions of product cordial labeling is not satisfied. Hence $G(3A_v, k)$ is not product cordial.

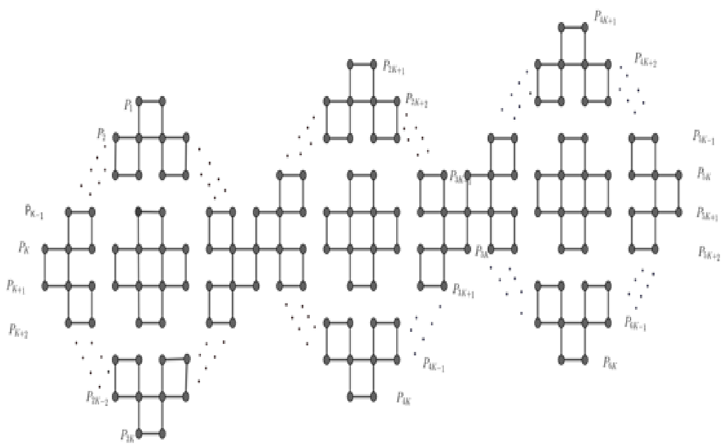
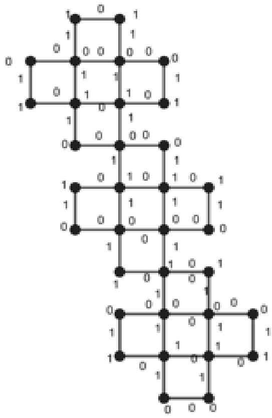


Figure 14. **Cordial labeling** $G(3A_v, k)$

□

Example 2.9. *Triple Vertex Merged Aztec Diamond Graph $G(2A_v, 3)$ is Cordial*



	0	1
|V|	17	17
|E|	24	24

Figure 15. **Cordial labeling** $G(3A_v, 2)$

Theorem 2.10. *Any n-chain vertex merged Aztec diamond graph $G(nA_v, k)$ is cordial but not product cordial.*

Proof. The $n-$chain vertex merged Aztec diamond graph consists of n Aztec diamond graph. For $n = odd$, the labeling is similar to $G(3A_v, k)$ and for $n = even$, the labeling is similar to $G(2A_v, k)$. Hence for any n finite positive integer n the n-chain vertex merge Aztec diamond graph is cordial but not product cordial. □

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