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The Dominator Coloring of Central and Middle Graph of Some Special Graphs

Research Article*

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Abstract: Let $G = (V, E)$ be an undirected graph. A dominator coloring of a graph G is a proper coloring in which every vertex of G dominates every vertex of at least one color class. The convention is that if $\{V\}$ is a color class then V dominates the color class $\{V\}$. The Dominator Chromatic number $\chi_d(G)$ is the minimum number of colors required for a dominator coloring of G. Here we investigate the Dominator Chromatic number of middle and central graph of Tadpole graph and Snake graph.

Keywords: Proper coloring, Dominator coloring, Dominator Chromatic number, Middle graph , Central graph, Tadpole graph and snake graph.

1. Introduction

Graph coloring is an assignment of colors to the vertices of a graph. A vertex coloring is said to be proper coloring if no two adjacent vertices of the graph receive the same color and the graph is called properly colored graph. There are many problems in graph coloring ,one such problem is dominator coloring problem. In the mathematical discipline of graph theory, the (m, n)-tadpole graph is a special type of graph consisting of a cycle graph on m (atleast 3) vertices and a path on n vertices, connected with a bridge. The Triangular snake T_n is obtained by from the path P_n by replacing each edge of the path by a triangle C_3. In this paper, we discuss about the dominator chromatic number for central and middle graph of Tadpole graph and also found the dominator chromatic number of snake graph.

1.1. Definitions

Definition 1.1 (Central Graph). *Let G be a simple and undirected graph and let its vertex set and edge set be denoted by V(G) and E(G). The Central graph of G, denoted by C(G) is obtained by subdividing each edge of G exactly once and joining all the non-adjacent vertices of G in C(G).*

Definition 1.2 (Proper Coloring). *A graph G having no two adjacent vertices receive the same color is said to be Proper coloring. The chromatic number $\chi(G)$, is the minimum number of colors required for a proper coloring of G. A color class is the set of all vertices, having the same color. The color class corresponding to the color i is denoted by V_i.*

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Definition 1.3 (Dominator Coloring)**.** *A dominator coloring of a graph G is a proper coloring in which every vertex of G dominates every vertex of at least one color class. The convention is that if $\{v\}$ is a color class, then v dominates the color class $\{v\}$. The dominator chromatic number $\chi_d(G)$ is the minimum number of colors required for a dominator coloring of G.*

Definition 1.4 (Middle Graph)**.** *The middle graph M(G) is a graph which is obtained by subdividing each edge of G exactly once and join all the newly added middle vertices of adjacent edges of G.*

Definition 1.5 (Snake Graph)**.** *A snake graph is an Eulerian path in the hypercube that has no chords. In other words, any hypercube edge joining snake vertices is a snake edge. It is denoted by T_n.*

Definition 1.6 (Tadpole Graph)**.** *The (m, n)-Tadpole is the graph is obtained by joining a cycle graph C_n to a path graph P_n with a bridge. It is also known as dragon graph. It is denoted by $\{"Tadpole", m.n\}$.*

2. Dominator Chromatic Number of Central and Middle Graph of Tadpole Graph

Dominator chromatic number of central graph and middle graph of Tadpole graphs is obtained in this section.

Theorem 2.1. *For the Tadpole graph $T_{m,n}$ where $n = 1$ and $m \geq 3$, then*

$$\chi_d[C(T_{m,1})] = \begin{cases} \lfloor \frac{m}{2} \rfloor + 2, & \text{when } m \text{ is odd} \\ \frac{m+2}{2}, & \text{when } m \text{ is even.} \end{cases}$$

Proof. Let $T_{m,1}$ be the tadpole graph with joining the cycle C_m and path P_n with $m = 3$, $n = 1$. Let $V(T_{m,1}) = \{V_1, V_2, V_3, ..., V_n\}$. By the definition of central graph $C(T_{m,1})$ is obtained by subdividing the each edge v_iv_{i+1}, $1 \leq i \leq n-1$ of $T_{m,1}$ exactly once by adding a new vertex c_i in $C(T_{m,1})$ and joining each vertex v_j with each vertex v_k, $j+2 \leq k \leq n$. Let $V_1 = \{v_1, v_2, ..., v_n\}$ and $V_2 = \{c_1, c_2, ..., c_n\}$. Then $V[C(T_{m,1})] = V_i \cup C_j \cup V_{i+1}$ for $1 \leq i \leq n-1$, $1 \leq j \leq n$. By applying the definition of dominator coloring of $C(T_{m,1})$ is as follows. Let $\chi_d[C(T_{m,1})]$ be the dominator chromatic number of the central graph of tadpole graph.

Case(i): If m is odd

Suppose $m = 3$, the graph is defined as $T_{3,1}$. Let us consists vertices v_1, v_2, v_3, v_4 and the newly added vertices c_1, c_2, c_3, c_4. Let $S = \{v_1, v_2, v_3\}$ be the dominating set of $T_{3,1}$. Now assign the dominator coloring to the graph $C(T_{3,1})$. Let us assign the color 1 to the vertices $\{v_1, v_2, v_3\}$ of the dominating set S and assign the minimum color classes from $2, 3..., n$ to the remaining non-adjacent vertices of the graph. By the definition of dominator coloring all the vertices in the Set S dominates the color class of $\{v_1, v_2, v_3\}$ but $\{v_1, v_2, v_3\}$ dominates all the color classes. Therefore the dominator chromatic number of $C(T_{3,1})$ is 3. By proceeding this way for order m. We get the successive sequence of dominator chromatic number $\lfloor \frac{m}{2} \rfloor$ +2 colors for dominator coloring in $C(T_{m,1})$. Therefore the dominator chromatic number of $C(T_{m,1})$ is $\lfloor \frac{m}{2} \rfloor + 2$, when m is odd.

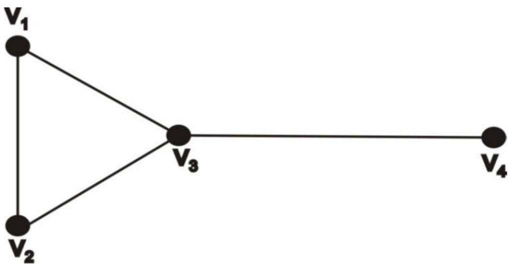
Case(ii): If m is even

Suppose $m = 4$, the graph is defined as $T_{6,1}$. Let us consists vertices v_1, v_2, v_3, v_4, v_5, v_6 and the newly added vertices c_1, c_2, c_3, c_4, c_5, c_6, c_7. Let S=$\{v_1, v_2, c_3, c_4, c_5, c_7\}$ be the dominating set of $C(T_{6,1})$. Now assign the dominator coloring to the graph $C(T_{6,1})$. Let us assign the color 1 to the vertices $\{v_1, v_2, c_3, c_4, c_5, c_7\}$ of the dominating set S and assign the minimum color classes from $2, 3..., n$ to the remaining non-adjacent vertices of the graph. By the definition of dominator coloring all the vertices in the Set S dominates the color class of $\{v_1, v_2, c_3, c_4, c_5, c_7\}$ but $\{v_1, v_2, c_3, c_4, c_5, c_7\}$ dominates all the color classes. Therefore the dominator chromatic number of $C(T_{6,1})$ is 4. By proceeding this way for order m. We get the successive sequence of dominator chromatic number $\frac{m+2}{2}$ colors for dominator coloring in $C(T_{m,1})$. Therefore the dominator chromatic number of $C(T_{m,1})$ is $\frac{m+2}{2}$, when m is even. Hence

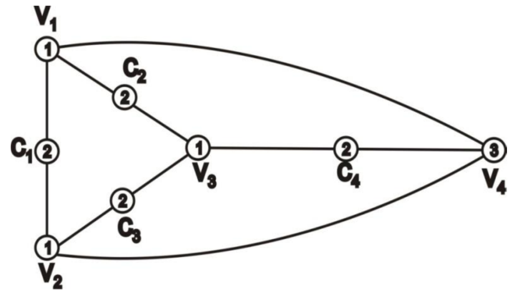
$$\chi_d[C(T_{m,1})] = \begin{cases} \lfloor \frac{m}{2} \rfloor + 2, & \text{when m is odd} \\ \frac{m+2}{2}, & \text{when m is even.} \end{cases} \quad , \quad \text{where } \ n = 1.$$

□

Example 2.2. *In figure, Tadpole graph of $T_{3,1}$ and its central is depicted with a dominator coloring. For $C(T_{3,1})$ when m is odd*

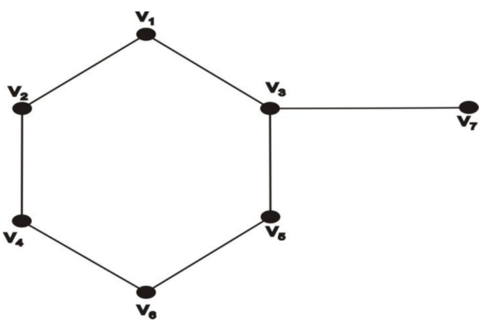


(a) $T_{3,1}$

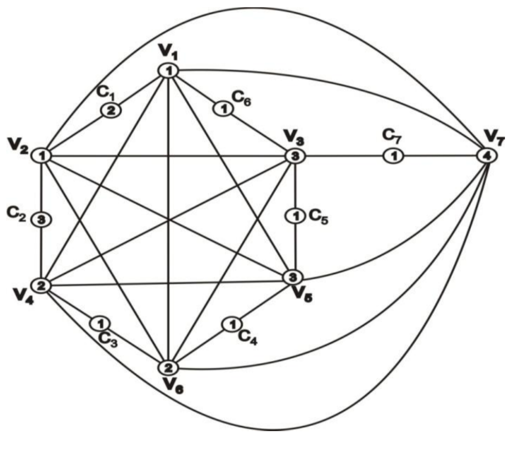


(b) $C(T_{3,1})$

The color classes of $C(T_{3,1})$ are $V_1 = 1 = \{v_1, v_2, v_3\}$, $V_2 = 2 = \{c_1, c_2, c_3, c_5\}$, $V_3 = 3 = \{v_4\}$. The dominator chromatic number is, Therefore, $\chi_d[T_{3,1}] = 3$. In figure, Tadpole graph of $T_{6,1}$ and its central is depicted with a dominator coloring. For $C(T_{6,1})$ when m is even



(a) $T_{6,1}$



(b) $C(T_{6,1})$

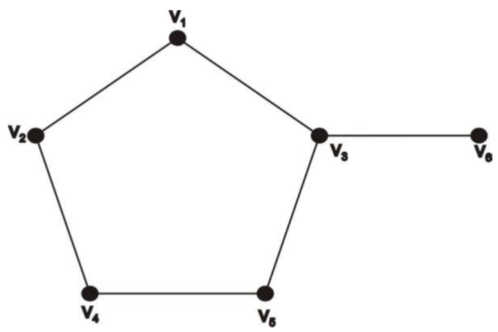
The color classes of $C(T_{6,1})$ are $V_1 = 1 = \{v_1, v_2, c_3, c_4, c_5, c_7\}$, $V_2 = 2 = \{v_4, c_1, c_6, v_6\}$, $V_3 = 3 = \{v_5, c_2, v_3\}$, $V_4 = 4 = \{v_7\}$. Therefore, $\chi_d[T_{6,1}] = 4$.

Theorem 2.3. *For the Tadpole graph $T_{m,n}$ where $n = 1$ and $m \geq 3$, then $\chi_d[M(T_{m,1})] = 4$.*

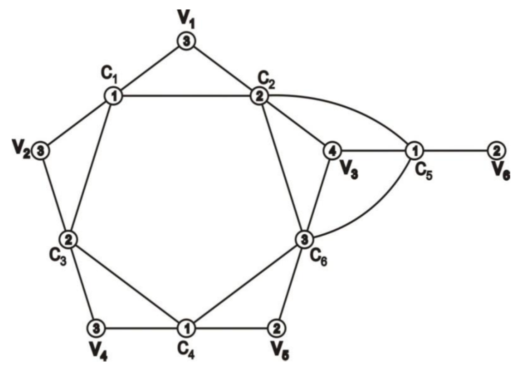
Proof. Let $T_{m,1}$ be the tadpole graph with joining the cycle C_m and path P_n, with $m \geq 3$, $n \geq 1$. Let $V(T_{m,1}) = \{V_1, V_2, V_3, ..., V_n\}$. By the definition of middle graph $M(T_{m,1})$ is a graph which is obtained by subdividing each edge of $T_{m,1}$ exactly once and join all the newly added middle vertices of adjacent edges of $T_{m,1}$. Let $V_1 = \{v_1, v_2, ..., v_n\}$, and

$V_2 = \{c_1, c_2, ..., c_n\}$. Then $V[M(T_{m,1})] = V_i \cup C_j \cup V_{i+1}$, for $1 \le i \le n-1$, $1 \le j \le n$. By applying the definition of dominator coloring of $M(T_{m,1})$ is as follows. Let $\chi_d[M(T_{m,1})]$ be the dominator chromatic number of the middle graph of tadpole graph. Suppose $m = 3$, the graph is defined as $T_{3,1}$. Let us consists vertices v_1, v_2, v_3, v_4 and the newly added vertices c_1, c_2, c_3, c_4. Let $S = \{c_4, c_1\}$ be the dominating set of $T_{3,1}$. Now assign the dominator coloring to the graph $M(T_{3,1})$. Let us assign the color 1 to the vertices $\{c_4, c_1\}$ of the dominating set S and assign the minimum color classes from $2, 3..., n$ to the remaining non-adjacent vertices of the graph. By the definition of dominator coloring all the vertices in the Set S dominates the color class of $\{c_4, c_1\}$ but $\{c_4, c_1\}$ dominates all the color classes. Therefore the dominator chromatic number of $M(T_{3,1})$ is 4. By proceeding this way for order m, we get the successive sequence of dominator chromatic number 4 colors for dominator coloring in $M(T_{m,1})$. Hence $\chi_d[M(T_{m,1})] = 4$, where $n = 1$. □

Example 2.4. *In figure, Tadpole graph of $T_{5,1}$ and its middle is depicted with the dominator coloring.*



(a) $T_{5,1}$



(b) $M(T_{5,1})$

The color classes of $M(T_{5,1})$ are $V_1 = 1 = \{c_5, c_4, c_1\}$, $V_2 = 2 = \{v_6, v_5, c_3, c_2\}$, $V_3 = 3 = \{c_6, v_2, v_1, v_4\}$ and $V_4 = 4 = \{v_3\}$. Therefore $\chi_d[M(T_{5,1})] = 4$.

3. Dominator Chromatic Number of Central and Middle Graph of Snake Graph

Dominator chromatic number of central and middle graph of Snake graph is obtained in this section.

Theorem 3.1. *For the Snake graph T_n of order $n \ge 2$, then*

$$\chi_d[C(T_n)] = \begin{cases} \left(\frac{n+2}{2}\right) + 1 & \text{when } n \text{ is even} \\ \left\lfloor \frac{n+2}{2} \right\rfloor + 1, & \text{when } n \text{ is odd.} \end{cases}$$

Proof. By the definition of triangular snake graph is a connected graph all of whose blocks are triangles. A triangular snake graph is triangular cactus whose block-cut point graph is a path. And it is obtained from a path $P = v_1$, v_2,...,v_{n+1} by joining v_i and v_{i+1} to a new vertex u_1, u_2, u_3,...,u_n. By the definition of central graph $C(T_n)$ is obtained by subdividing the each edge $v_i v_{i+1}$, $1 \le i \le n-1$ of T_n exactly once by adding a new vertex c_i in $C(T_n)$ and joining each vertex v_j with each vertex v_k, $j+2 \le k \le n$.

Let $V_1 = \{v_1, v_2, ..., v_n\}$, $U = \{u_1, u_2, ..., u_n\}$ and $V_2 = \{c_1, c_2, ..., c_n\}$. Then $V[M(T_n)] = V_i \cup C_j \cup U_k \cup V_{i+1}$, for $1 \le i \le n-1$, $1 \le j \le n$, $1 \le k \le n$. By applying the definition of dominator coloring of $C(T_n)$ is as follows. Let $\chi_d[C(T_n)]$ be the dominator chromatic number of the central graph of snake graph.

Case(i): If n is odd

Suppose $n = 3$, the graph is defined as T_3. Let us consists vertices v_1, v_2, v_3, v_4, u_1, u_2, u_3 and the newly added vertices c_1, c_2, c_3, c_4, c_5, c_6, c_7 c_8, c_9. Let $S = \{v_1, v_2, c_5, u_1, c_7, c_8, c_9\}$ be the dominating set of T_3. Now assign the dominator coloring to the graph $C(T_3)$. Let us assign the color 1 to the vertices $\{v_1, v_2, c_5, u_1, c_7, c_8, c_9\}$ of the dominating set S and assign the minimum color classes from $2, 3..., n$ to the remaining non-adjacent vertices of the graph. By the definition of dominator coloring all the vertices in the Set S dominates the color class of $\{v_1, v_2, c_5, u_1, c_7, c_8, c_9\}$ but $\{v_1, v_2, c_5, u_1, c_7, c_8, c_9\}$ dominates all the color classes.

Therefore the dominator chromatic number of $C(T_3)$ is 3. By proceeding this way for order n, we get the successive sequence of dominator chromatic number $\lfloor \frac{n+2}{2} \rfloor + 1$ colors for dominator coloring in $C(T_n)$. Therefore the dominator chromatic number of $C(T_n)$ is $\lfloor \frac{n+2}{2} \rfloor + 1$, when n is odd.

Case(ii): If n is even

Suppose $n = 2$, the graph is defined as T_2. Let us consists vertices v_1, v_2, v_3, u_1, u_2 and the newly added vertices c_1, c_2, c_3, c_4, c_5, c_6. Let $S = \{u_1, v_1, v_2, c_6\}$ be the dominating set of $C(T_2)$. Now assign the dominator coloring to the graph $C(T_2)$. Let us assign the color 1 to the vertices $\{u_1, v_1, v_2, c_6\}$ of the dominating set S and assign the minimum color classes from $2, 3..., n$ to the remaining non-adjacent vertices of the graph. By the definition of dominator coloring all the vertices in the Set S dominates the color class of $\{u_1, v_1, v_2, c_6\}$ but $\{u_1, v_1, v_2, c_6\}$ dominates all the color classes.

Therefore the dominator chromatic number of $C(T_2)$ is 3. By proceeding this way for order n, we get the successive sequence of dominator chromatic number $\left(\frac{n+2}{2}\right) + 1$ colors for dominator coloring in $C(T_n)$. Therefore the dominator chromatic number of $C(T_n)$ is $\left(\frac{n+2}{2}\right) + 1$, when n is even. Hence

$$\chi_d[C(T_n)] = \begin{cases} \left(\frac{n+2}{2}\right) + 1 & \text{when n is even} \\ \lfloor \frac{n+2}{2} \rfloor + 1, & \text{when n is odd.} \end{cases}$$

□

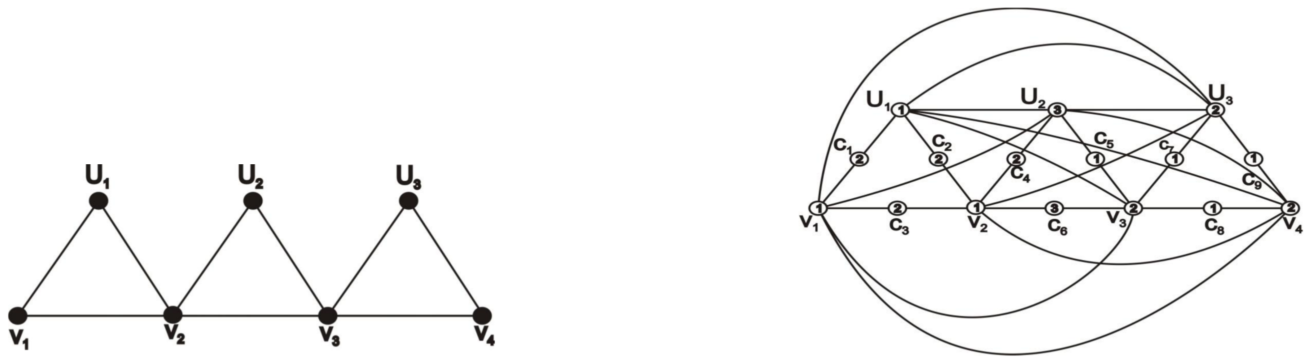
Example 3.2. *In figure, snake graph of T_2 and its central is depicted with a dominator coloring. When $n = 2$ for even*

(a) T_2

(b) $C(T_2)$

The color classes of $C(T_2)$ are, $V_1 = 1 = \{u_1, v_1, v_2, c_6\}$, $V_2 = 2 = \{c_1, c_3, c_2, v_3, u_2\}$, $V_3 = 3 = \{c_4, c_5\}$. Therefore, $\chi_d[T_2] = 3$. In figure, snake graph of T_3 and its central is depicted with a dominator coloring. When $n = 3$ for odd.

The color classes of $C(T_3)$ are, $V_1 = 1 = \{u_1, v_1, v_2, c_5, c_7, c_8, c_9\}$, $V_2 = 2 = \{c_1, c_2, c_3, c_4, v_3, v_4, u_3\}$, $V_3 = 3 = \{u_2, c_6\}$. Therefore $\chi_d[T_3] = 3$.



(a) (T_3) (b) $C(T_3)$

Theorem 3.3. *For the Snake graph T_n of order $n \geq 2$, then $\chi_d[M(T_n)] = 5$.*

Proof. By the definition of triangular snake graph is a connected graph all of whose blocks are triangles. A triangular snake graph is triangular cactus whose block-cut point graph is a path. And it is obtained from a path $P = v_1,\ v_2,...,v_{n+1}$ by joining v_i and v_{i+1} to a new vertex $u_1,\ u_2,\ u_3,...,u_n$. By the definition of middle graph $M(T_n)$ is a graph which is obtained by subdividing each edge of T_n exactly once and join all the newly added middle vertices of adjacent edges of T_n. Let $V_1 = \{v_1, v_2, ..., v_n\}$, $U = \{u_1, u_2, ..., u_n\}$ and $V_2 = \{c_1, c_2, ..., c_n\}$. Then $V[M(T_n)] = V_i \cup C_j \cup U_k \cup V_{i+1}$, for $1 \leq i \leq n-1$, $1 \leq j \leq n$, $1 \leq k \leq n$. By applying the definition of dominator coloring of $M(T_n)$ is as follows.

Let $\chi_d[M(T_n)]$ be the dominator chromatic number of the middle graph of tadpole graph. Suppose $n = 3$, the graph is defined as T_3. Let us consists vertices $v_1,\ v_2,\ v_3,\ v_4,\ u_1,\ u_2,\ u_3$ and the newly added vertices $c_1,\ c_2,\ c_3,\ c_4,\ c_5,\ c_6,\ c_7,\ c_8,\ c_9$. Let $S = \{c_1, c_5, c_9, v_3\}$ be the dominating set of T_3. Now assign the dominator coloring to the graph $M(T_3)$.

Let us assign the color 1 to the vertices $\{c_1, c_5, c_9, v_3\}$ of the dominating set S and assign the minimum color classes from $2, 3..., n$ to the remaining non-adjacent vertices of the graph. By the definition of dominator coloring all the vertices in the set S dominates the color class of $\{c_1, c_5, c_9, v_3\}$ but $\{c_1, c_5, c_9, v_3\}$ dominates all the color classes. Therefore the dominator chromatic number of $M(T_3)$ is 5. By proceeding this way for order n, we get the successive sequence of dominator chromatic number 5 colors for dominator coloring in $M(T_n)$. Therefore the dominator chromatic number of $M(T_n)$ is 5. Hence $\chi_d[M(T_n)] = 5$, where $n \geq 2$. □

Example 3.4. *In figure, snake graph of T_3 and its middle is depicted with the dominator coloring*

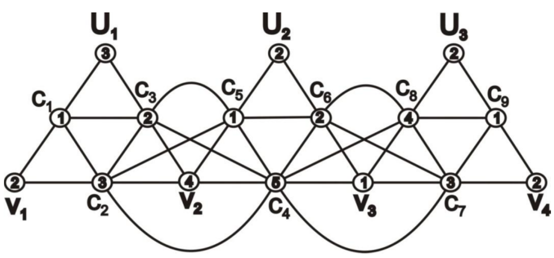


Figure 1: $M(T_3)$

The color classes of $M(T_3)$ are $V_1 = 1 = \{c_1, c_5, c_9, v_3\}$, $V_2 = 2 = \{c_6, c_3, u_3, v_4, v_1, u_2\}$, $V_3 = 3 = \{u_1, c_2, c_7\}$ and $V_4 = 4 = \{v_2, c_8\}$, $V_5 = 5 = \{c_4\}$. Therefore $\chi_d[M(T_3)] = 5$.

4. Conclusion

In this paper, we have tried to obtain the central and middle graph of Tadpole and Snake graph, and evaluated the dominator coloring of that graph and also found the dominator chromatic number of that graph.

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